

Earth Science Applications of Space Based Geodesy

DES-7355

Tu-Th 9:40-11:05

Seminar Room in 3892 Central Ave. (Long building)

Bob Smalley

Office: 3892 Central Ave, Room 103

678-4929

Office Hours – Wed 14:00-16:00 or if I'm in my office.

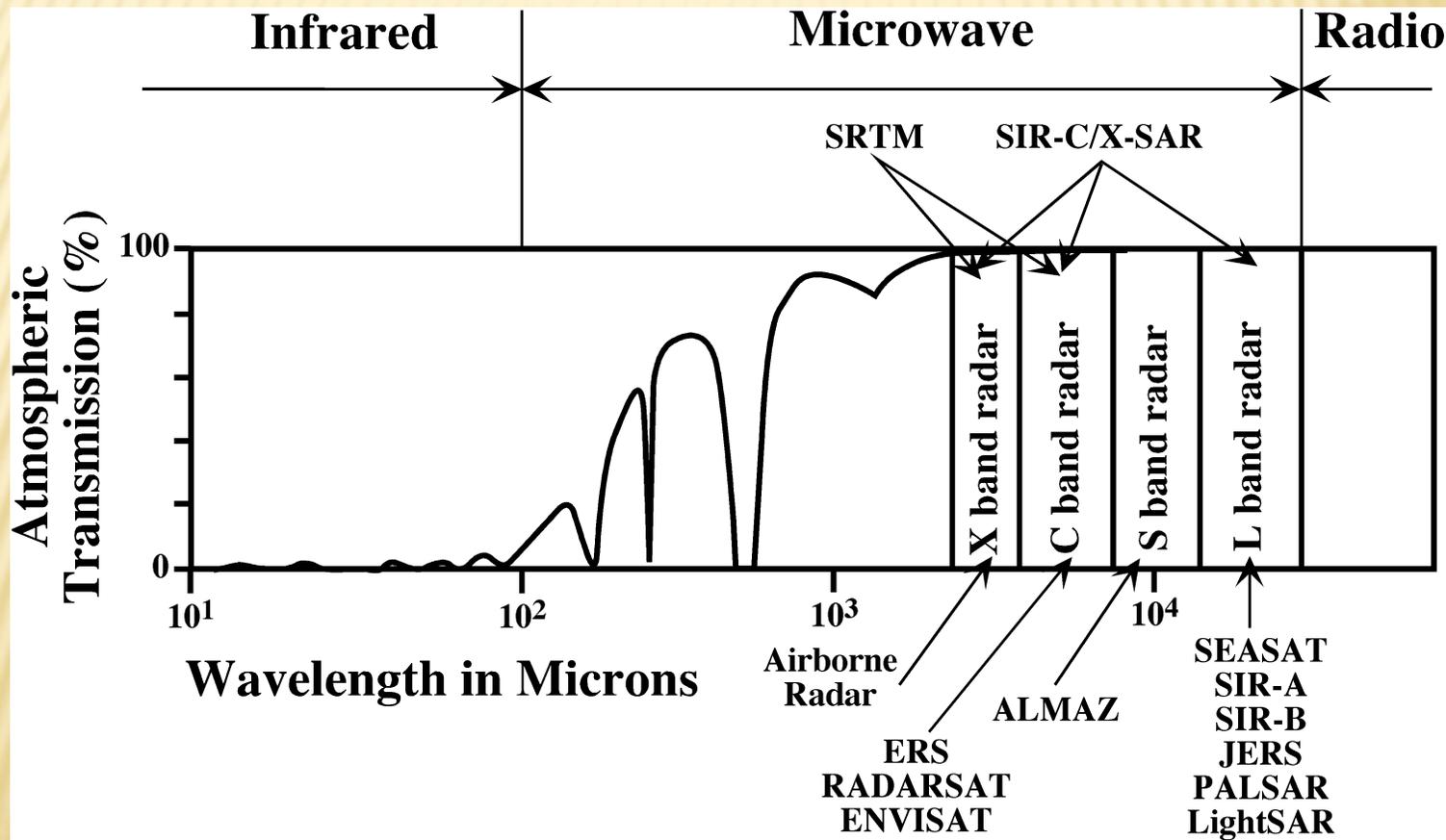
http://www.ceri.memphis.edu/people/smalley/ESCI7355/ESCI_7355_Applications_of_Space_Based_Geodesy.html

Class 23

IMAGING RADAR - GENERAL

- ✘ RADAR = RAdio Detection and Ranging
- ✘ Uses the microwave region of the electromagnetic spectrum.
- ✘ Wavelengths used in imaging radar range between 1 mm and 1 m
- ✘ Longer wavelengths are used for communication and navigation.

MICROWAVE REGION

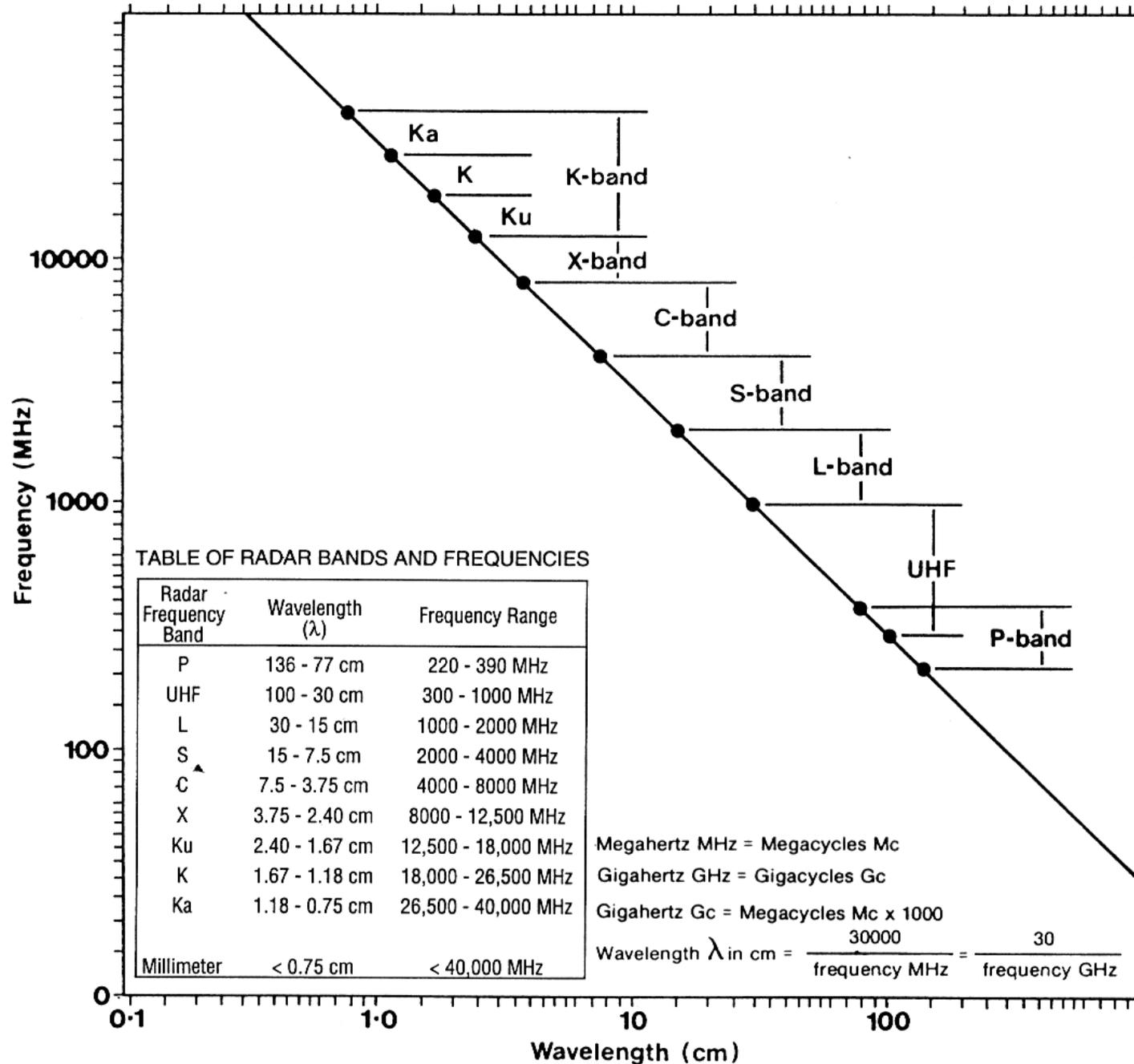


RADAR BANDS

Wavelength Range and Descriptions

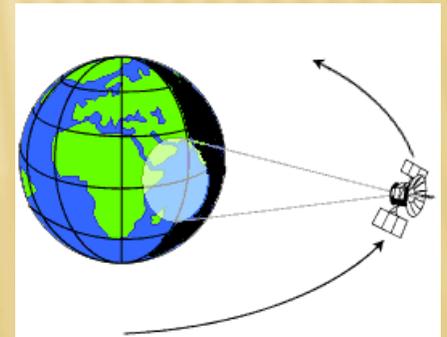
- ✘ Ka, K, and Ku Bands
 - + very short wavelengths used in early airborne radar systems but uncommon today
- ✘ X-band
 - + used extensively on airborne systems for military reconnaissance and terrain mapping.
- ✘ C-band
 - + on many airborne research systems (CCRS Convair-580 and NASA AirSAR) and spaceborne systems (including ERS-1 and 2 and RADARSAT).
- ✘ S-band
 - + used on board the Russian ALMAZ satellite.
- ✘ L-band
 - + used onboard American SEASAT and Japanese JERS-1 satellites and NASA airborne system.
- ✘ P-band
 - + longest radar wavelengths, used on NASA experimental airborne research system

RADAR BANDS



IMAGING RADAR - ADVANTAGES

- ✘ Active system (works day or night).
 - + There is also passive microwave imaging (radiometer) mode. This senses surface radio-emission, which can be converted to radiant temperatures.
- ✘ Not affected by cloud cover or haze if $\lambda > 2$ cm. It operates independent of weather conditions. Water clouds have a significant effect on radar with wavelength $\lambda < 2$ cm.
- ✘ Unaffected by rain $\lambda > 4$ cm.
- ✘ Can penetrate well-sorted dry sand in hyper-arid regions to a depth of about 2 m.



TERMINOLOGY

- ✘ RAR: Real Aperture Radar
- ✘ SAR: Synthetic Aperture Radar
- ✘ SLAR: Side-looking airborne radar (could be RAR or SAR).
- ✘ SIR: Shuttle imaging radar (a SAR)
 - + 3 missions: SIR-A (1981), SIR-B (1984) and SIR-C (1994)
- ✘ INSAR: Interferometric SAR. Can be satellite or airborne.
- ✘ SRTM: Shuttle Radar Topography Mission (an INSAR mapping mission)

Synthetic Aperture Radar – Systems and Signal Processing

APERTURE

Optics : Diameter of the lens or mirror. The larger the aperture, the more light a telescope collects. Greater detail and image clarity will be apparent as aperture increases.

2.4m Hubble Space Telescope

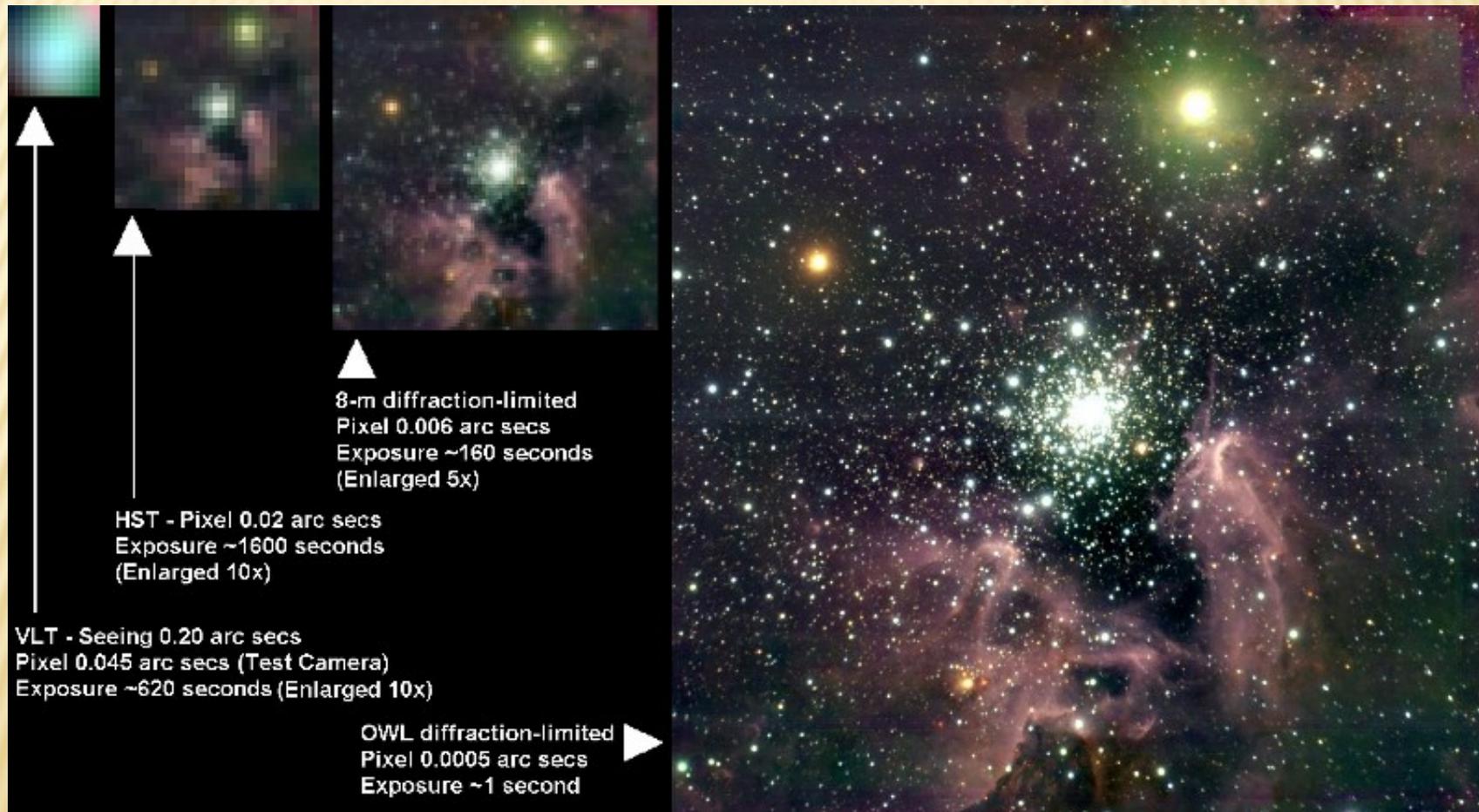
10m Keck, Hawaii

16.4m VLT (Very Large Telescope), Chile

50m Euro50

100m OWL (Overwhelmingly Large T.)

