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A PRELIMINARY EARTHQUAKE LOCATION METHOD BASED ON A HYPERBOLIC APPROXIMATION TO TRAVEL TIMES

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ABSTRACT

We present a fast, two-step method for preliminary earthquake location using the direct arrivals recorded by a local network by assuming that travel times follow a hyperbolic relationship. For a layered medium, hypocentral coordinates (x_e, y_e, h), arrival times (t), origin time (T_o), travel times of either P or S waves (τ), and station coordinates (x, y) are approximately related by an equation describing a hyperbolic surface: $\tau = t - T_o = \sqrt{(x - x_e)^2 + (y - y_e)^2 + h^2}/v$ (1), where v is the rms velocity between the surface and depth h. This equation is a good approximation when the angle between the vertical and the ray path is small. In the first step T_o and the coefficients of the hyperboloid that best fit the observed arrival times are determined by nonlinear inversion. The coordinates of the minimum of the hyperboloid give the event epicenter. In the second step all the unknown parameters are computed simultaneously by nonlinear inversion of equation (1). Testing with synthetic data shows that the method performs better than might have been expected given the approximations involved. The determination of epicentral locations is very robust even in the presence of noise. When the events are under or near the network and only P arrivals are used, the epicenters are mislocated by a few kilometers at most. When S arrivals are included, depths and origin times are also reliably estimated. This method can be applied when approximate, but expeditious, earthquake locations are required. It can be used, for example, as part of an automatic event location program, first to produce a set of P and S arrival times uncontaminated by gross errors, and then to generate initial estimates of hypocentral location and origin time to be used by standard, and more time-consuming, locating programs. Furthermore, in optimal cases, it is possible to obtain reliable and independent estimates of rms velocities.

INTRODUCTION

The approximate location method presented here was motivated by our work on the automatic location of events recorded by a local network (Smalley *et al.*, 1989a, b). As discussed in the "Applications" section, any automated location program should satisfy at least three conditions: (1) it should produce a consistent set of phase arrival times, i.e., uncontaminated by gross errors; (2) if the network will record events with a large range of epicenters (e.g., local and regional events) and depths (e.g., both crustal and Benioff-zone events), the program should discriminate among them; and (3) for real-time data analysis the program has to be fast. Our approach to this problem is based on geometric considerations and can be considered an extension of the plane wave approximation, which is routinely used to find the azimuth of distant events recorded by a seismic network or array. Under the plane wave approximation, contours of constant travel times for the direct waves are straight lines. For an event near or under the network, however, these contours are concentric arcs of circles centered at the epicenter (assuming a laterally homogeneous medium). The arrival times define a surface of revolution centered at the event epicenter whose shape depends on the event depth and the velocity of the

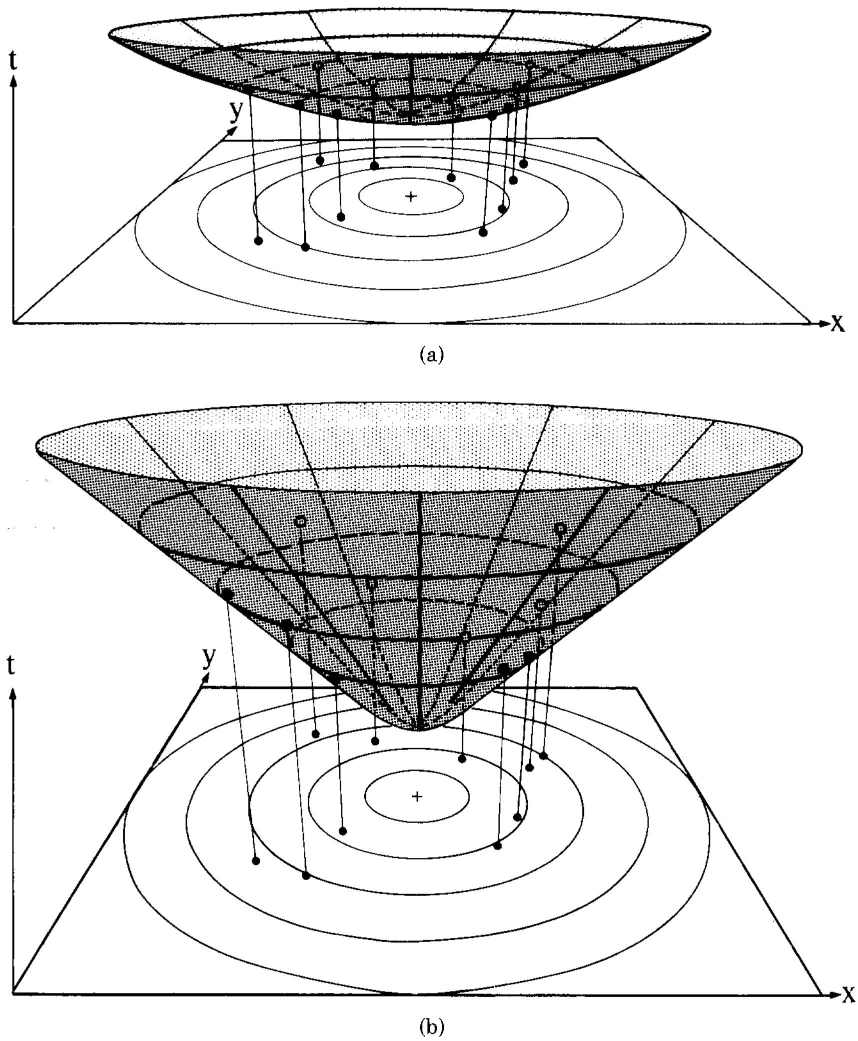

FIG. 1. Illustration of the geometrical motivation for the approximate hyperbolic location technique. The bottom parts of (a) and (b) are inclined map views showing the stations (filled circles) of a network, the event epicenter (cross), and contours of equal arrival time. The top parts of (a) and (b) represent arrival time surfaces for intermediate-depth and crustal events, respectively. The shape of the surfaces is independent of the origin time. For a laterally homogeneous medium, the contours are circular and each surface is centered at the epicenter.

medium (Fig. 1). This suggests that the epicenter could be determined just by finding the coordinates of the minimum of the surface that fits the observed arrival times.

