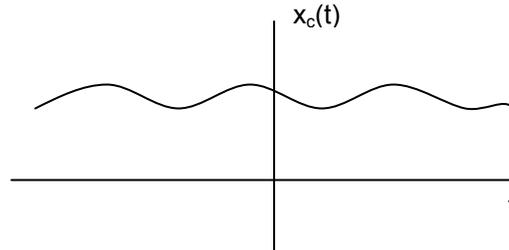
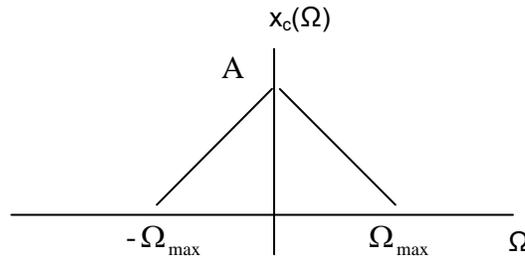


Sampling Example in Fourier Space

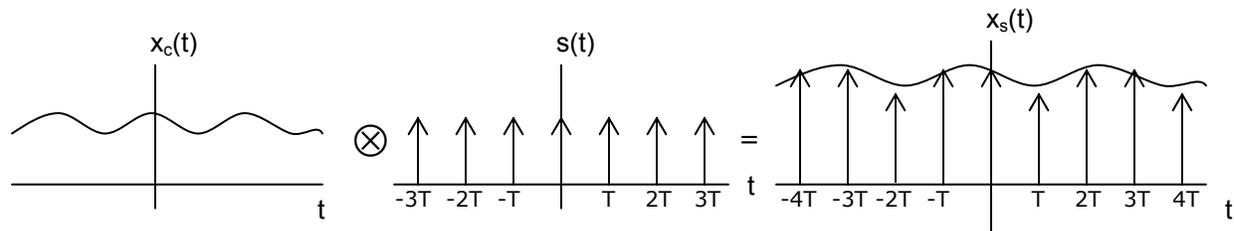
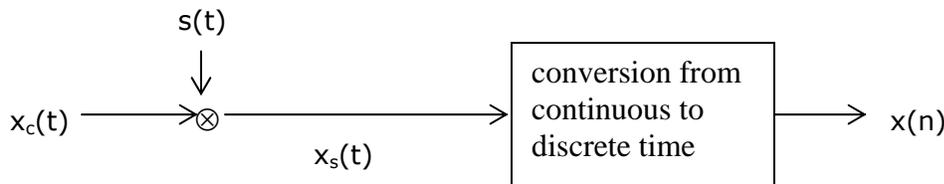
Suppose we have a signal in continuous time.



It has the following frequency response:



Suppose we want to sample the continuous time signal into discrete time using a frequency $f = \frac{1}{T}$. The following block diagram illustrates what we would like to do.



$$x_c(t) = \text{continuous time signal} \quad s(t) = \sum_{-\infty}^{\infty} \delta(t - nT) \quad x_s(t) = \text{sampld continuous signal}$$

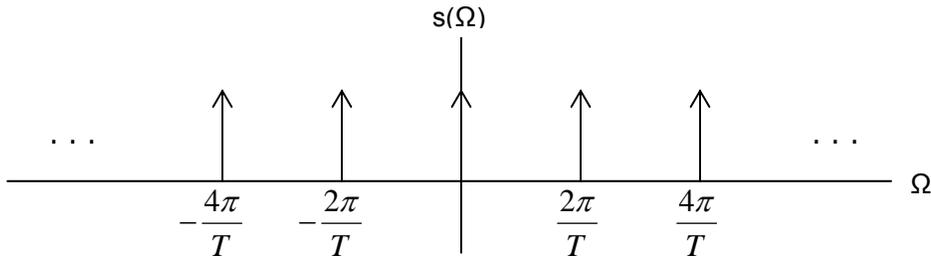
"impulse train"

Note: When you multiply a continuous time signal with an "impulse train," the result is another "impulse train" that has the amplitude of the continuous time signal at every $n \cdot T$,