OVERWHELMINGLY LARGE TELESCOPE



What is InSAR?

- Method using an orbiting satellite that emits and receives radio energy in the form of waves
- Carries two types of information.
 - Information about surface from which radio waves reflected carried in strength and intensity of signal
 - Changes in roundtrip distance seen through phase changes of the radio waves.

Synthetic Aperture Radar – Systems and Signal Processing

REAL APERTURE VS. SYNTHETIC APERTURE

- Real Aperture :
resolution $\sim R\lambda/L$
- Synthetic Aperture:
resolution $\sim L/2$

Irrespective of R
Smaller, better?!
- Carl Wiley (1951)

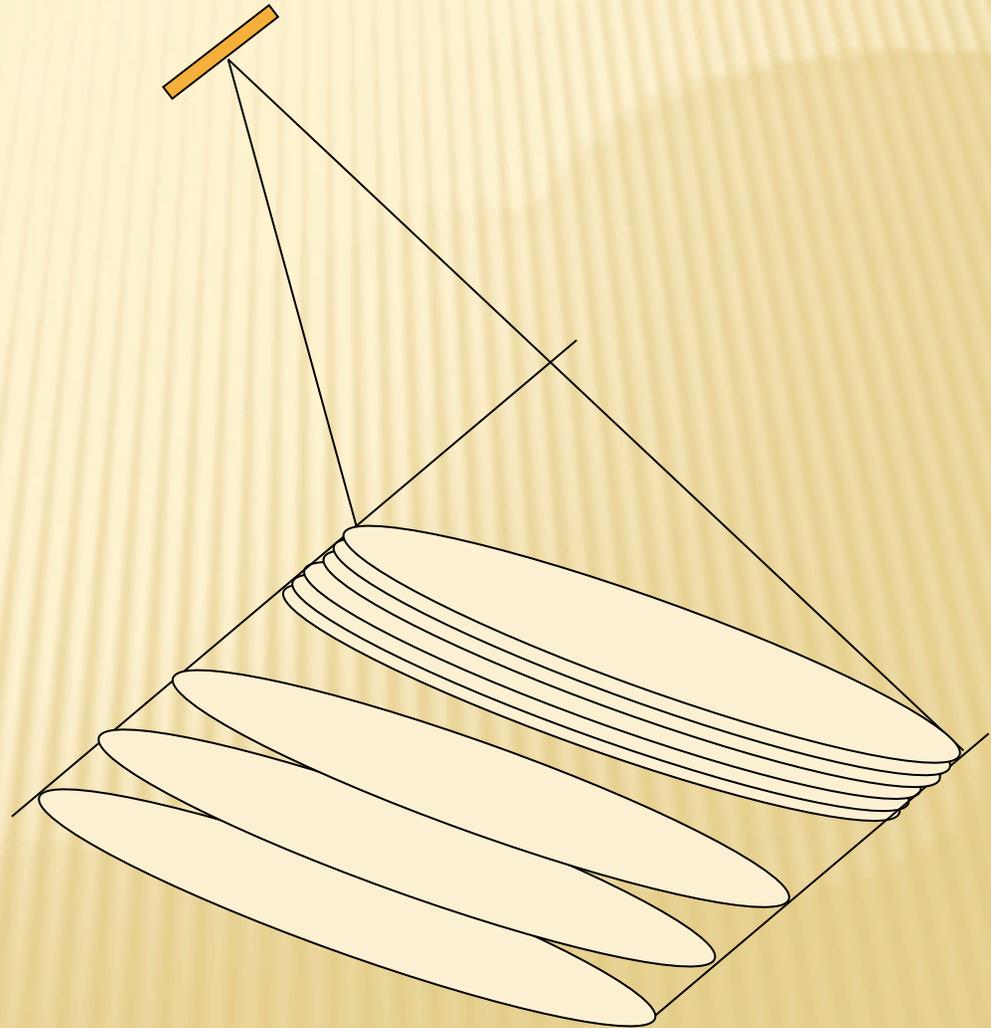


IMAGE ACQUISITION

ERS-1/2 SAR

L: 10 m, D: 1 m

Altitude: 785 km, sun-synchronous orbit

Ground Velocity: 6.6 km/s

Look Angle: Right 17° - 23° (20.355° mid-swath)

Slant Range: 845 km (mid-swath) Frequency: C- Band (5.3GHz,
5.6 cm)

Footprint : 100 km x 5 km

Incidence Angle: 19° - 26° (23° mid-swath)

Sampling Rate: 18.96 MHz

Pulse duration: $37.1 \mu\text{s}$

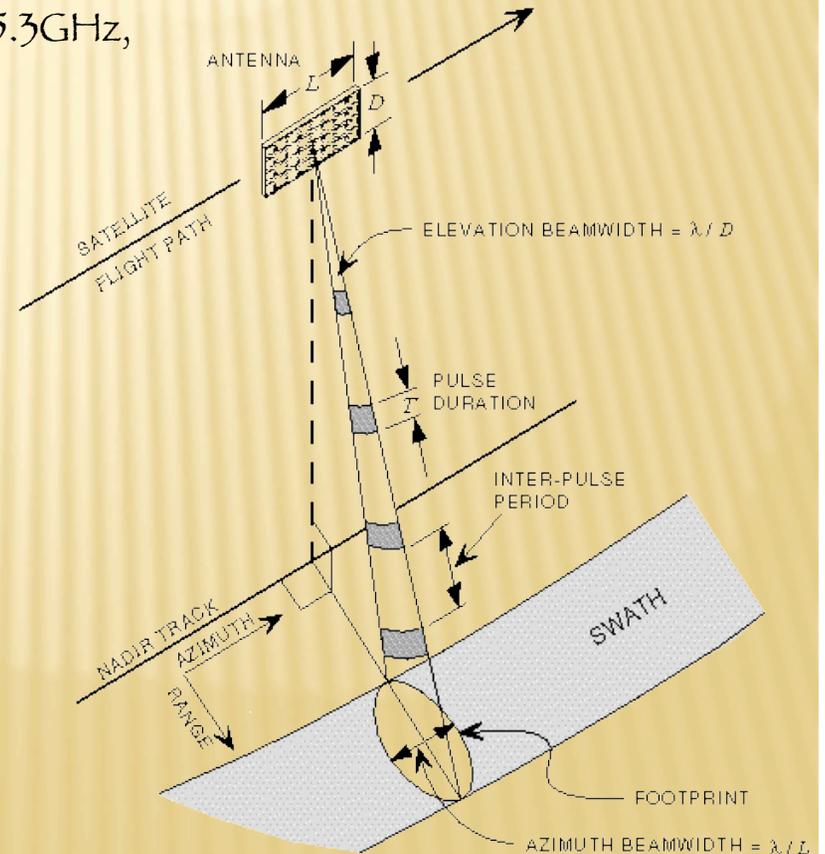
Range gate: $\sim 6000 \mu\text{s}$

Sampling Duration: $\sim 300 \mu\text{s}$ (5616 samples)

Inter-pulse period: $\sim 600 \mu\text{s}$ (upto 10 pulses)

Pulse Repetition Frequency: 1700 Hz

Data Rate: 105 Mb/s (5 bits/sample)



SAR SYSTEMS

Spaceborne SAR

SEASAT-A (USA, 1978), SIR-A (USA, 1981), SIR-B (USA, 1984), SIR-C/X-SAR (USA, Germany, Italy, 1994), ALMAZ-1 (Russia, 1991-1993), ERS-1 (EU, 1991-2000), ERS-2 (EU, 1995-), JERS-1 (Japan, 1992-1998), Radarsat-1 (Canada, 1995-), SRTM (USA/Germany, 2000), ENVISAT (EU, 2002),

RADARSAT-2 (Canada, 2005), PALSAR (Japan, 2004), LightSAR (US)*, TerraSAR (Germany)*, MicroSAR (EU)*

Airborne SAR

TOPSAR (JPL, USA), IFSARE (ERIM/Intermap, USA), DO-SAR (Dornier, Germany), E-SAR (DLR, Germany), AeS-1 (Aerosensing, Germany), AER-II (FGAN, Germany), C/X-SAR (CCRS, Canada), EMISAR (Denmark), Ramses (ONERA, France), ESR (DERA, UK)

Planetary SAR

Magellan (US, 1990-1994), Titan Radar Mapper (US, 2004), Arecibo Antenna, Goldstone antenna

* Under development

SAR SYSTEM MODES

- × Target – the Earth or planets
- × Vehicle – stationary, airborne, satellite, or spaceship
- × Mode – monostatic and/or bistatic
- × Carrier frequency – X, C, S, L, and/or P bands
- × Polarisation – HH, VV, VH, HV (single-pol, dual-pol, full-pol)
- × Imaging geometry – strip, scan, spot

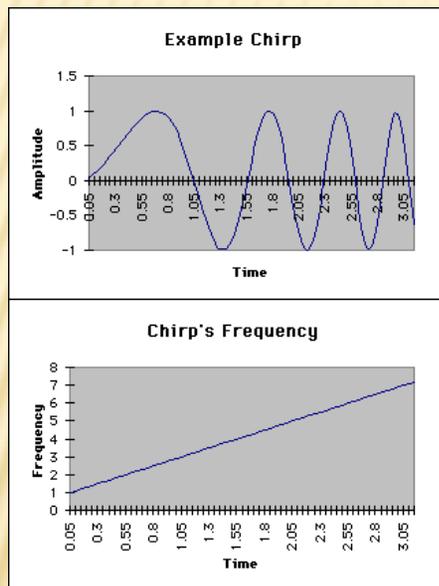
- × <examples>
- × SIR-C/X-SAR: space shuttle, mono, L/C/X, full-pol.
- × ERS-1/2, Envisat: Earth satellite, mono, C, VV.
- × SRTM: space shuttle, mono/bistatic, C/X, HH/VV.
- × Arecibo Antenna: planetary, stationary, mono/bi, multi-bands, multi-pol.
- × Magellan, Cassini SAR: Venus and Titan, mono, S, HH.
- × AIRSAR/TOPSAR: airborne, mono/bi, L/C/P, full-pol
- ×

RANGE COMPRESSION

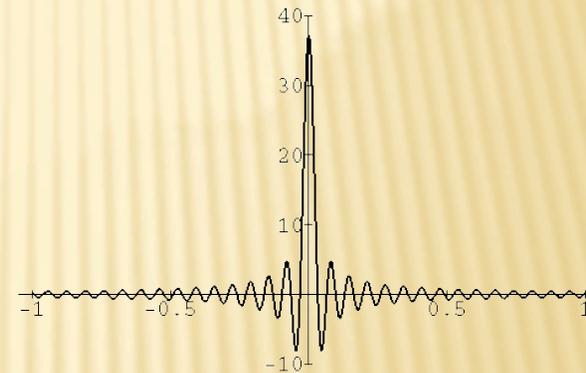
Linear Chirp Signal

$$f(t) = \exp[2\pi i(f_0 + 0.5at)t], \quad 0 \leq t \leq T$$

Chirp autocorrelation Function



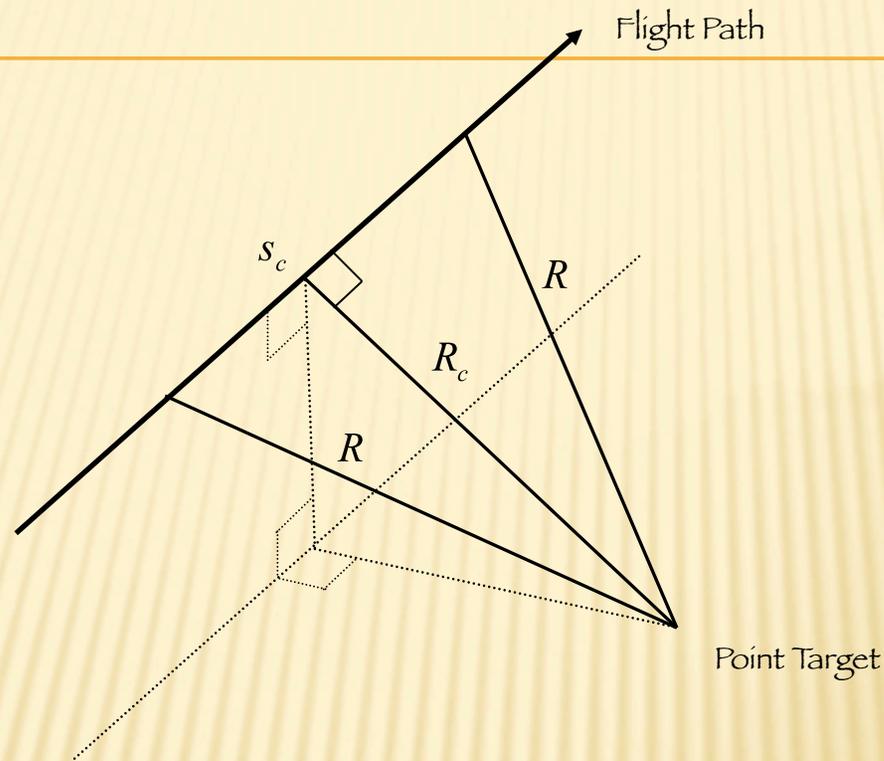
Matched Filtering



For ERS-1/2,
Pulse duration (T): $37.1 \mu\text{s}$
Bandwidth : 15.5 MHz
Half power width of autocorrelation function: $0.065 \mu\text{s}$
Pulse Compression Ratio: 575 (ERS-1/2)
Ground Range Resolution: 12.5 m