This rather general, and simple, geometric concept was quantified by further assuming a homogeneous medium, in which case the surface defined by the arrival times is a hyperboloid. With this assumption, it is possible to develop a method that is able to determine approximate epicenters of events a few hundred kilometers distant from a local network as well as the hypocenters and origin times of closer events. In addition, when the event epicenter is near the center of the network, the method gives a good estimate of the rms velocity of the medium between the hypocenter and the surface.

Development of the Method

Consider a half-space with velocity v, and a seismic event with hypocenter (x_e, y_e, h), and origin time T_o. The ray path for a direct wave is, therefore, a straight line and its arrival time at a station located at (x, y) and zero elevation is given by

$$t = T_o + \sqrt{(x - x_e)^2 + (y - y_e)^2 + h^2}/v \tag{1a}$$

which can be written as

$$\tau^2 = (t - T_o)^2 = a_1(x^2 + y^2) + a_2 x + a_3 y + a_4 \tag{1b}$$

where τ is travel time and the coefficients a_i absorb the constant terms in equation (1a). These equations show that $\tau(x, y)$ can be represented by a hyperboloid centered at (x_e, y_e).

In a layered medium, the expression for $\tau(x, y)$ is more complex, because ray paths are no longer straight. The effect of layering can be estimated by noting that a similar problem has already been solved in reflection seismology. For a layer over a half-space model with layer thickness H, and velocity v, the travel time for reflected waves is given by $\mathcal{T}(r) = \sqrt{r^2 + (2H)^2}/v$, where r denotes the source-receiver distance along the seismic profile. This expression and the square root term of equation (1a) are equivalent if $h = 2H$. When layers are present, Taner and Koehler (1969) have shown that the hyperbolic relation should be replaced by a power series

$$[\mathcal{T}(r)]^2 = c_1 + c_2 r^2 + c_3 r^4 + c_4 r^6 + \cdots$$

where the coefficients c_i depend on the layer thicknesses and velocities. Coefficient c_2, in particular, is equal to the inverse of the rms velocity squared. In routine work, the series is truncated, keeping only the constant and quadratic terms, and used to compute stacking velocities (Al-Chalabi, 1979). This hyperbolic approximation, however, is valid only for small angles between the vertical and the ray path (Carrion, 1987).

Based on the results from reflection seismology, we approximate the expression for travel times for an earthquake source in a layered medium by equations (1a) and (1b) and use them to determine x_e, y_e, h, T_o, and v, where v is now an approximation to the rms velocity of the medium between the surface and depth h. The range of applicability of the approximation is then determined by analyzing synthetic data.

The location method proceeds in two steps: first we determine the epicentral coordinates via equation (1b), and then T_o, h and v, as well as x_e and y_e, are determined by nonlinear inversion using equation (1a). This stepwise approach was taken because the first step is designed to be a stand-alone unit, to be used as part of an automatic location program (see the "Applications" section), and also to provide essential initial estimates for the second step.

Epicentral Location

To determine the event epicenter, we first find the coefficients a_j of equation (1b) and then use the relations $x_e = -a_2/2a_1$ and $y_e = -a_3/2a_1$, which give the coordinates of the minimum of the hyperboloid. If T_o were known, the problem would be very simple, but since it is not, we approximate T_o by $T_o{}^* = t_{\min}^{\text{obs}} - t_o$, where $t_{\min}^{\text{obs}}$ is the earliest arrival time and t_o is a constant to be determined iteratively. Initially, t_o is set equal to 3 sec (which is the travel time for a rather shallow generic event under a network). Then equation (1b) is replaced by

$$[\tau_i{}^*]^2 = (t_i^{\text{obs}} - T_o{}^*)^2 = b_1(x_i^2 + y_i^2) + b_2 x_i + b_3 y_i + b_4 \tag{2}$$

where i indicates the station. For computational purposes, x_i and y_i are measured from the station with the earliest arrival. The coefficients b_j, however, are no longer simply related to the hypocentral parameters. Let $\vec{T}$ be the vector of the square of the approximate travel times, $\vec{B}$ the vector of unknown coefficients b_j, $\mathbf{C}$ the matrix whose ith row is given by $(x_i^2 + y_i^2, x_i, y_i, 1)$, and $\mathbf{W}$ a (diagonal) matrix of weights based on the quality of the phase picks. In matrix form, including weights, equation (2) is written as

$$\mathbf{WC}\vec{B} = \mathbf{W}\vec{T}. \tag{3}$$

$\vec{B}$ can be obtained from equation (3) by least squares, computing either the solution to the standard system of normal equations or the generalized inverse solution (Dongarra *et al.*, 1979; Lee and Stewart, 1981). We use the second option, because it has better numerical properties, gives critical information on the condition number of matrix $\mathbf{C}$, and requires little additional computational time.