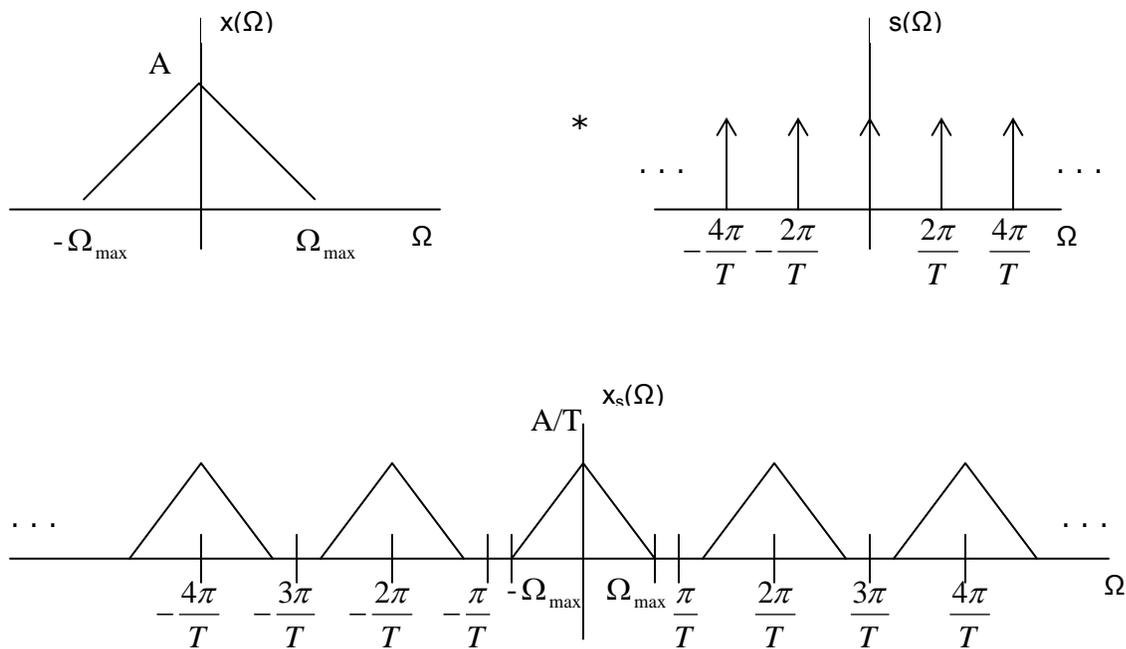
where n is some integer from $(-\infty, \infty)$ and T is the sampling period or the spacing between delta functions in the impulse train.

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \xrightarrow{\mathcal{F}} s(\Omega) = \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\Omega - k \frac{2\pi}{T})$$

The Fourier Transform of an impulse train in time is also an impulse train in frequency.

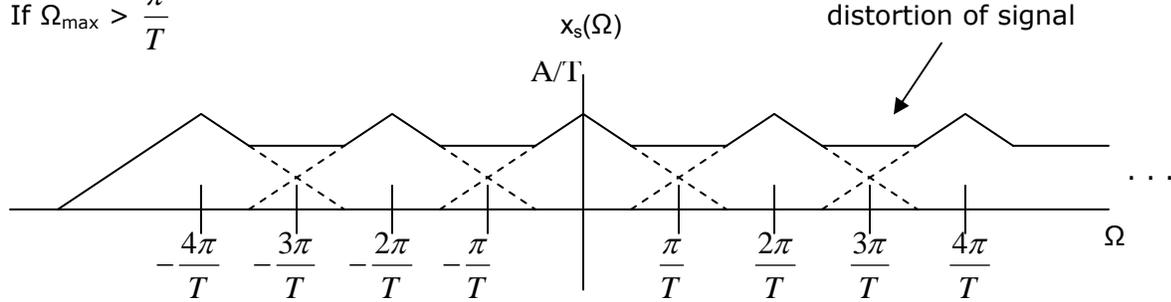


A multiplication of $s(t) \otimes x(t)$ is a convolution in frequency $s(\Omega) \circledast x(\Omega)$. Therefore,



Note: $\Omega_{\max} < \frac{\pi}{T}$, or else the signal will "alias" or superimpose at the edges (which we want to avoid).

If $\Omega_{\max} > \frac{\pi}{T}$

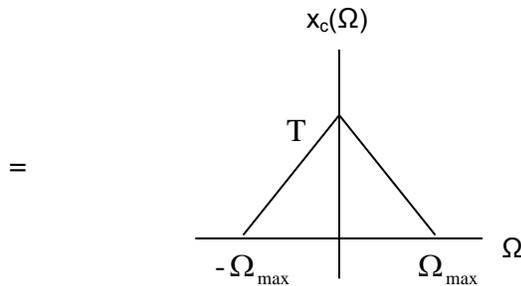
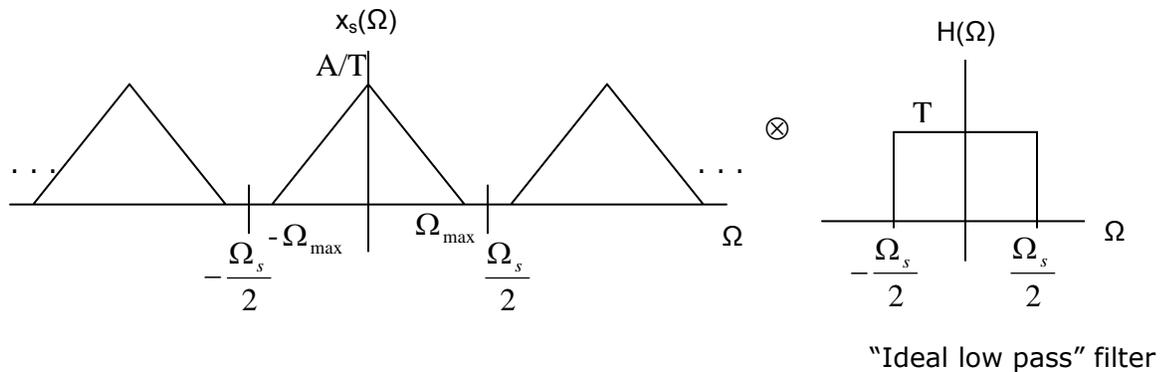