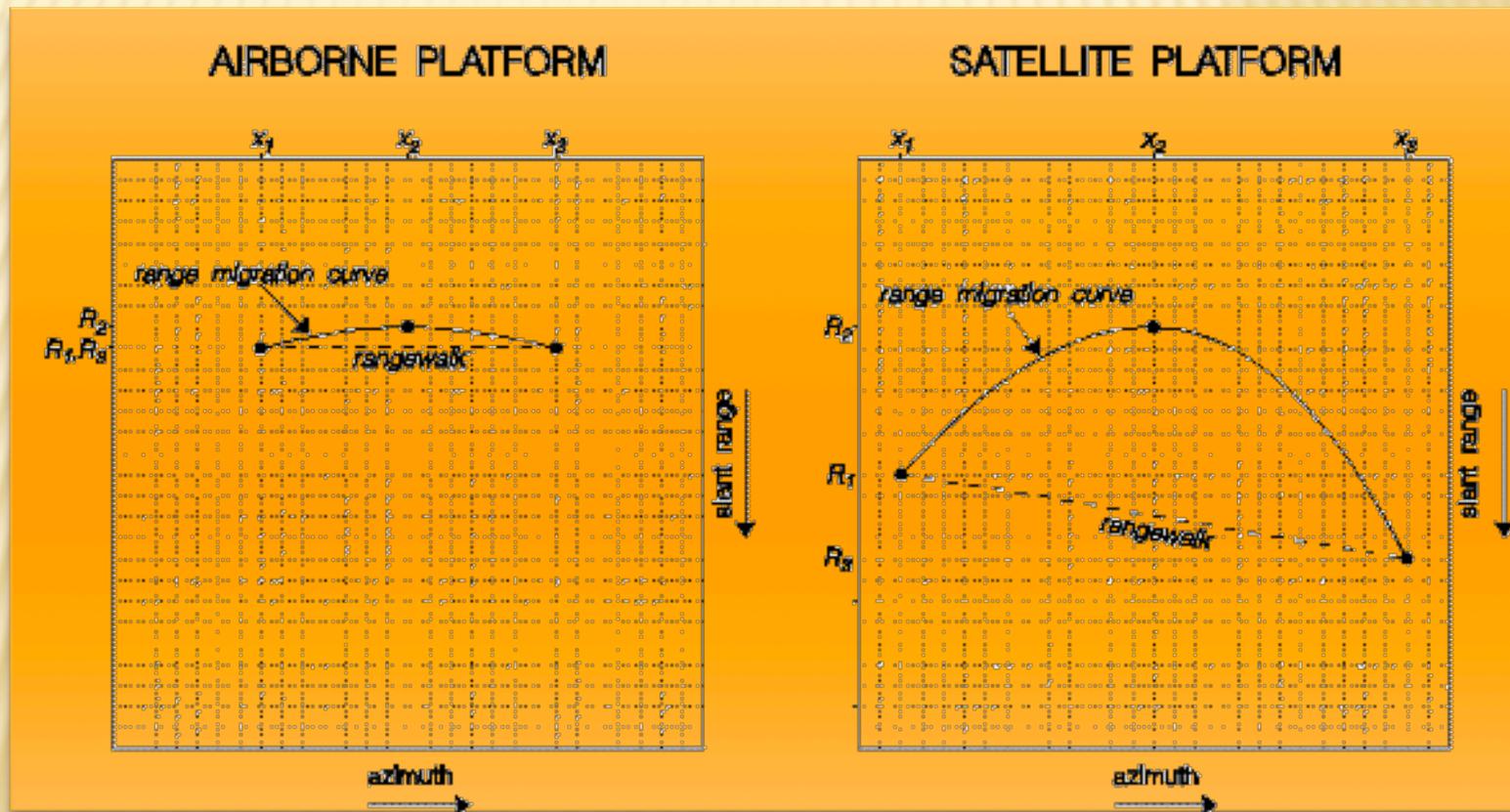
RANGE MIGRATION



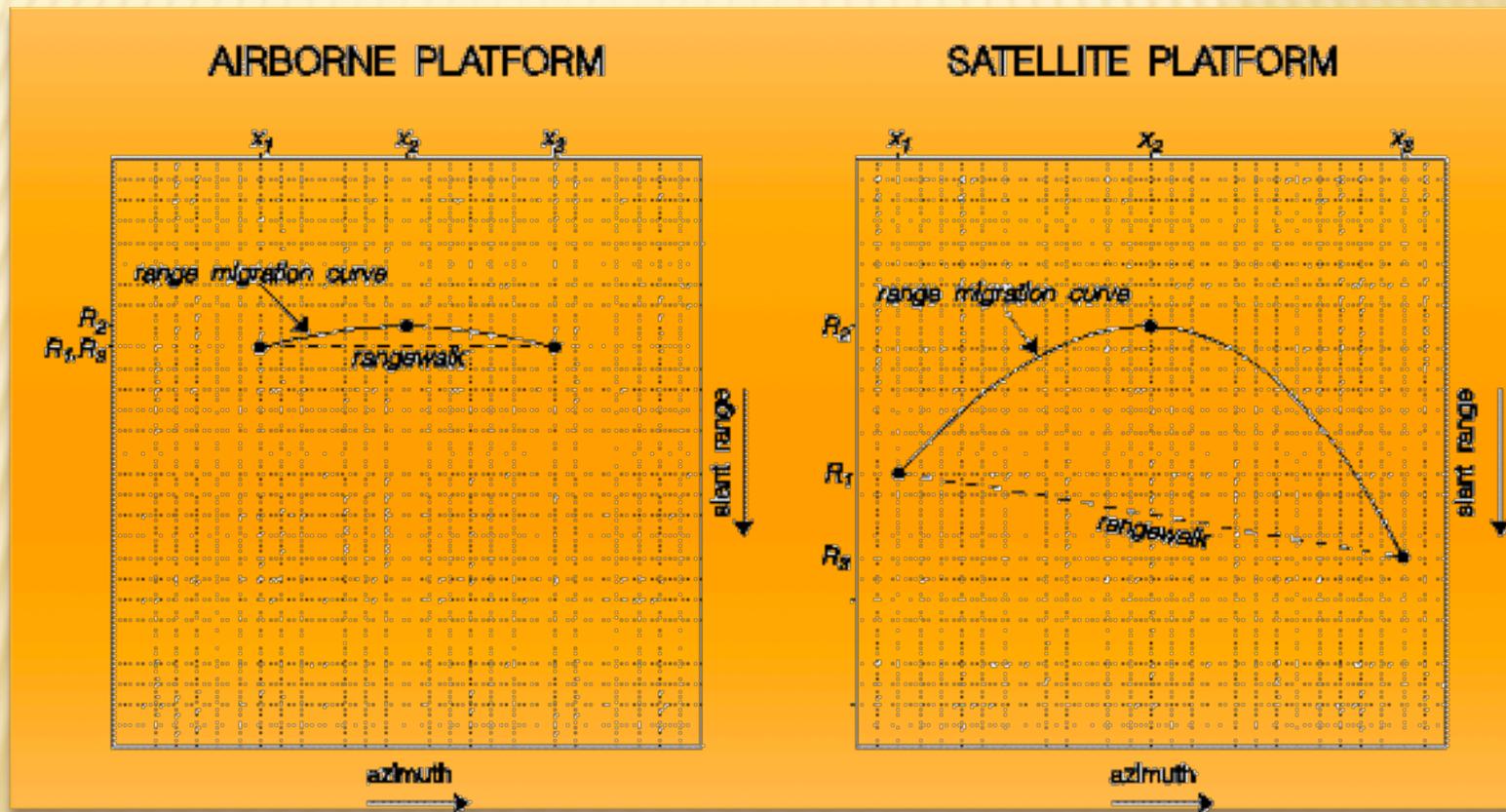
$$R(s) - R_c = \underbrace{-(\lambda f_{Dc}/2)(s - s_c)}_{\text{Linear (Range Walk)}} - \underbrace{(\lambda f_R/4)(s - s_c)^2}_{\text{Quadratic (Range Curvature)}}$$

→ Azimuth FFT →

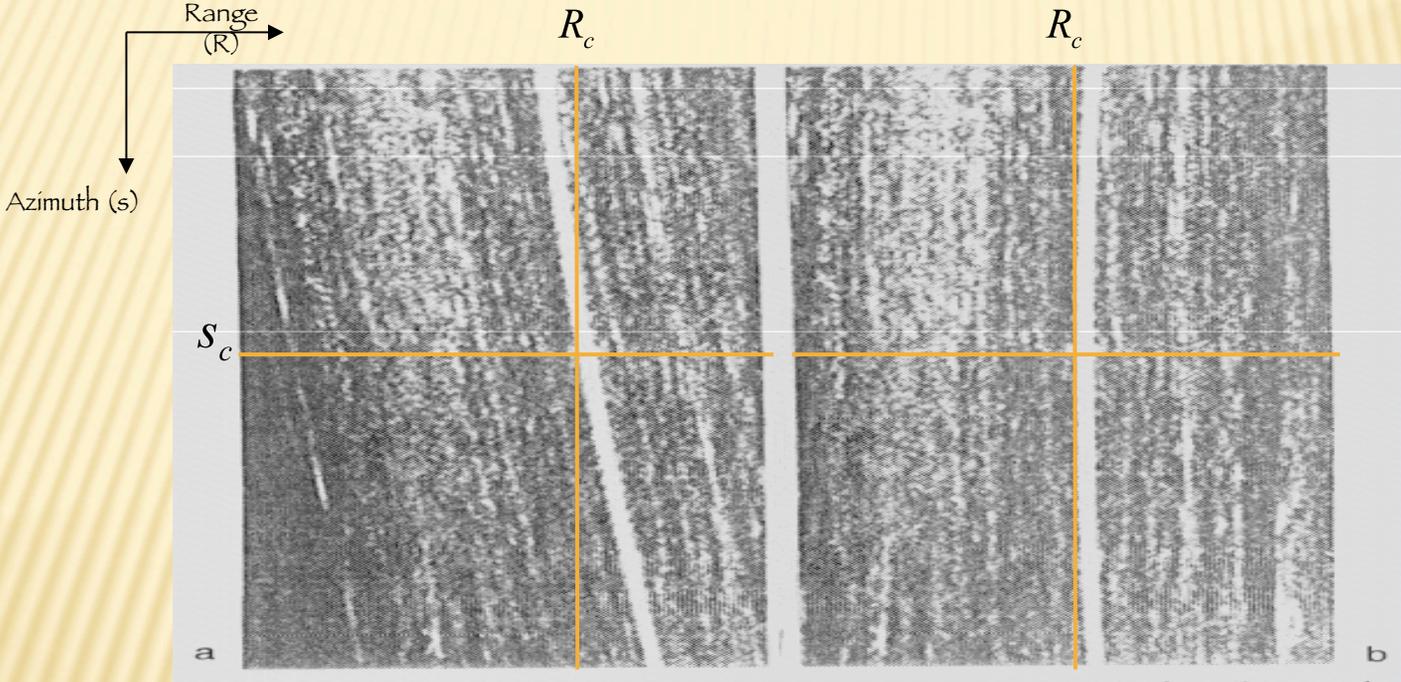
Left side: range mitigation curve and rangewalk for airborne SAR – consider earth flat and stationary, range mitigation curve relatively flat.



Right side: range mitigation curve and rangewalk for spaceborne SAR – have to take into account that earth is sphere and “moving” (from rotation of earth).