Once $\vec{B}$ has been computed, the pair (x_m, y_m) that minimizes $[\tau^*]^2$ is determined. If the event is not under the network or if it is relatively deep, the initial $T_o{}^*$ estimate may be too late and $[\tau^*(x_m, y_m)]^2$ will be negative. In this case, the previous process is repeated by replacing $T_o{}^*$ by $(T_o{}^* - t_o)$ until $[\tau_{\min}^*]^2$ becomes positive. Note that since $\mathbf{C}$ remains unchanged, there is no need to compute its generalized inverse at each iteration.

The coefficients a_j of the hyperboloid and T_o are determined by standard iterative least-squares nonlinear inversion using equation (1b) written as

$$t_i(T_o, \vec{A}) = T_o + \sqrt{a_1(x_i^2 + y_i^2) + a_2 x_i + a_3 y_i + a_4} \tag{4}$$

where $\vec{A}$ is the vector of coefficients a_j. First-order Taylor expansion of t about the initial estimates $T_o{}^*$ and $\vec{A}^* = \vec{B}$ gives

$$\delta t_i = t_i^{\text{obs}} - t_i(T_o{}^*, \vec{A}^*) = dT_o + \sum_j \frac{\partial t}{\partial a_j} da_j \tag{5}$$

where dT_o and the da_j's are the corrections to be applied to the initial estimates. The partial derivatives are computed from equation (4). In matrix form, including weights, equation (5) becomes

$$\mathbf{WD}\vec{X} = \mathbf{W}\vec{R} \tag{6}$$

where $\mathbf{D}$ is the matrix of derivatives, $\vec{X}$ is the vector of unknown corrections and $\vec{R}$ is the vector of residuals δt_i. $\mathbf{D}$ has a column of ones and its other four columns are obtained from matrix $\mathbf{C}$ [equation (3)] by dividing each of its rows by twice the corresponding computed travel times.

Equation (6) is also solved by computing the generalized inverse solution. The initial estimates are then updated and a new solution vector is computed. If $\vec{X}^{(k)}$ solves equation (6) for the kth iteration, then the new initial estimates are given by $(T_o^{(k)} + dT_o^{(k)})$ and $(a_j^{(k)} + da_j^{(k)})$. At this stage, $\mathbf{W}$ can be modified to downweight arrivals with residuals above a predefined threshold. This process is repeated until some stopping condition is met. Once convergence is achieved, the epicenter is determined by minimizing equation (1b). In addition, estimates of h and v are computed from coefficients a_1 and a_4, to be used in the next step.

This method will not be applicable if the first arrivals include a mixture of direct and head waves, because the corresponding travel-time curves are different. We note, however, that when the station-epicenter distance r is so large that $r \gg h$, equation (1a) can be replaced by $t \approx T_o + r/v$. This is the equation for the travel time of head waves for a layer over a half-space model (with T_o related to the critical angle and the layer thickness and velocity) and corresponds to a truncated cone centered at the event epicenter. Therefore, it can be expected that, when all the arrivals are head waves, the epicenter can be estimated by this method. Testing with synthetic data shows that this is the case.

Hypocentral Location

In this step, we solve for T_o, x_e, y_e, h, and v by iterative nonlinear inversion using equation (1a). Proceeding as before and using T_o^*, x_e^*, y_e^*, h^*, and v^* determined in the previous step as initial values, we can write

$$\begin{aligned}\delta t_i &= t_i^{\text{obs}} - t_i(T_o^*, x_e^*, y_e^*, h^*, v^*)\\ &= dT_o + \frac{\partial t}{\partial h}\,dh + \frac{\partial t}{\partial x_e}\,dx_e + \frac{\partial t}{\partial y_e}\,dy_e + \frac{\partial t}{\partial v}\,dv\end{aligned} \tag{7}$$

where dT_o, dx_e, dy_e, dh, and dv are corrections to be applied to the initial estimates, and the partial derivatives are computed from equation (1a). In matrix form, including weights, equation (7) becomes

$$\mathbf{WG}\vec{X} = \mathbf{W}\vec{R} \tag{8}$$

where $\mathbf{G}$ is the matrix of derivatives, $\vec{X}$ is the vector of unknown corrections, and $\vec{R}$ is the vector of residuals.

Equation (8) is solved by computing the generalized inverse solution. Once $\vec{X}$ has been determined, the initial estimates are updated and a new solution vector is computed. The process is repeated until some stopping condition is met.