Previously, we defined sampling frequency as $f = \frac{1}{T}$. Now we define sampling rate as

$\Omega_s = 2\pi f = \frac{2\pi}{T}$. Therefore, $|\Omega_{\max}| < \frac{\Omega_s}{2}$ for aliasing to be prevented. $\frac{\Omega_s}{2}$ is then known as

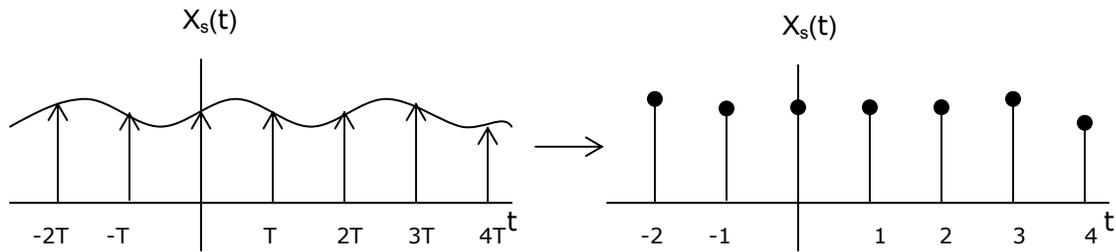
the Nyquist frequency. Any signal whose $|\Omega_{\max}| < \frac{\Omega_s}{2}$ is known as a "band-limited" signal.

The original signal from the first page can then be recovered by an ideal low pass filter.



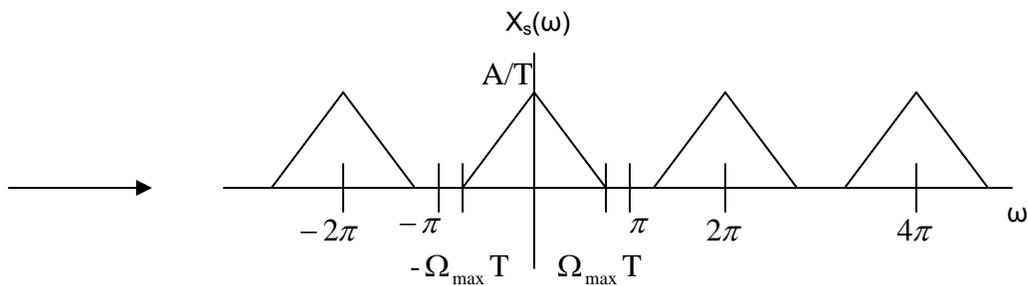
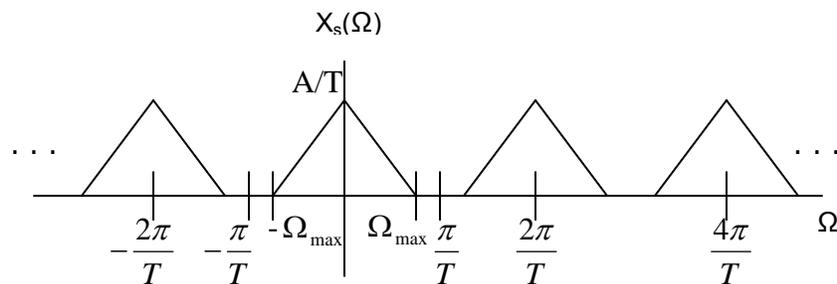
For those of you who are curious about the difference between "continuous time" and "discrete time," feel free to read on.

Now we want to go from continuous time to discrete time. This just means turning a dimensionalized time into a non-dimensionalized unit of time.



In time we multiply by $\frac{1}{T}$ to change dimensionalized time to "non-dimensionalized" time.

In frequency we multiply by $\frac{1}{f}$, or T , to reflect this change.



This is an important point, because if you leave out the sampling frequency parameter in `pwelch` you will notice that your frequency is given in radians. This is because the frequency is given in "discrete time" if you leave out the sampling frequency parameter.