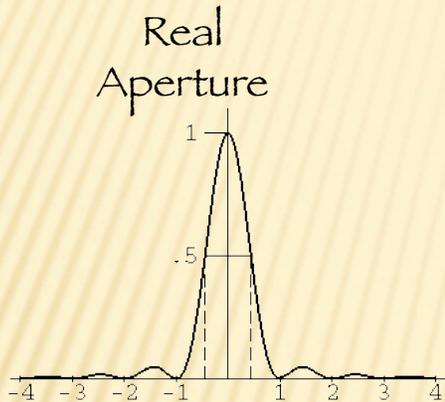
RANGE MIGRATION COMPENSATION



After Range Walk Compensation



AZIMUTH COMPRESSION



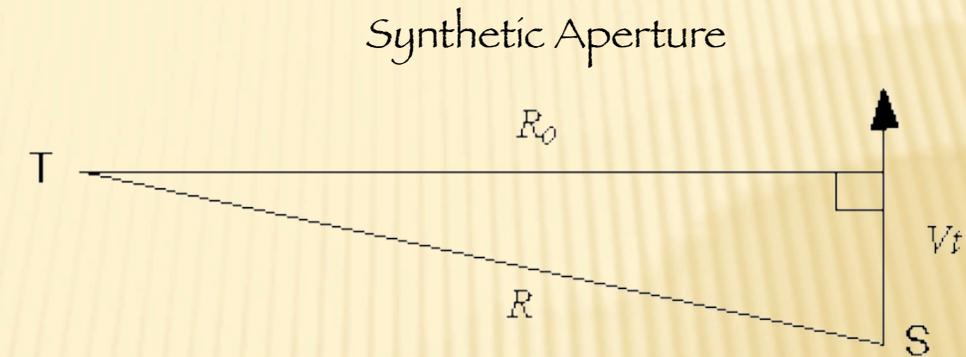
$$\gamma = \lambda / L$$

λ : wavelength

L: Antenna length

Azimuth footprint width:

5 km (ERS-1/2)



Doppler Shift (Linear Chirp Pulse)

$$g(s) = \exp[2\pi i(f_0 - \frac{2V^2}{\lambda R} s)s], \quad -\frac{S}{2} \leq s \leq \frac{S}{2}$$

For ERS-1/2,

Coherent Integration Time (S): 600 ms (5 km footprint)

Bandwidth: 1260 Hz

Half power width of autocorrelation function: 0.8 ms

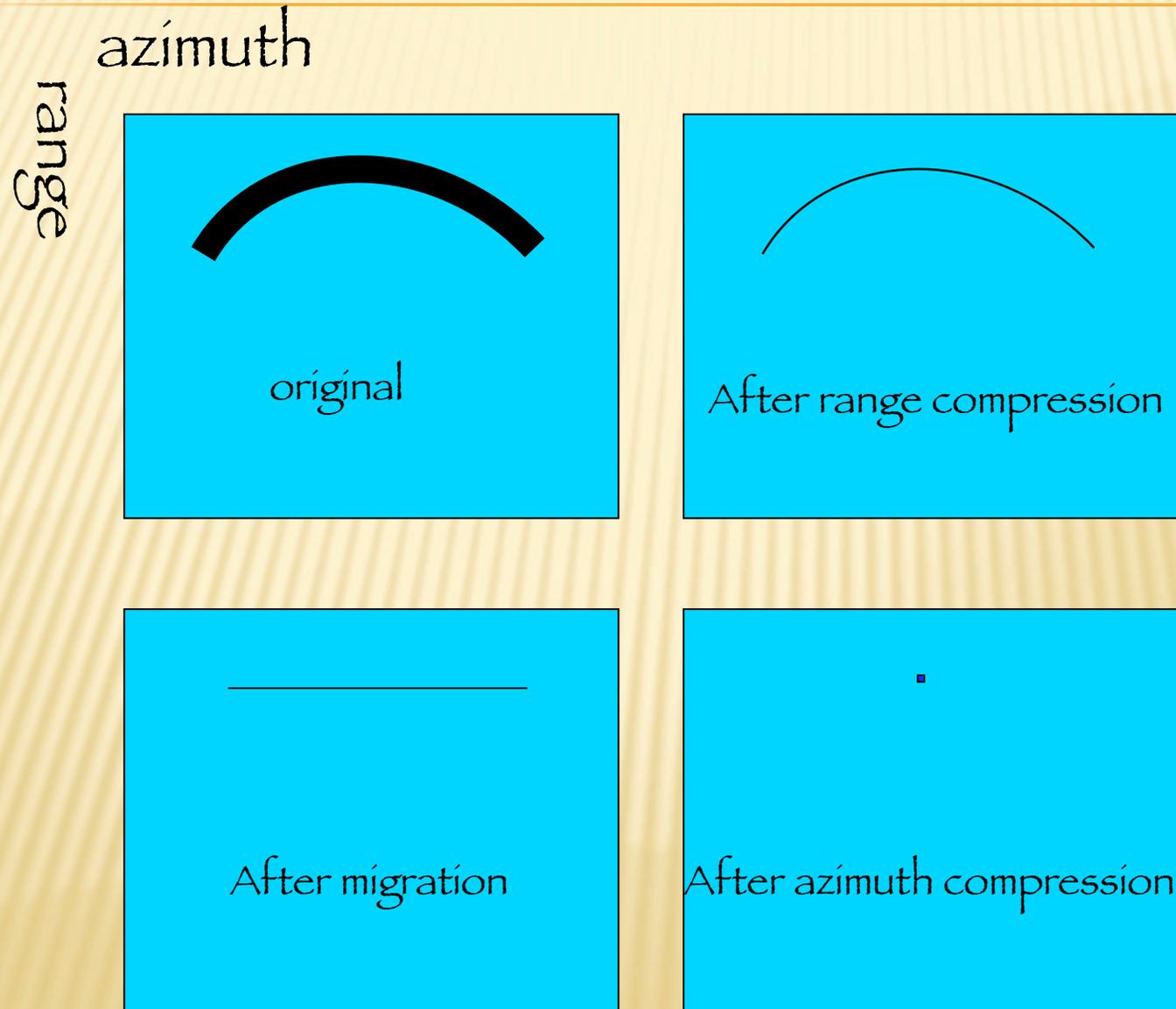
Pulse Compression Ratio: 756 (ERS-1/2)

Azimuth Resolution: 5 m

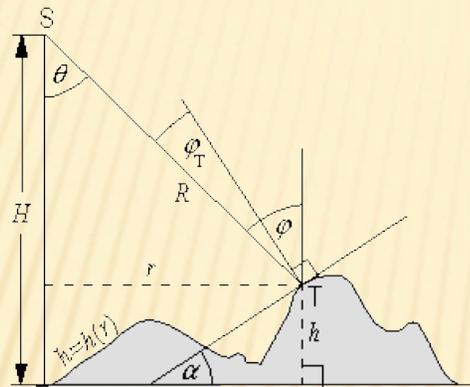
Matched Filtering

→ Azimuth Matched Filtering → Output

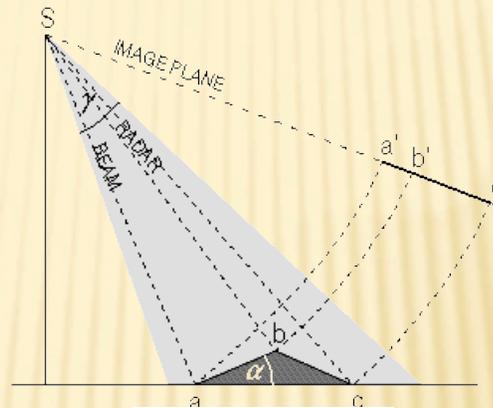
SAR FOCUSING – POINT TARGET



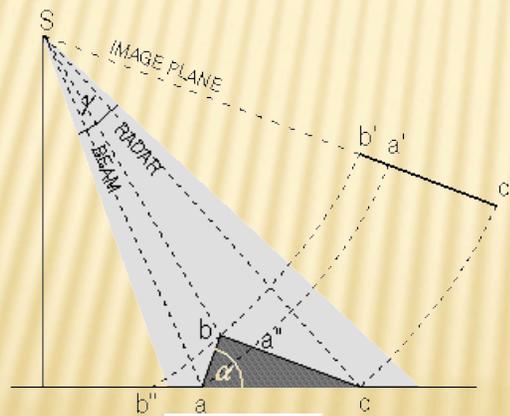
GEOMETRIC DISTORTION



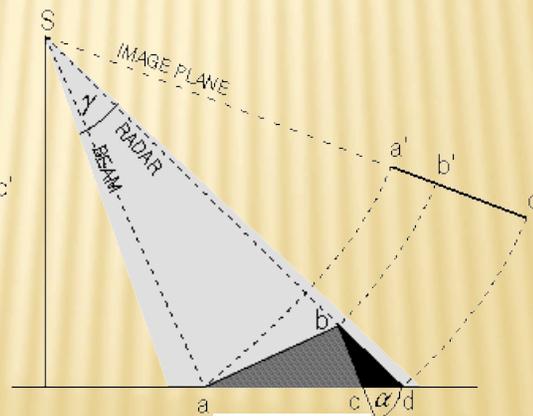
Terrain Imaging Geometry



Foreshortening

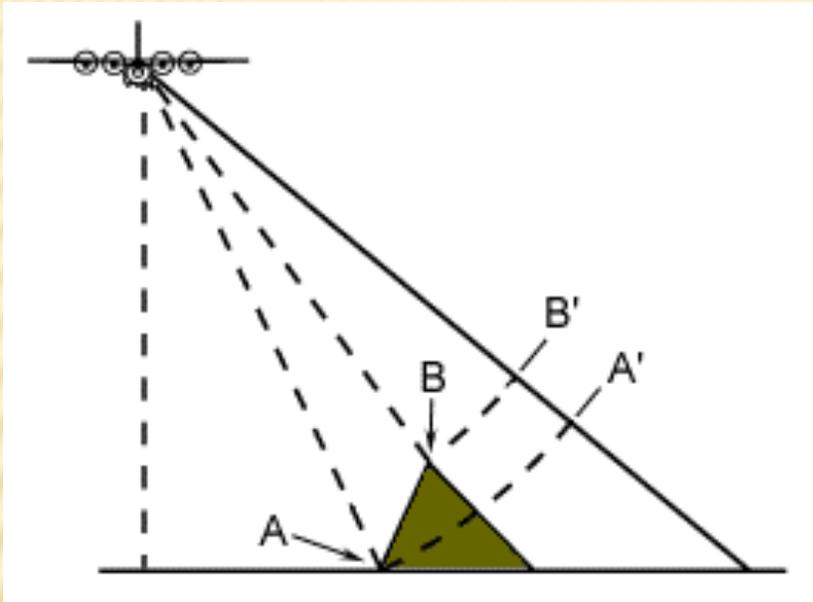


Layover



Shadow

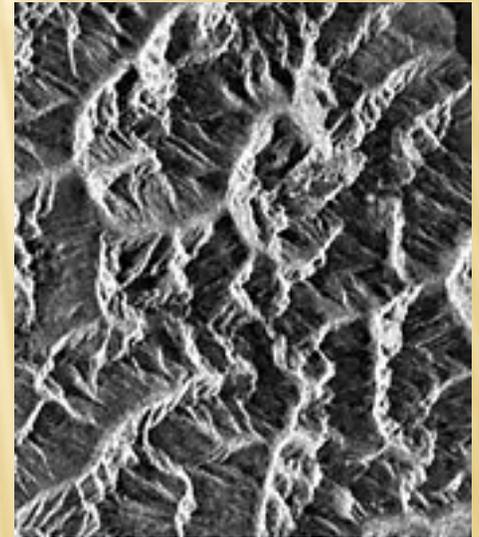
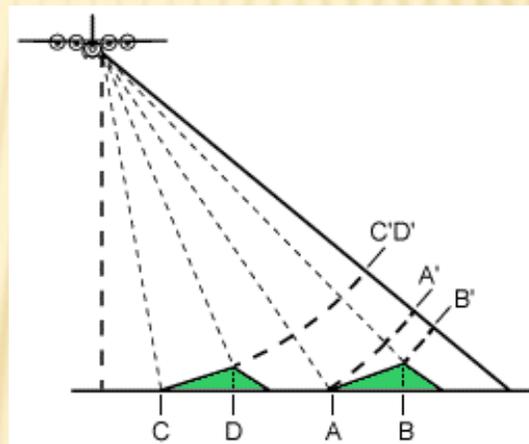
RADAR IMAGE GEOMETRY - LAYOVER



Layover occurs when the radar beam reaches the top of a tall feature before it reaches the base. The top of the feature is displaced towards the radar sensor and is displaced from its true ground position - it 'lays over' the base.

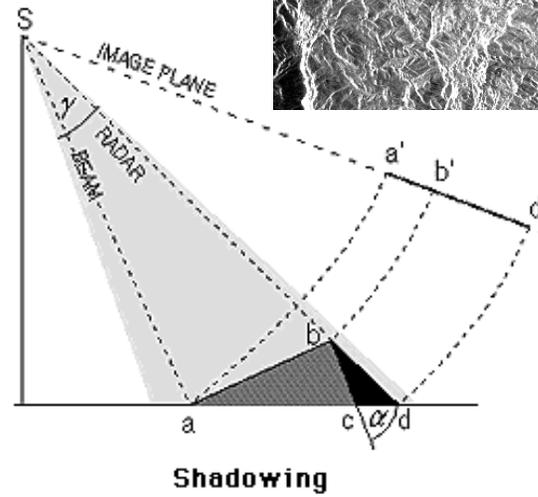
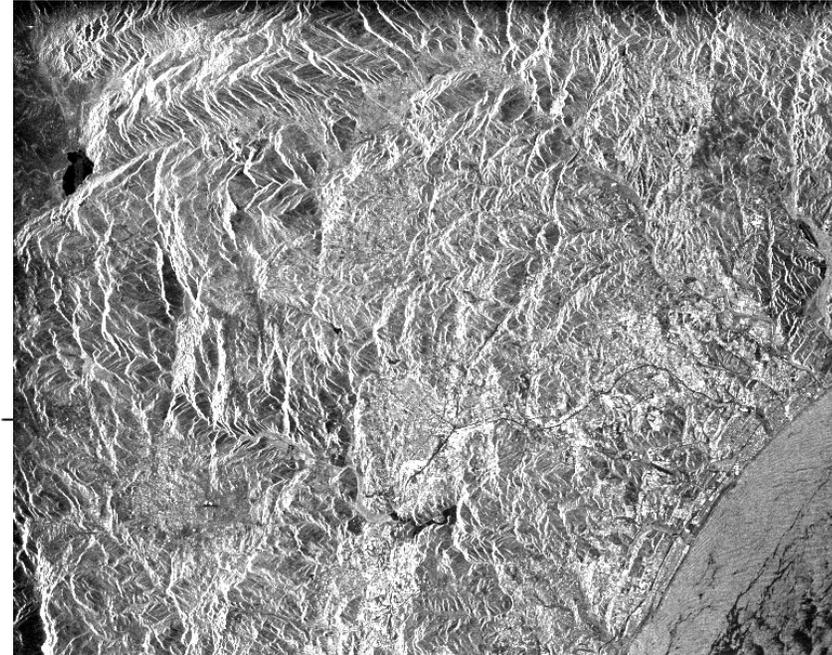
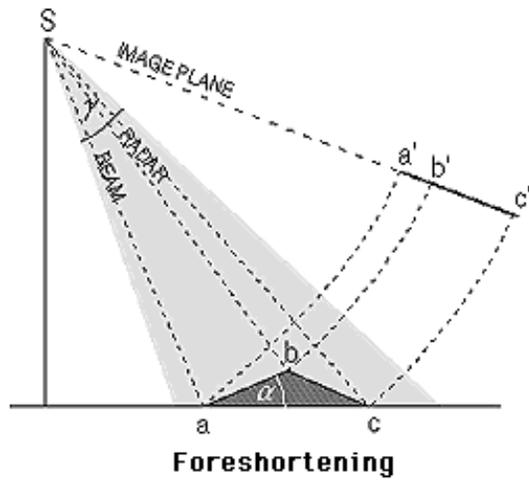
FORESHORTENING

Even if there is no layover, radar returns from facing steep slopes will make the terrain look steeper than it is. This is known as 'foreshortening'. Features which show layover in the near range will show foreshortening in the far range.