Although the method described is straightforward, there are some numerical aspects worth noting. When solving inverse problems, it is important to analyze the condition number of the matrices involved. The concept of condition number, however, is meaningful only when it is computed for a matrix properly scaled, so that all its elements are of similar magnitude (Dongarra *et al.*, 1979). In our case, all the matrices are characterized by very large condition numbers, but at the same time the magnitude of their elements may vary substantially from column to column. The reason for this variation is the different dimension of the quantities involved (e.g., [*distance*]) and [*distance*]^2 in $\mathbf{C}$). Therefore, a possible scaling strategy is to make the problem nondimensional. For the case of matrix $\mathbf{C}$, this is simply done by dividing all the x_i and y_i values by the average distance from the center of coordinates. For the other matrices, each column is scaled by the average of the absolute values of its elements. After the vector solution has been computed, the effect of scaling has to be removed. If the ith column of the matrix is multiplied by s_i, then the ith row of the solution vector should also be multiplied by s_i. This approach produced a dramatic reduction in $\mathscr{K}(\mathbf{C})$, $\mathscr{K}(\mathbf{D})$, and $\mathscr{K}(\mathbf{G})$, where $\mathscr{K}$ indicates condition number of the matrix in parentheses. $\mathscr{K}(\mathbf{C})$ before scaling was about 2×10^4, and after scaling it decreased to less than 20. Since this matrix depends on the station locations only, its condition number is independent of the event epicenters. $\mathscr{K}(\mathbf{D})$, on the other hand, decreased from about 10^6 to between 3×10^2 and 3×10^3. $\mathscr{K}(\mathbf{D})$ becomes large when the column of ones and the column of partial derivatives corresponding to da_4 are nearly linearly dependent, a condition that arises when $1/\tau_i$ (which is the coefficient of da_4) is similar for all the stations. In such a situation, the matrix will be intrinsically ill-conditioned, a problem that cannot be solved by scaling. Note, however, that only after proper scaling is it possible to determine whether a matrix is ill-conditioned or not. The relation between $\mathscr{K}(\mathbf{D})$ and event location is discussed in the next section.

When only P arrivals are used, $\mathbf{G}$ can become ill-conditioned when the takeoff angle θ is similar for all ray paths. In this case, the column of $\mathbf{G}$ corresponding to $\partial t/\partial h$ (equal to $\cos\theta/v$) and the column of ones will be approximately linearly dependent. As is well known from standard earthquake location, this problem is alleviated by the inclusion of S arrivals, but this requires an estimate of the v_P/v_S ratio, because only P-wave velocities are computed. Furthermore, if the events are far enough away, the derivatives of t with respect to h, x_e, and y_e can become nearly linearly related, which makes $\mathscr{K}(\mathbf{G})$ large even when S arrivals are used. For the tests described below with P and S arrivals, $\mathscr{K}(\mathbf{G})$ before scaling was on the order of 10^4 to 10^5 for distant events and decreased to about 500 for events under the network. Using P arrivals only, these numbers were one order of magnitude larger. After scaling, $\mathscr{K}(\mathbf{G})$ ranged between 30 and 1,000 and between 300 and 5,000, respectively. Although this reduction in condition number is substantial, large values indicate that there will be an unavoidable trade-off between origin time and depth and even between depth and epicenter.

We also note that, although the hyperbolic approximation method presented here was developed independently, there are at least two published papers which make use of the same ideas (F. Followill, personal comm.). Von Seebach (1872; see Macelwane, 1936) assumed that the arrival time curve for the crustal P was a hyperbola. He obtained the velocity from the slope of the asymptote; the time at the focus and at the epicenter from the time intercepts of the asymptote and the hyperbola, respectively; and the depth from the product of the difference of these times and the computed velocity. Clearly, the basis for this method is equation (1b), but the computation of times is not correct. Inglada (1928; see Bullen and Bolt, 1985) assumed a homogeneous medium of known velocity to locate events recorded by a local network. His method is based on the subtraction of equation (1b) (with the coefficients written explicitly) for different pairs of stations, which give a linear system of equations in the hypocentral parameters and T_o. If the velocity was not known, the linearity would be lost and the method could not be applied.

The approximate determination of epicenters has also received attention recently. Garza *et al.* (1977, 1979) assumed a constant velocity medium and derived a set of equations involving arrival times and hypocentral coordinates. Anderson (1981) presented a fast method to determine preliminary epicenters based on the order of arrival of a wave front at the stations of a local network. In both methods, epicenters are computed without knowledge of the velocity structure of the medium, but depths and origin times cannot be determined.

TESTS OF THE METHOD

The hyperbolic approximation method was tested with data recorded by networks with different geometries deployed in regions with known lateral velocity variations such as Arkansas (Pujol *et al.*, 1989b), Italy (Aster and Meyer, 1988), and the Andes in San Juan, Argentina (Pujol *et al.*, 1989a). In each case, the values of x_e, y_e, T_o, and h were compared with those obtained by a standard earthquake location program, with very good agreement.

The range of applicability of the method, however, is best analyzed with synthetic data. There are two factors that affect the method. One is its approximate nature and the other is the presence of errors in the data, although the latter is dominant. Therefore, the method was tested with synthetic arrival times with and without errors. Representative examples are described below.

Test 1. Arrival times were computed for the network (20 stations) and epicenters of Figure 2. The hypocentral depths were fixed at 100 km and the origin time was taken as 50 sec. The layered velocity model, to be used in all the tests, had thicknesses of 1.2, 1.7, 3.3, 6.8, and 27 km, and P velocities of 3.7, 4.7, 5.7, 6.1, and 6.6 km/sec, respectively. The velocity of the half-space was 8.2 km/sec and the v_P/v_S ratio 1.73. Error-free data were used. Figure 3a summarizes the results of the epicentral location (only P arrivals are used). For events near the center of the network the epicentral error is on the order of meters and remains under 10 km for events within about 300 km from the center of the network. For more distant events, the epicentral error increases slowly with distance as a result of the breakdown of the hyperbolic approximation. It should be noted that the mislocation of the events is not random, as the computed epicenters are consistently placed away from the network. The rms residual for all events is 0.01 sec or less. The behavior of $\mathscr{K}(\mathbf{D})$ as a function of distance is interesting. For events under the network, it is rather large, between 500 and 1,300. It decreases to about 250 for events on the edge of the network and then increases to about 3,000 for the farthest events. Large condition numbers indicate that T_o and coefficient a_4 are linearly related, but since x_e and y_e do not depend on a_4, this dependence does not affect the computation of the epicenter.

The results for the determination of hypocenters depends on the type of arrivals used. Using only P arrivals, epicenters are located almost perfectly but the computed depths are consistently shallower (Fig. 3b), although for events under the network the difference with the true depths is rather small. This difference increases with distance, to a maximum 30 km at 700 km distance. All the origin times are late by 2 to 4 sec, and the velocities are between 7.7 and 8.1 km/sec, somewhat higher than the rms velocity of the model, equal to 7.4 km/sec. The rms residual for all events is less than 0.01 sec. The inclusion of S arrivals improves the solution substantially (Fig. 3c), with the depths much closer to the true values over a larger range of distances, although the epicenters are recovered with slightly larger error than before. For all events, the origin time is within 0.05 sec of the true value and the rms residual is 0.03 sec or less. Furthermore, for events under the network, the computed P-wave velocities differ from the rms velocity of the model by 2 per cent or less.

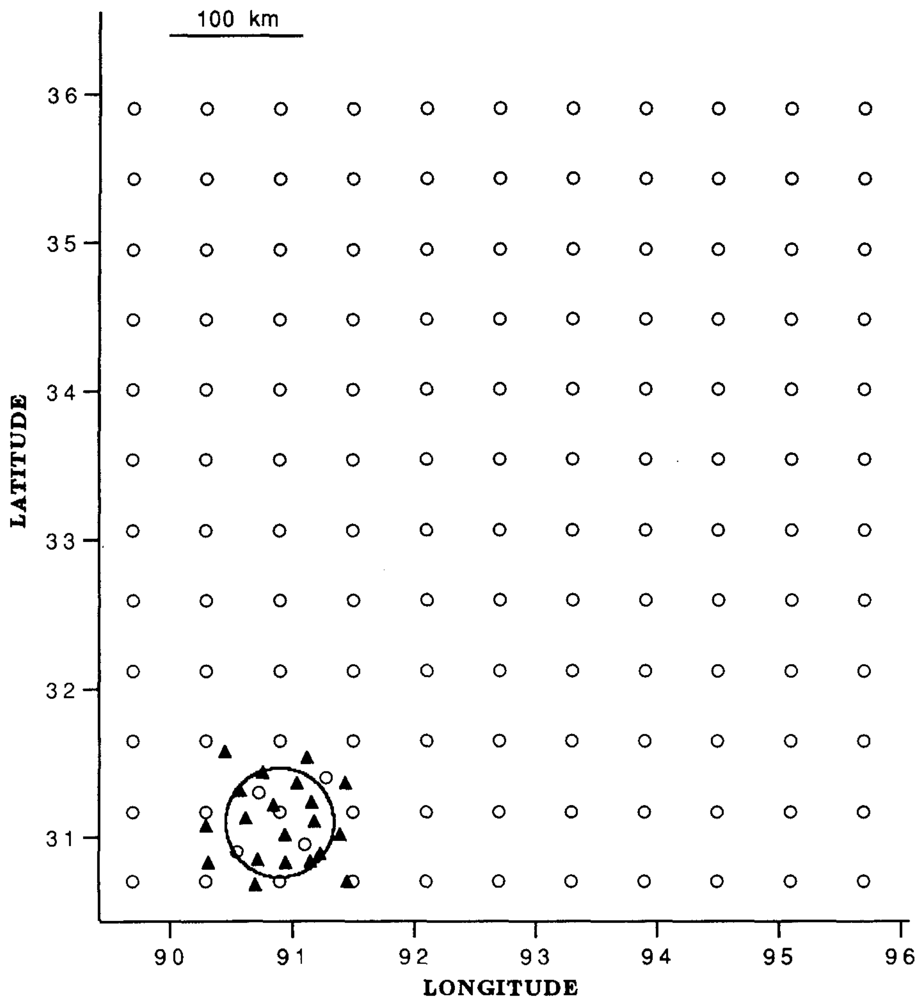

FIG. 2. Distribution of stations (solid triangles) and epicenters (open circles) for Tests 1, 2, and 3. For Test 4 only the stations inside the circle were used.

This test shows that the hypocentral determination is affected by a complicated relationship between model errors and the presence of linear dependencies in the data. The exclusive use of P arrivals results in extremely good epicenters, but with a clear trade-off between origin times, velocities, and depths. When S data are included the overall solution is more reliable over a larger range of distances, but at the expense of some error in the epicenters. These questions are not discussed in detail, however, because the presence of errors in the data is the dominant factor in the quality of the solution, as shown in the next test.