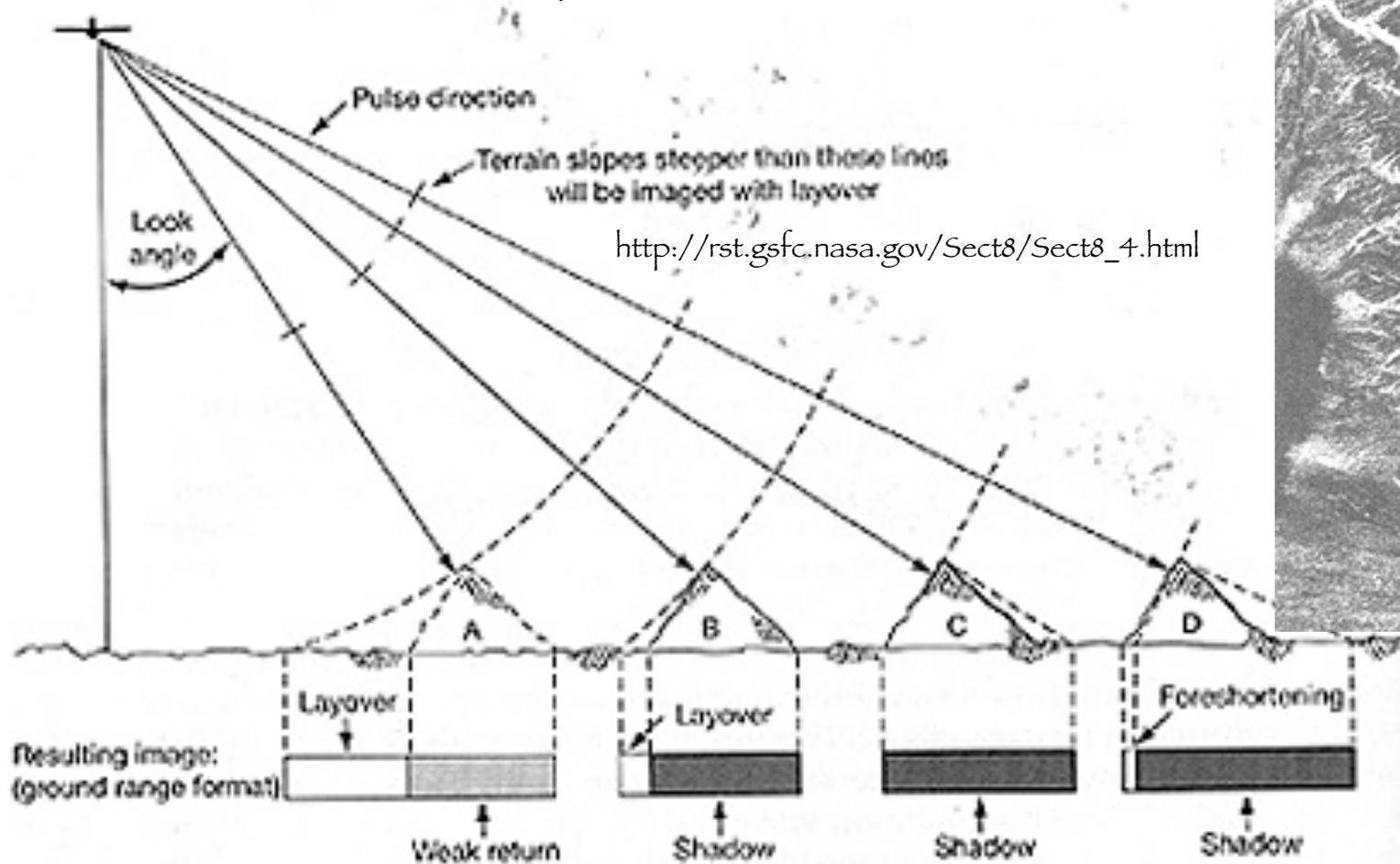


Foreshortening occurs because radar measure distance in the slant-range direction such that the slope A-B appears as compressed in the image (A'B') and slope C-D is severely compressed (C'D')

Geometric Distortion

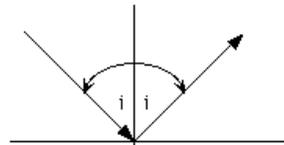


Topo irregularities can have a significant effect on the image. Slopes facing the radar are subject to a distorted appearance. The terms layover and foreshortening apply to this appearance. In most instances, layover and foreshortening produce the same end result visually.

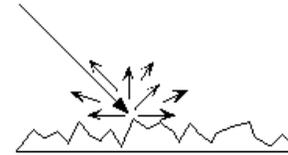


SCATTERING MECHANISMS

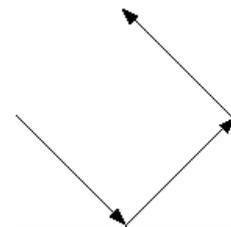
Scattering Mechanisms



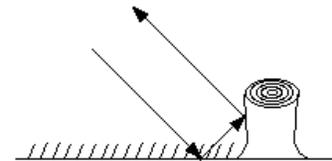
Reflection off a smooth surface
The angle of incidence, i , equals the angle of reflection.



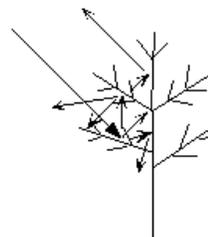
Scattering off a rough surface
The variation in surface height is on the order of the incoming signal's wavelength.



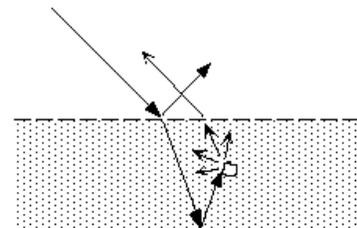
Double Bounce
(Corner Reflector)



Double Bounce
One possible natural occurrence - reflecting off two smooth surfaces, grass and a freshly-cut tree's stump



Volumetric Scattering
Example scattering in a tree



Volumetric Scattering
In this example the incident radiation is both reflected and refracted/transmitted through a layer of dry snow. The refracted radiation then reflects off underlying ice, scatters off a chunk of ice in the snow, and finally refracts back toward the receiver.

RULE OF THUMB IN SAR IMAGES

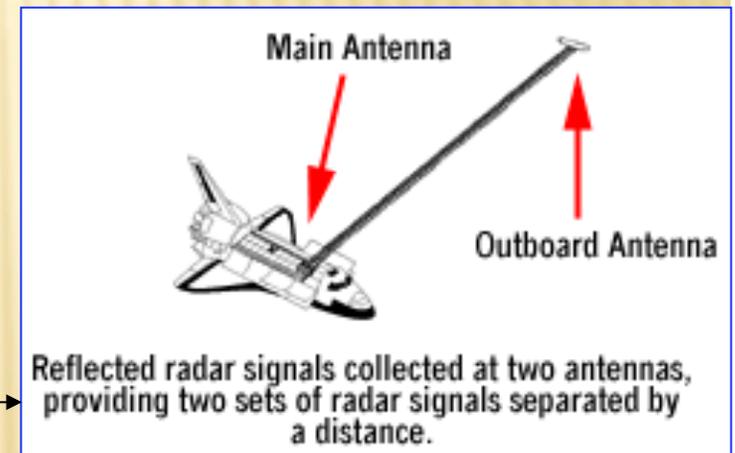
- Backscattering Coefficient
 - Smooth – Black
 - Rough surface – white
- Calm water surface – black
- Water in windy day – white
- Hills and other large-scale surface variations tend to appear bright on one side and dim on the other.
 - Human-made objects - bright spots (corner reflector)
 - Strong corner reflector- Bright spotty cross (strong sidelobes)

1. INTERFEROMETRIC SYNTHETIC APERTURE RADAR (INSAR OR IFSAR)

- ✘ Is a process whereby radar images of the same location on the ground are recorded by
 - + Two antennas of one platform separated by a few meters (single pass), or
 - + The same radar system at different times (multi-pass or repeat-pass)
- ✘ Applications on
 - + Elevation (DEM) derivation (single or multi pass)
 - ✘ Can be as accurate as DEM from traditional optical photogrammetric techniques. However, InSAR operate through clouds, day or night.
 - ✘ The first worldwide DEM (99.97%) was acquired in 2000 by SRTM in 2000, not by the photogrammetry
 - + Surface displacement study (multi-pass only)

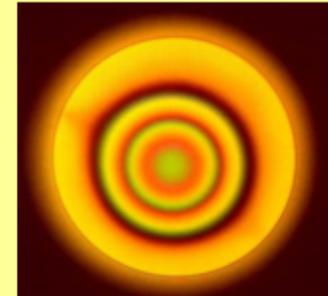
EXAMPLES

- ✘ One SAR with 2 antennas (single-pass)
 - + AIRSAR/TOPSAR
 - ✘ Along track interferometric mode (ATI) (L and C)
 - ✘ Ocean current and waves
 - ✘ Cross track interferometric mode (TXI) (L or C)
 - ✘ DEM (3-5 m or 1 m)
 - + Shuttle Radar Topographic Mission (SRTM)
 - ✘ C band and X band antennas separated by 60 m
- ✘ One SAR in different times (multi-pass)
 - + SIR-C
 - + ERS 1,2



Interferometry

- Study of interference patterns from two sets of electromagnetic waves reflecting from surface
 - E.g., thin film of oil on water
- When we measure the radar pulse, we consider not just the amplitude but the phase too



$$\text{Phase (cycles)} = \frac{\text{distance}}{\text{wavelength}}$$

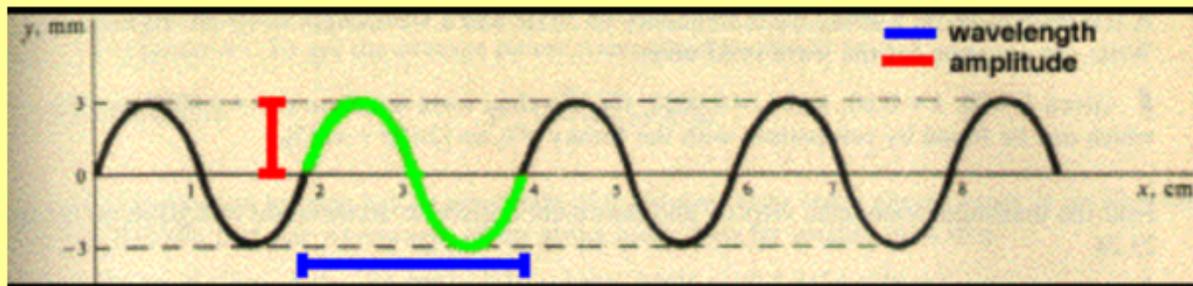


Figure 1: Wave Graph

5

Phase is total number of cycles at any given distance (or target) from transmitter, including fractional part. One cycle of phase is equal to 360 degrees (or 2π).

Can make use of this information with 2 receiving antennas

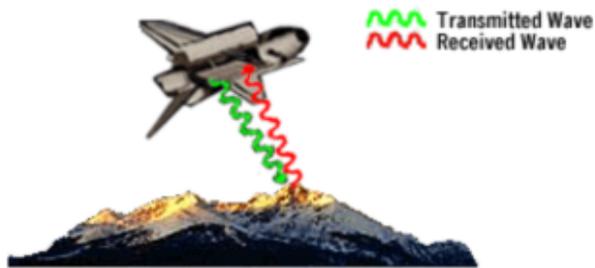


Figure 2: One Receiver

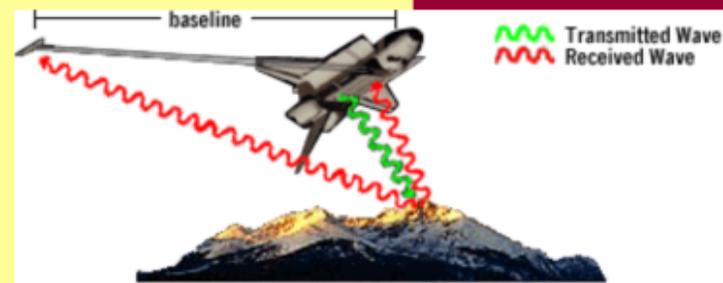
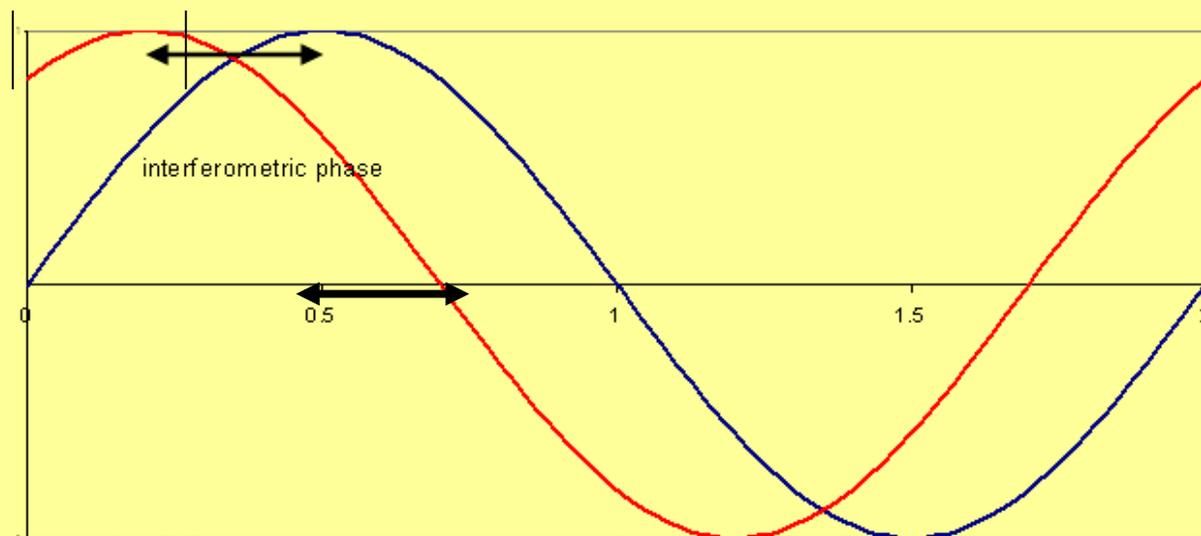
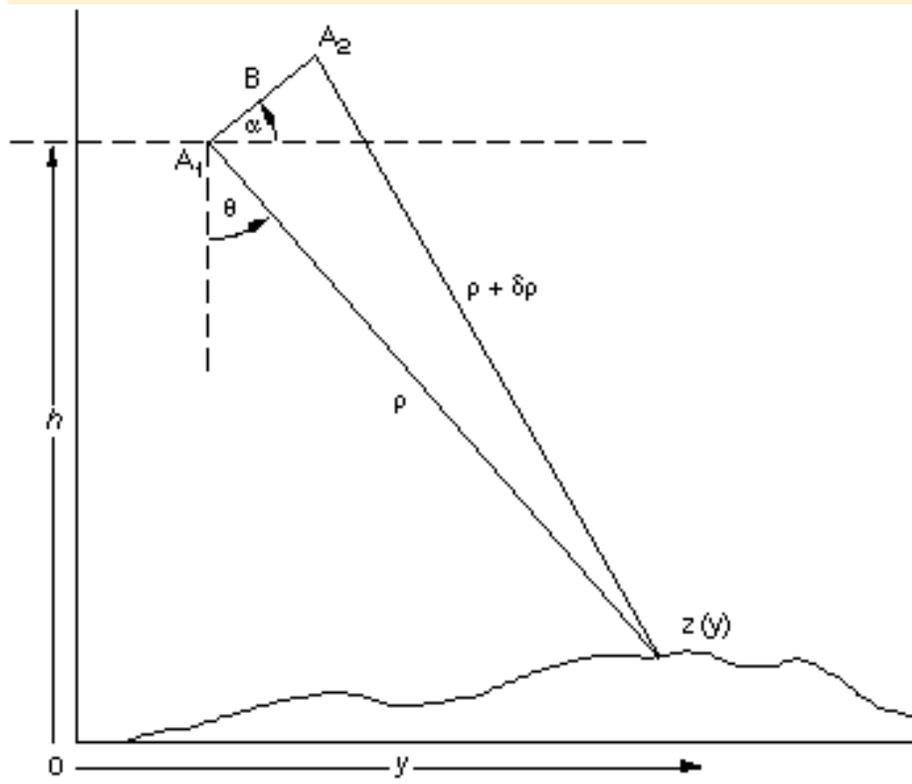


Figure 3: Two Receivers



6

phase difference called “interferometric phase” - determined by subtracting measured phase at each end of baseline, is actually the distance difference from each receiver to same target.



CALCULATE ALTITUDE

$$z(y) = h - \rho \cdot \cos \theta \quad (1)$$

$$(\rho + \delta\rho)^2 = \rho^2 + B^2 - 2\rho B \cos(90 - \theta + \alpha) = \rho^2 + B^2 + 2\rho B \sin(\alpha - \theta) \quad (2)$$

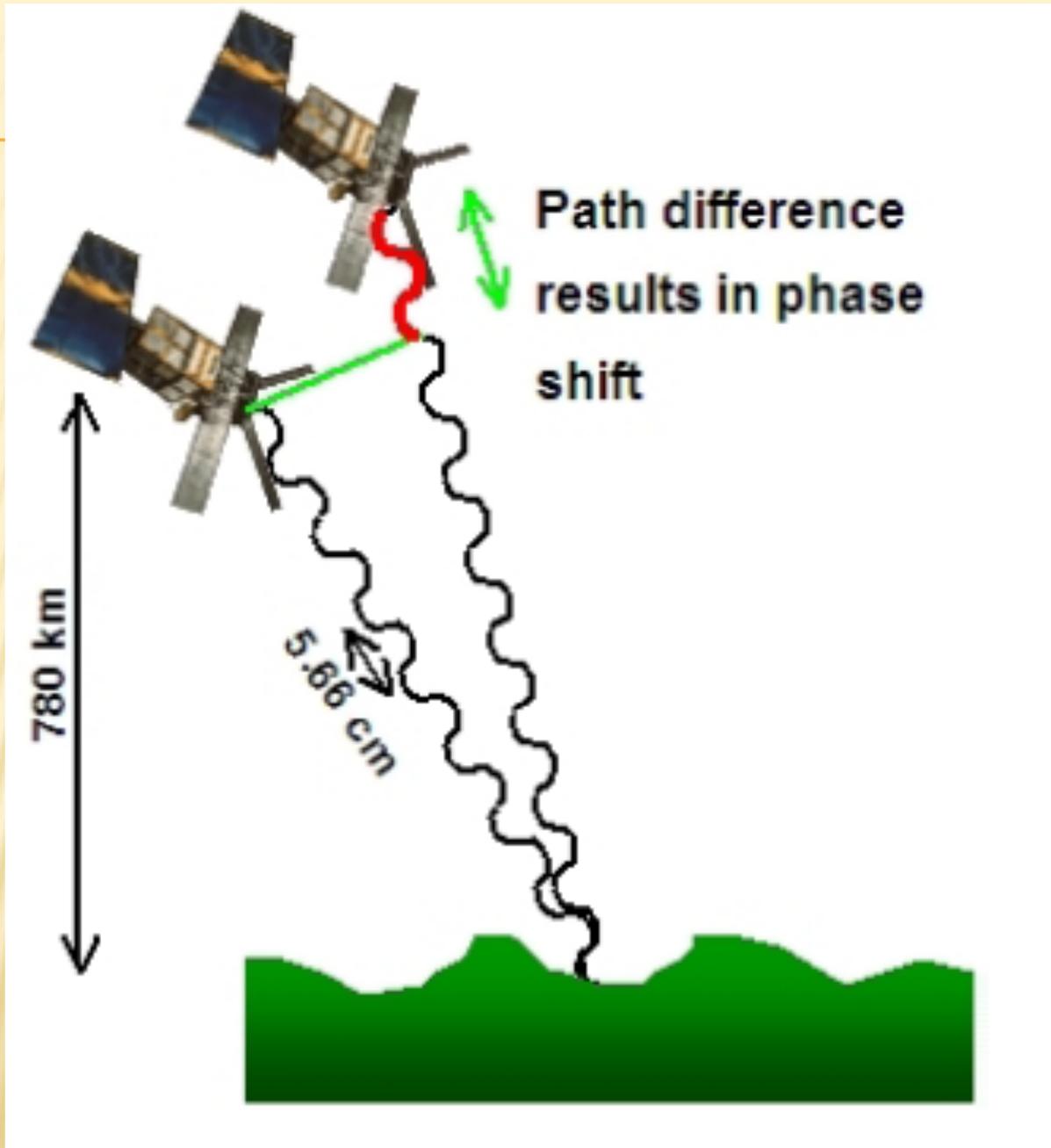
$$\delta\rho = \frac{\varphi}{2\pi} \times \lambda \quad (3)$$

(Phase difference)

$$z(y) = h - \frac{B^2 - \left(\frac{\lambda\varphi}{2\pi}\right)^2}{\frac{\lambda\varphi}{\pi} - 2B \sin(\alpha - \theta)} \cdot \cos \theta \quad (4)$$

Repeat-pass interferometry

- If we don't have a single vehicle with 2 antennas, we can use repeat-pass interferometry
 - Same vehicle, different days, *slightly* different position
 - This also allows us to detect subtle *changes* in surface elevation between the passes

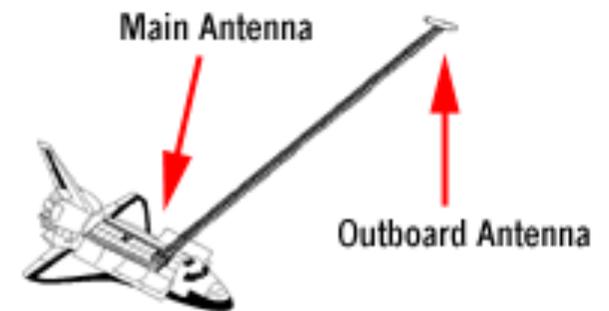


NASA's Shuttle Radar Topography Mission

- SIR-C/X-SAR flew two 10-day missions
 - April and October 1994
 - L-band, C-band, and X-band
- Instrument was then converted to an interferometer
 - C- and X-band only
 - 2nd receiving antenna added
- SRTM flew a 10-day mission in February 2000
 - Collected data on most of the land surface between 60°N–54°S

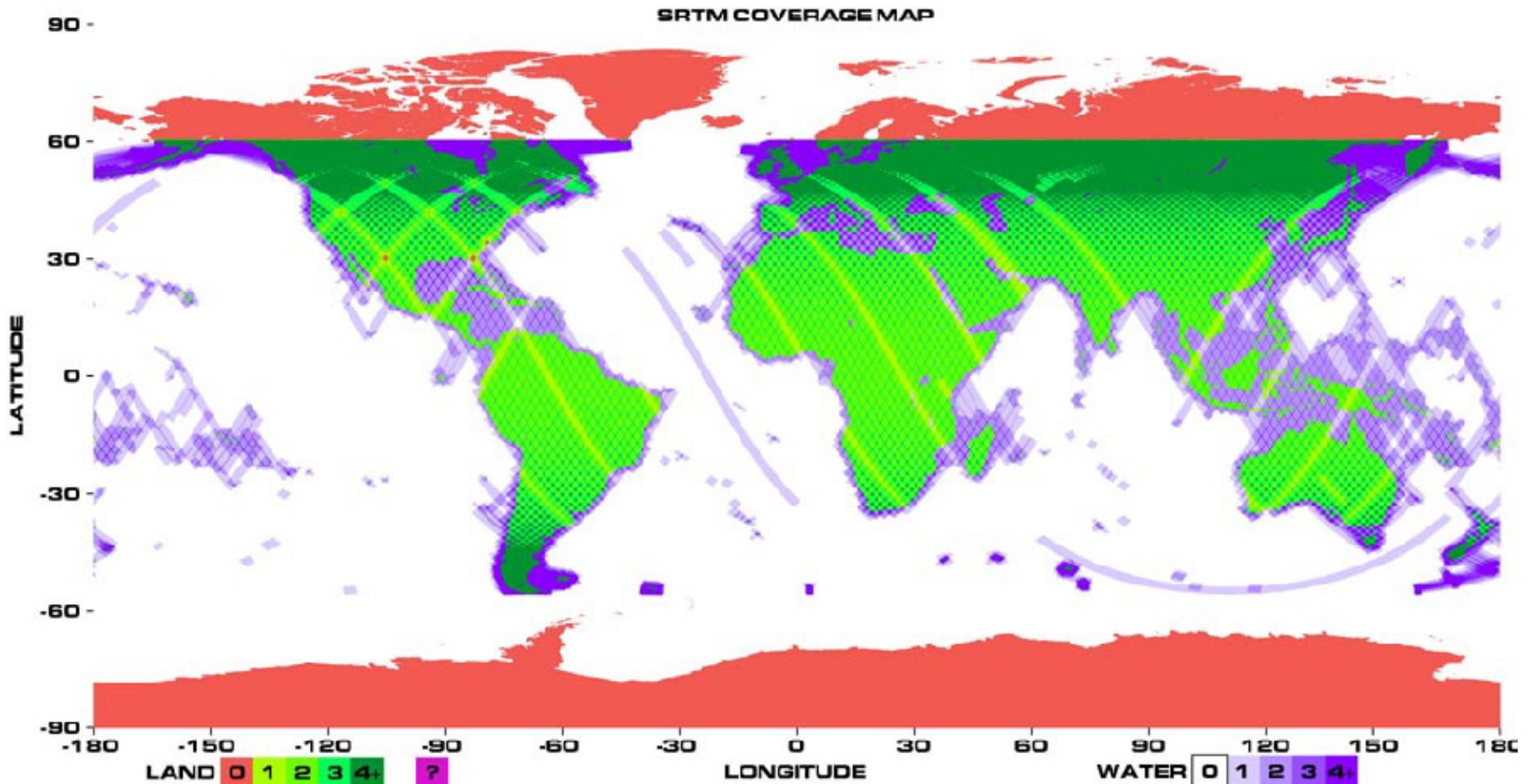
Both C (5.6 cm) and X (3 cm) bands in the Main Antenna transmit and receive radar signals, but in the Outboard Antenna only receive signals.