Test 2. Pseudo-random errors between −0.1 and 0.1 sec were added to the arrival times computed for the previous test. All the arrivals were assigned the same weights. Each synthetic event has a different error realization to assure that the results of the test have some statistical validity. For this test, results become unreliable for events beyond 400 to 500 km from the center of the network and depend on the phase arrivals available. When only P arrivals are used, the epicentral

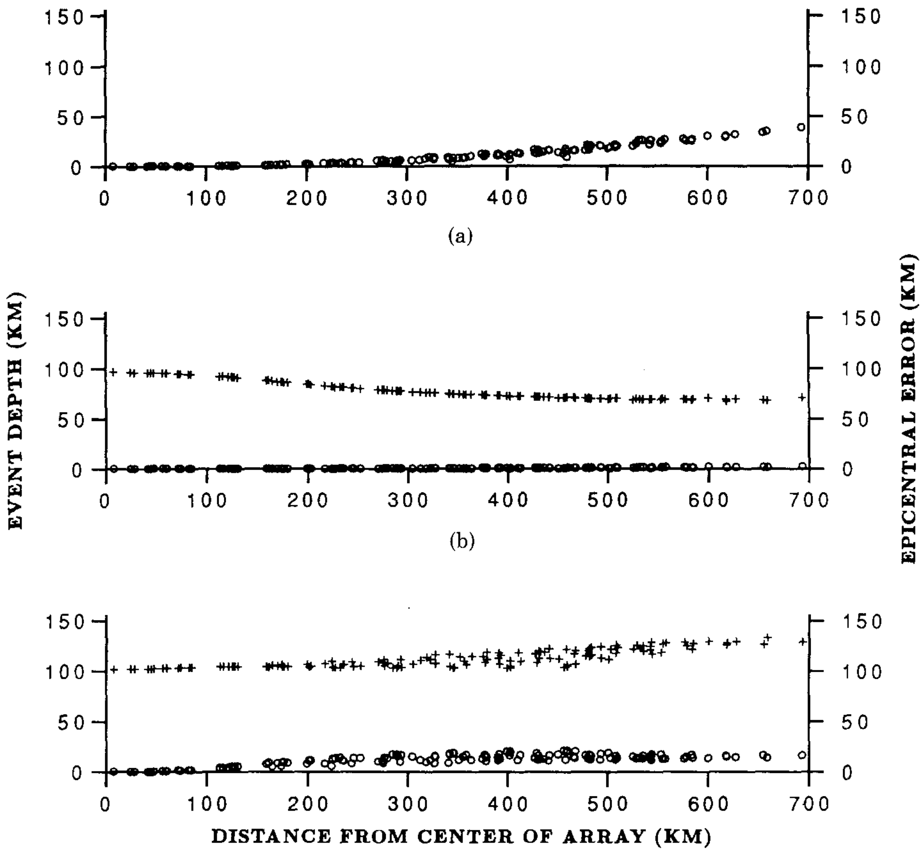

FIG. 3. Summary of results of Test 1. Depth of the events 100 km. Error-free data. Circles denote epicentral errors and crosses denote computed event depths. (a) Errors in the epicenters computed by the epicentral location routine. (b) Depths and errors in the epicenters computed by the hypocentral location routing using P arrivals only. (c) Depths and errors in the epicenters computed by the hypocentral location routine using both P and S arrivals.

and hypocentral location routines produce basically the same epicenters (Fig. 4a). Depths are within 30 km for events as far as 200 km, but they are not reliable at larger distances. To improve the depth determination, S arrivals are required (Fig. 4b), but even then some of the depths are very shallow. These erroneous depths, however, are easily detected because the computed velocities (on the order of 8 km/sec) are clearly too high for those depths. This disagreement between depth and velocity has been observed in numerous other tests and is a diagnostic of unreasonable shallow depth.

Test 3. The only difference between this test and the first one is the depth of the events, which was fixed at 25 km. In this case, the error in the epicenters determined by the epicentral location routine (Fig. 5) is almost the same as in Test 1 except for events between 100 and 200 km from the center of the network, in which case the errors are very large. These errors arise because the first arrivals are composed of a roughly 50–50 mixture of direct and head waves. When the two types of arrivals are mixed, the epicenters (as well as the hypocenters) are

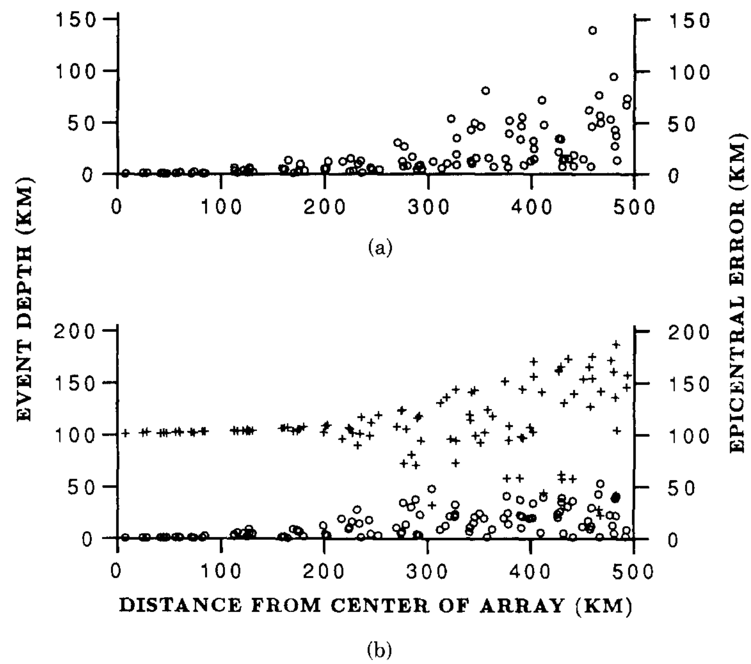

FIG. 4. Summary of results of Test 2. Depth of the events is 100 km. Errors added to the arrival times. Symbols as in Figure 3. (a) Errors in the epicenters computed by the epicentral location routine. (b) Depths and errors in the epicenters computed by the hypocentral location routine using both P and S arrivals.