Hongjie



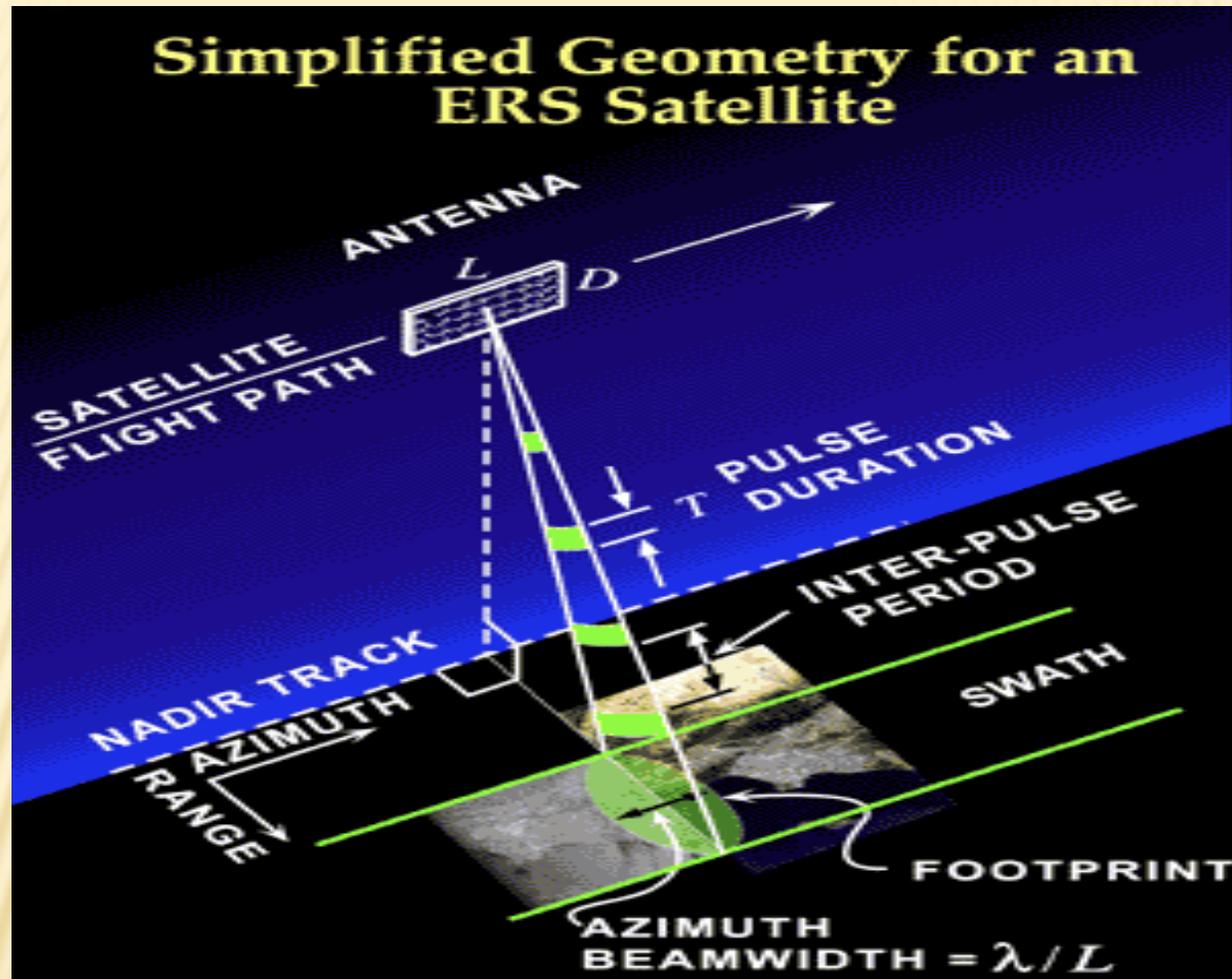
Reflected radar signals collected at two antennas, providing two sets of radar signals separated by a distance.

SRTM COVERAGE MAP



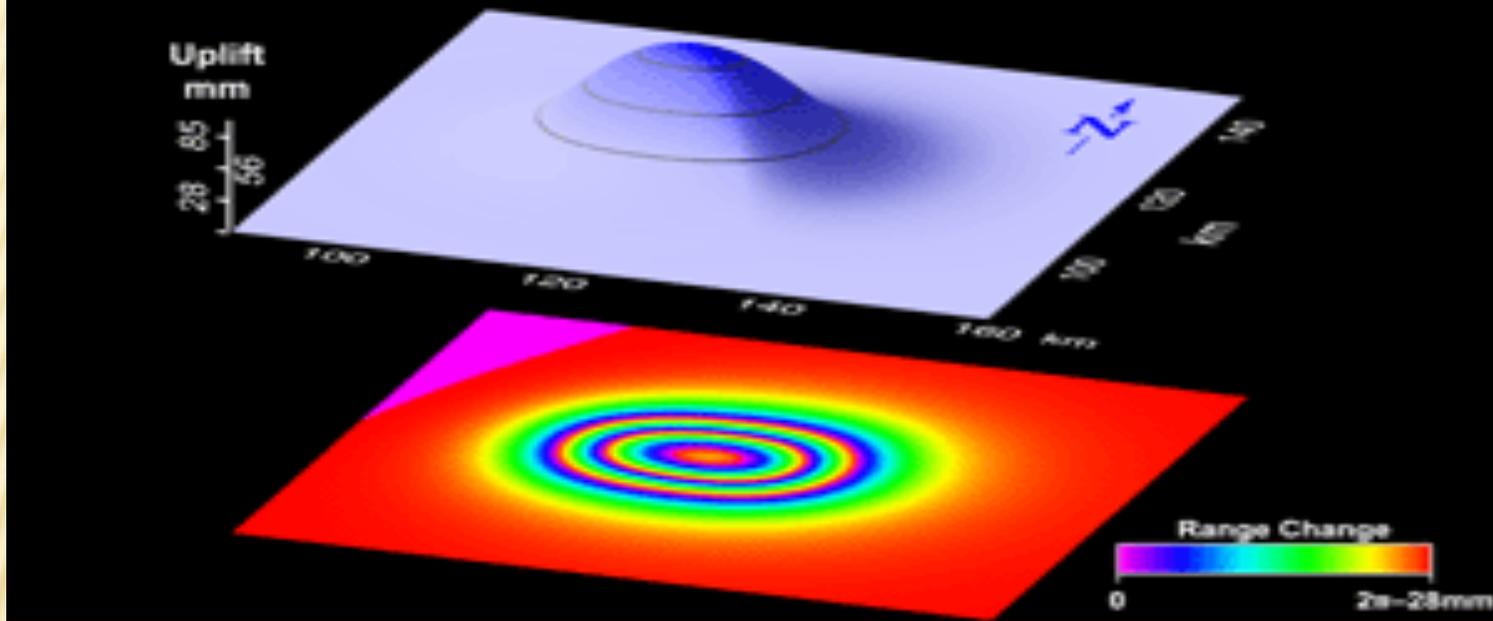
Hongjie

To download from here <http://seamless.usgs.gov/>



3-Dimensional view of InSAR geometry.

~10 cm of uplift produces ~3 fringes of deformation



Colors on an interferogram map into elevation changes.
Fringes – each one signifies change of 1 wavelength.

SAR satellites

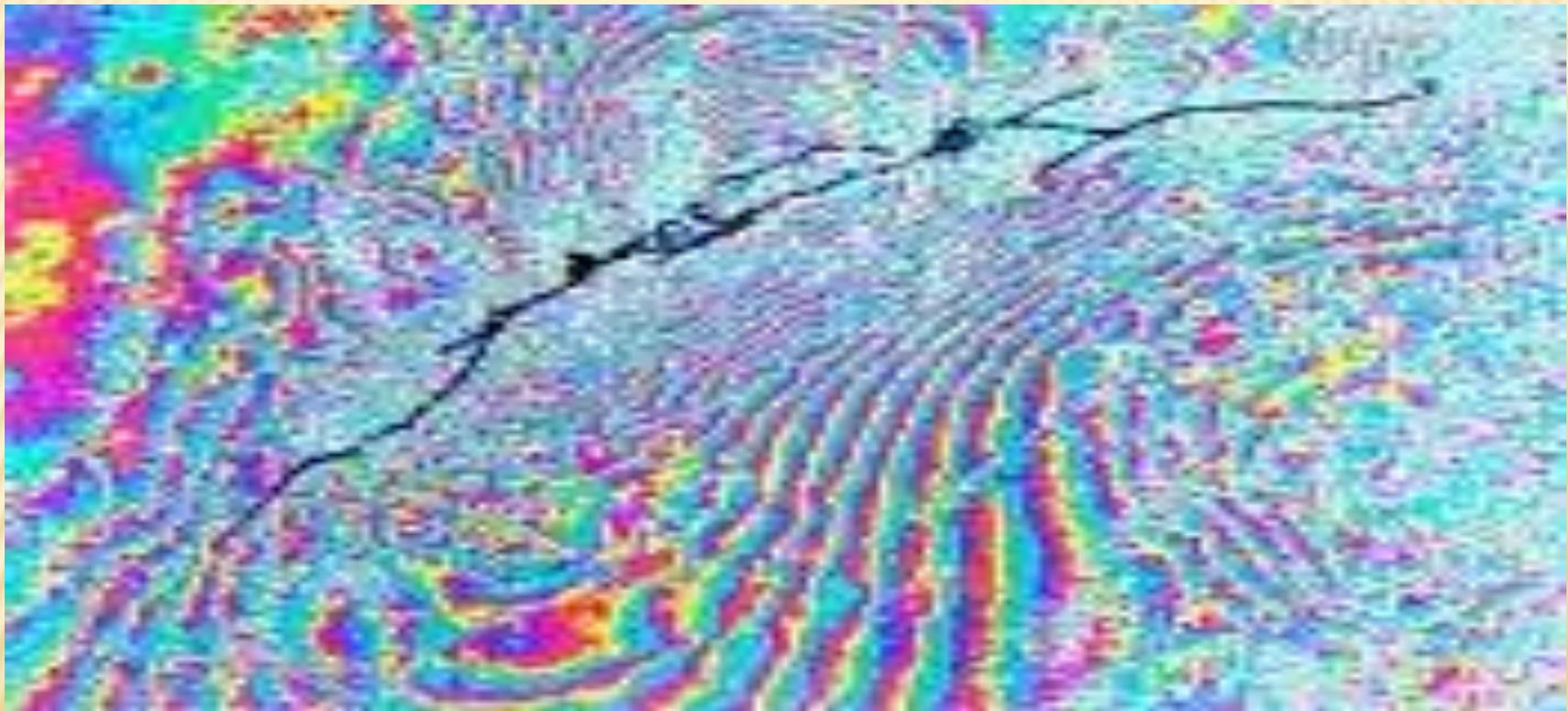
	repeat cycle (days)	wave-length (cm)
European ERS-1/ERS-2 1992-2001(+)	35	6
Canadian Radarsat-1 1995-present	24	6
European Envisat 2003-present	35	6
Japanese ALOS launched Jan. 2006	48	24
German TerraSAR-X launched July 2007	11	3
Italian COSMO/SkyMed 2 launched 2007	16/2	3
Canadian Radarsat-2 launched Dec. 2007	24	6

SIDE INFO: ASTER GLOBAL DEM

- ✘ This is not from INSAR tech, but It has an along-track stereoscopic capability using its near infrared spectral band and its nadir-viewing and backward-viewing telescopes to acquire stereo image data with a base-to-height ratio of 0.6.
- ✘ 30 m in pixel size
- ✘ 30 m accuracy in horizontal and 20 m accuracy in vertical
- ✘ Free downloaded from
 - + <http://www.gdem.aster.ersdac.or.jp/>
 - + <https://wist.echo.nasa.gov/~wist/api/imswelcome/>

Interferogram

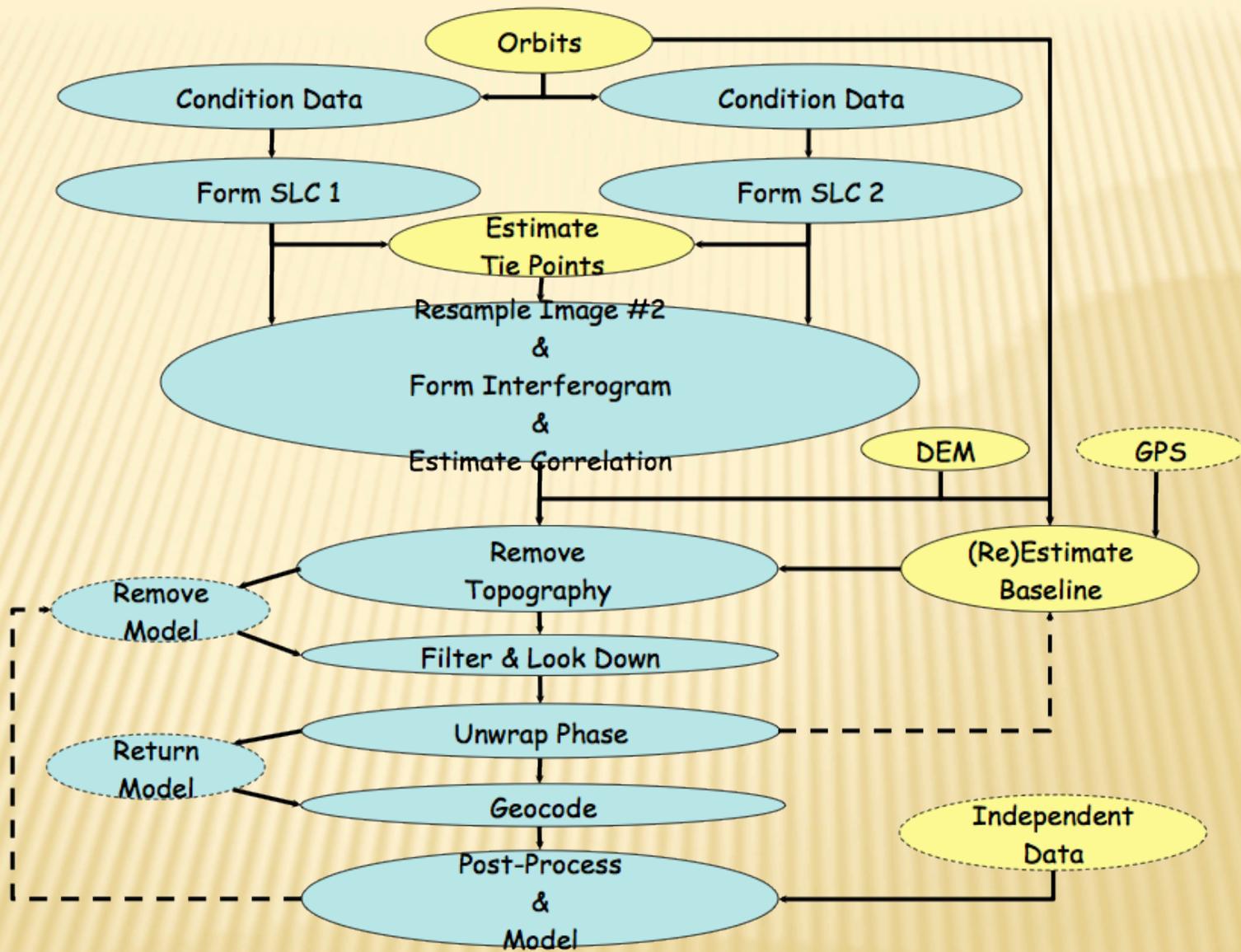
The data obtained from the satellite is mainly used to create an interferogram like the one showed in figure 1. An interferogram is actually a combination of 2 images of the exact same area with a time elapse in between. When these images are overlaid the change in phase shift can be color coded and will then show the deformation of a given area.