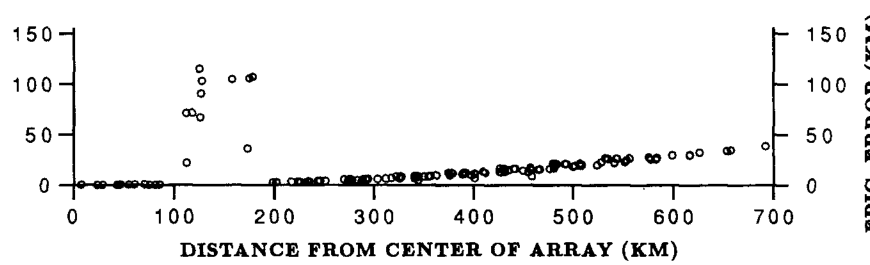

FIG. 5. Results of Test 3. Errors in the epicenters computed by the epicentral location routine. Depth of the events 25 km. Error-free data. Symbols as in Figure 3. The large epicentral errors between 100 and 200 km are caused by the mixing of direct and head wave arrivals.

determined incorrectly, as expected. The rms residuals are extremely large (several seconds, however, so these anomalous locations are easily detected. When all the first arrivals are head waves from the same interface (in this case the Moho), the epicenters are determined rather reliably.

Test 4. In this test, we use 12 stations (Fig. 2) and 200 events 25 km deep distributed on a regular grid. The farthest event is 120 km away from the center of the network. For the distant events, up to three head wave arrivals are included. Errors similar to those in Test 2 have also been added. The epicenters determined with the epicentral location routine (Fig. 6a) for events under or near the network are very close to the true ones. For the other events, errors can reach 13 km. The results of the hypocentral location when both P and S arrivals are included depend on the distance of the events from the center of the network. For events under the

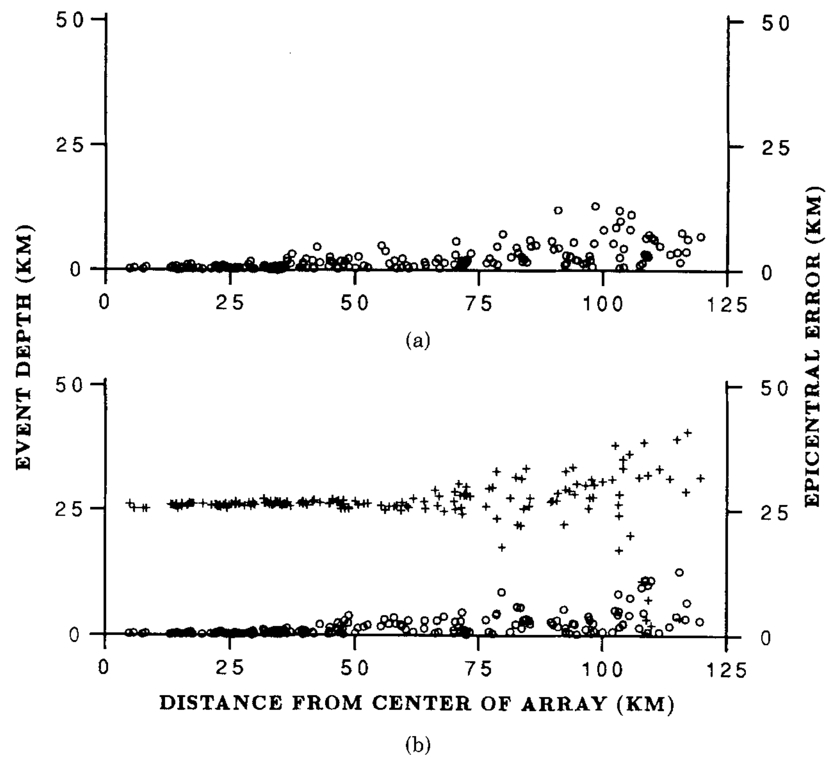

FIG. 6. Summary of results of Test 4. Depth of the events 25 km. Errors added to the arrival times. Symbols as in Figure 3. (a) Errors in the epicenters computed by the epicentral location routine. (b) Depths and errors in the epicenters computed by the hypocentral location routine using both P and S arrivals.

network, the hypocenters are recovered almost perfectly (Fig. 6b) with depth errors of less than 2 km. The difference in computed velocity and rms velocity ranges between 1 and 4 per cent for events near the center and the edge of the network, respectively. Origin times are witin 0.1 sec. For more distant events, the errors are somewhat higher. Depths are affected the most (Fig. 6b), but, as noted before, the few events with very shallow depths can be discarded because the velocities do not correspond to the depths. The errors in origin times and velocities for these events can reach 0.2 sec and 10 per cent, respectively.

APPLICATIONS

The described tests, as well as many others we performed, show that the hyperbolic approximation method is capable of giving rather good estimates of the epicenters of events up to a few hundred kilometers away from a network, when only P arrival times are available. If S arrival times are included, then the depth and origin time of the events can also be estimated. In fact, for events under or near the network the hypocentral parameters and origin time can be computed with rather small errors even in the presence of realistic noise, without knowledge of the velocity of the medium, and with very little computational effort. These features make the method well suited for the generation of preliminary locations for use by standard location programs such as HYPOINVERSE (Klein, 1978), which require an initial independent estimate of the depth. Having reliable initial locations is particularly important when a network records events from a wide range of depths and epicentral distances. Such a situation is encountered, for example, in the Andes in Argentina (Chiu *et al.*, 1987).

The availability of preliminary locations can also extend the range of applicability of the standard programs. Figure 7a summarizes the results of the location of the events of Test 2 obtained using HYPOINVERSE with an initial depth of 80 km. Most events beyond 380 km from the center of the network cannot be located. However, when the approximate locations and origin times are used as initial solutions, all the events are correctly located (Fig. 7b).

Another important application of the hyperbolic approximation method is in an automatic location program for use with a telemetered network (Smalley *et al.*, 1989a, b). The first step in the program involves the determination of the arrival times using an automatic phase picker (Allen, 1978). For high signal-to-noise ratio data, this process usually generates a consistent set of arrivals. However, for data of lower quality or data affected by radio noise (caused by lightning, for example) not all the arrivals are reliable. Under these conditions, a common problem is the identification of noise or the *S* arrival as the *P* arrival. One way to detect the erroneous picks is to locate the events with a program like HYPOINVERSE and to remove those picks with very large residuals. Such an approach has various disadvantages, because HYPOINVERSE requires an initial depth, which is not always known, and does not perform well for events outside the network, particularly when only *P* arrivals are available. Furthermore, since the program is very robust, it is

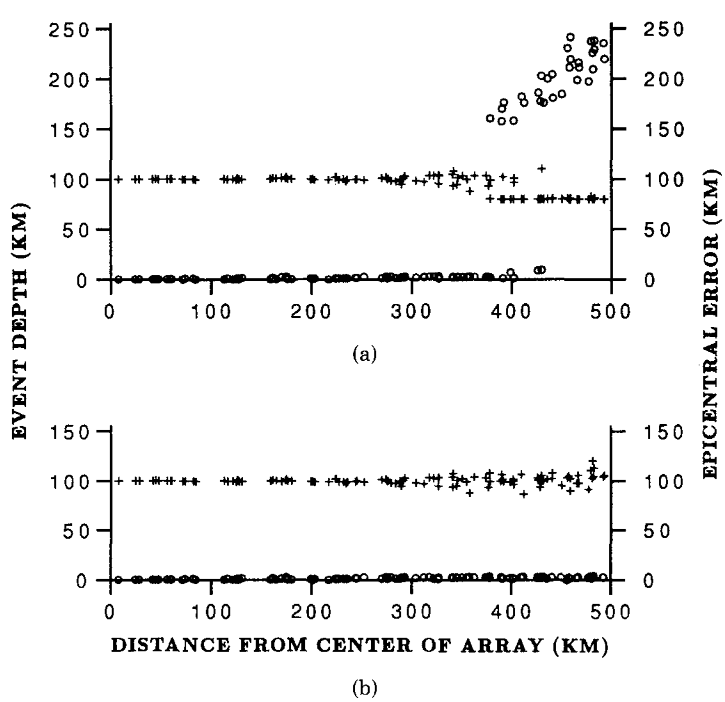

FIG. 7. Depths and errors in the epicenters for the events of Test 2 located with HYPOINVERSE. Both *P* and *S* arrivals were used. Symbols as in Figure 3. (a) An initial depth of 80 km was given. For events beyond about 380 km, depths remain unchanged, an indication that the program failed to converge. (b) The approximate locations and origin times were used as initial solutions.

rather slow, particularly because the program has to be run several times until consistent sets of P and S arrivals have been generated. Speed is a major consideration when locating large data sets or in the real-time processing of data. In the automatic location program, we apply equation (3) iteratively to select sets of P and S arrivals free from outliers. For example, errors of 30, −25, 10, −9, −7, and 5 sec added to six of the arrival times computed for Tests 1 and 3 were detected in a few iterations using P arrivals only. Once the set of arrival times has been culled, a preliminary hypocenter can be determined by the hyperbolic approximation method and used as input to the standard location program.

Finally, the hyperbolic approximation method can give reliable estimates of the rms velocity of the medium provided that the located events are near the center of the network and the P and S arrival times are available. This feature is important on two accounts. First, the computed velocities can be used to check velocity models, which are never perfectly known. Second, if the events occur in a region with a poorly known velocity structure and the hypocenters are distributed over a range of depths, it will be possible to generate a velocity model for the region, as the relation between rms velocities and interval velocities is known (Dix, 1955).

Conclusions

We have introduced an approximate location method based on a hyperbolic approximation to travel times which is fast and does not require *a priori* knowledge of velocities. As with any other location method, its performance depends on a number of factors: depth and distance of the events from the center of the network, aperture of the network and distribution of stations, magnitude of the errors, and whether only P or P and S arrivals are available. Tests with synthetic data affected by realistic errors show that epicenters are well located for a wide range of depths and distances from the network. This step was found to be rather independent of the velocity of the medium, as it is based on a geometric property of the travel times. Provided that P and S arrivals are available, the error in the determination of hypocentral depth and origin time depends on the departure of actual ray paths from the small-angle approximation. This, in turn, is a function of the event depth and network aperture. In general, the shallower the event, the smaller the network required for successful application of the method. Our results indicate that for reasonable networks events under or in the vicinity of a network will be located better than might be expected, considering the approximations involved. Furthermore, events near the center of the network can be used to obtain reliable and independent estimates of the rms velocity.

Clearly, this method is not intended to replace standard hypocenter location methods but can complement them in cases where a good initial estimate of the location is necessary. In this context, the most important contribution will probably be in real-time data processing and automatic location of events with a large range of epicenters and depths; characteristics that can be detected by the approximate method, thus reducing the need of manual processing.

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