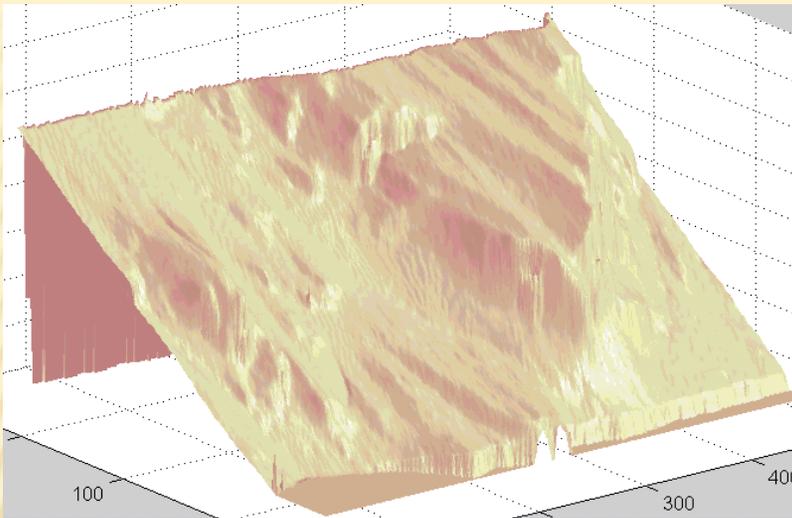
interferogram of the 1992 Lander's Earthquake

Getting from raw satellite data to an interferogram is not a easy process. Luckily, there is a software program known as ROI_PAC or Repeat Orbit Interferometric Package which helps a lot with this procedure. Using this software package there are just a few simple steps that must be followed.

- Acquire 2 frames of data collected at different times from the same location.
 - Acquire orbital data for orbits in question.
- Acquire a digital elevation model for area in question
 - Set up environment variables
 - Condition the raw data
 - Run through ROI_PAC

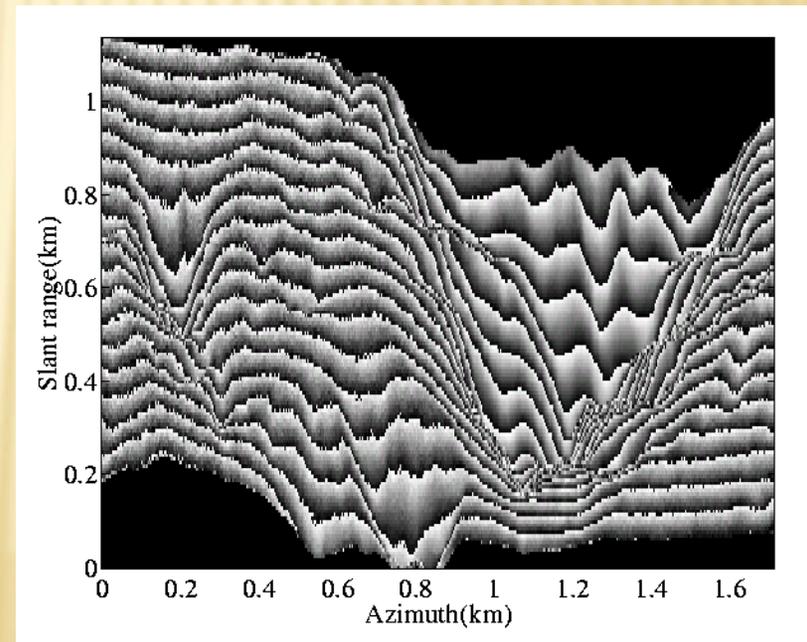
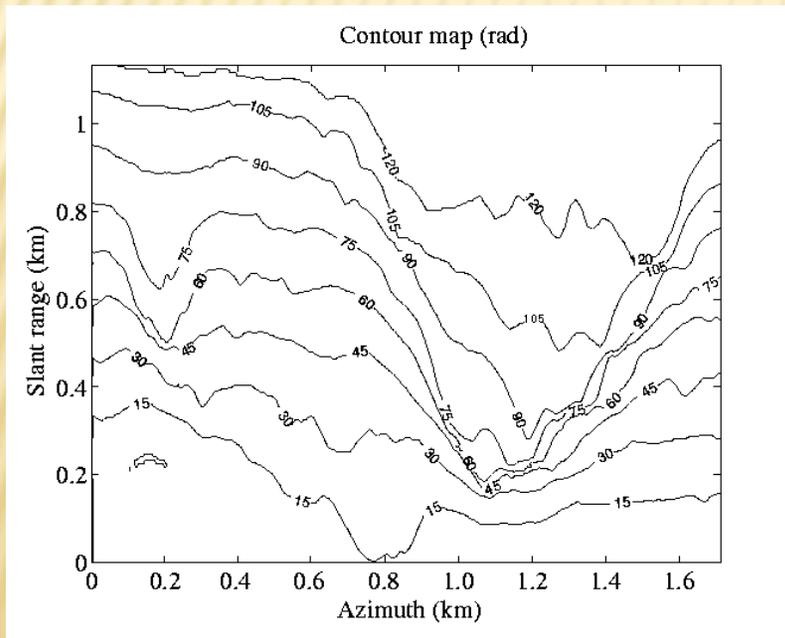


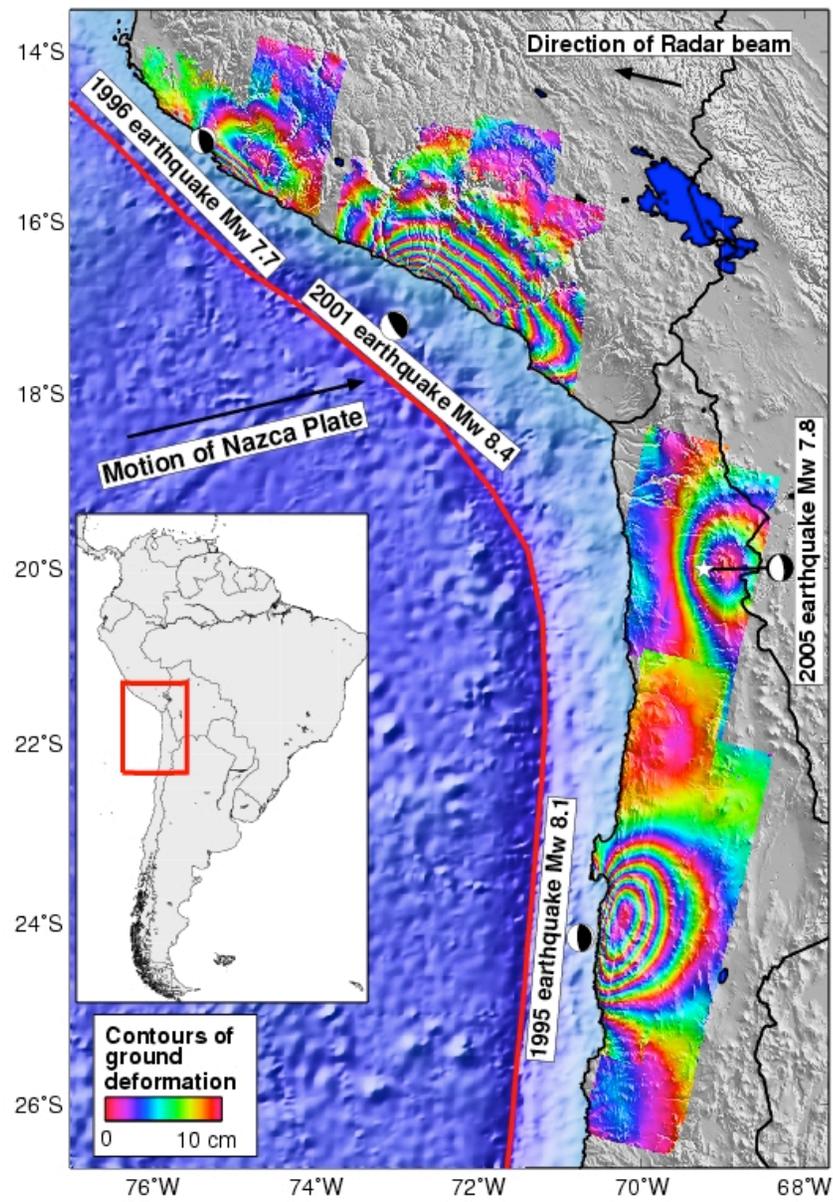
courtesy Mark Simons

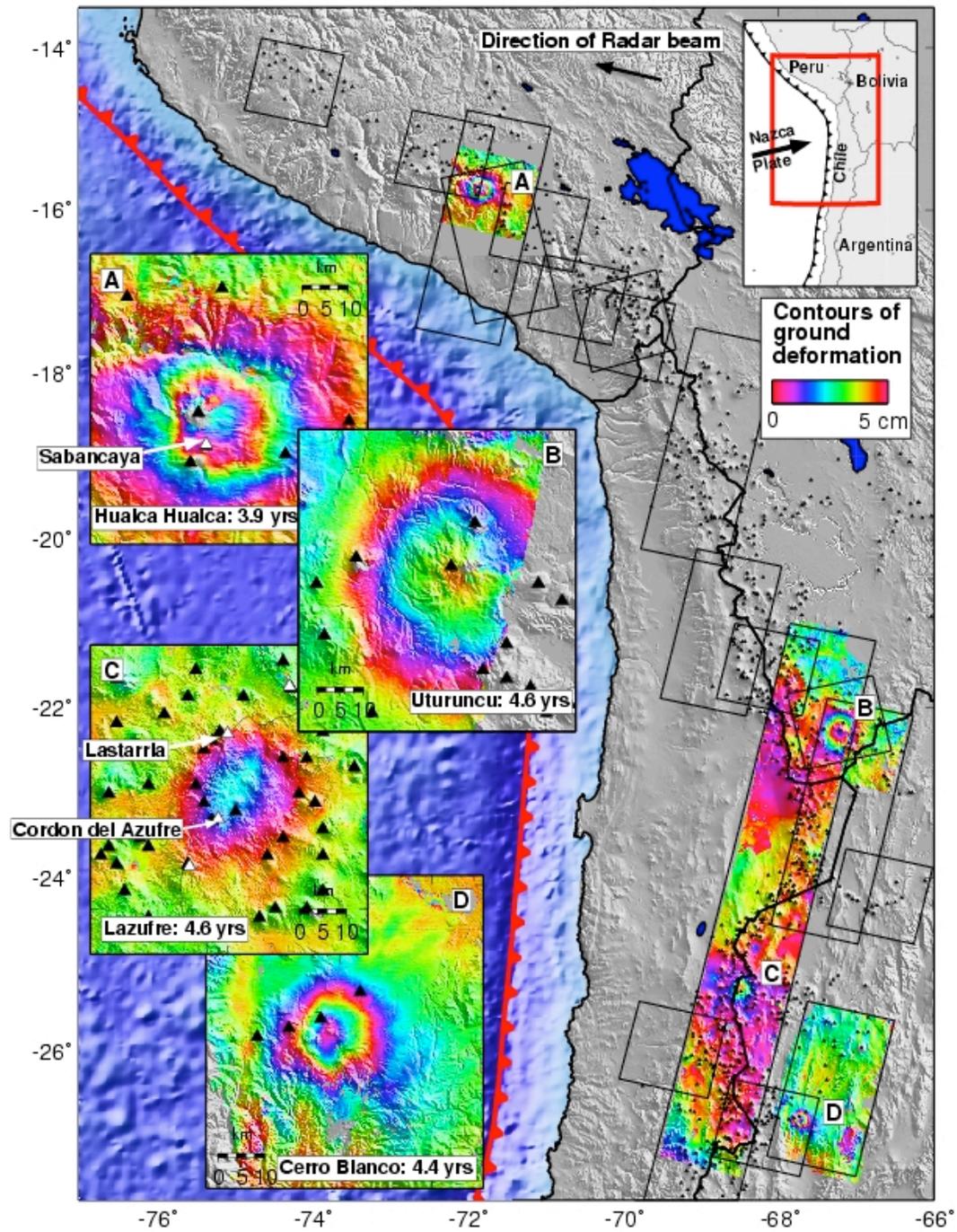


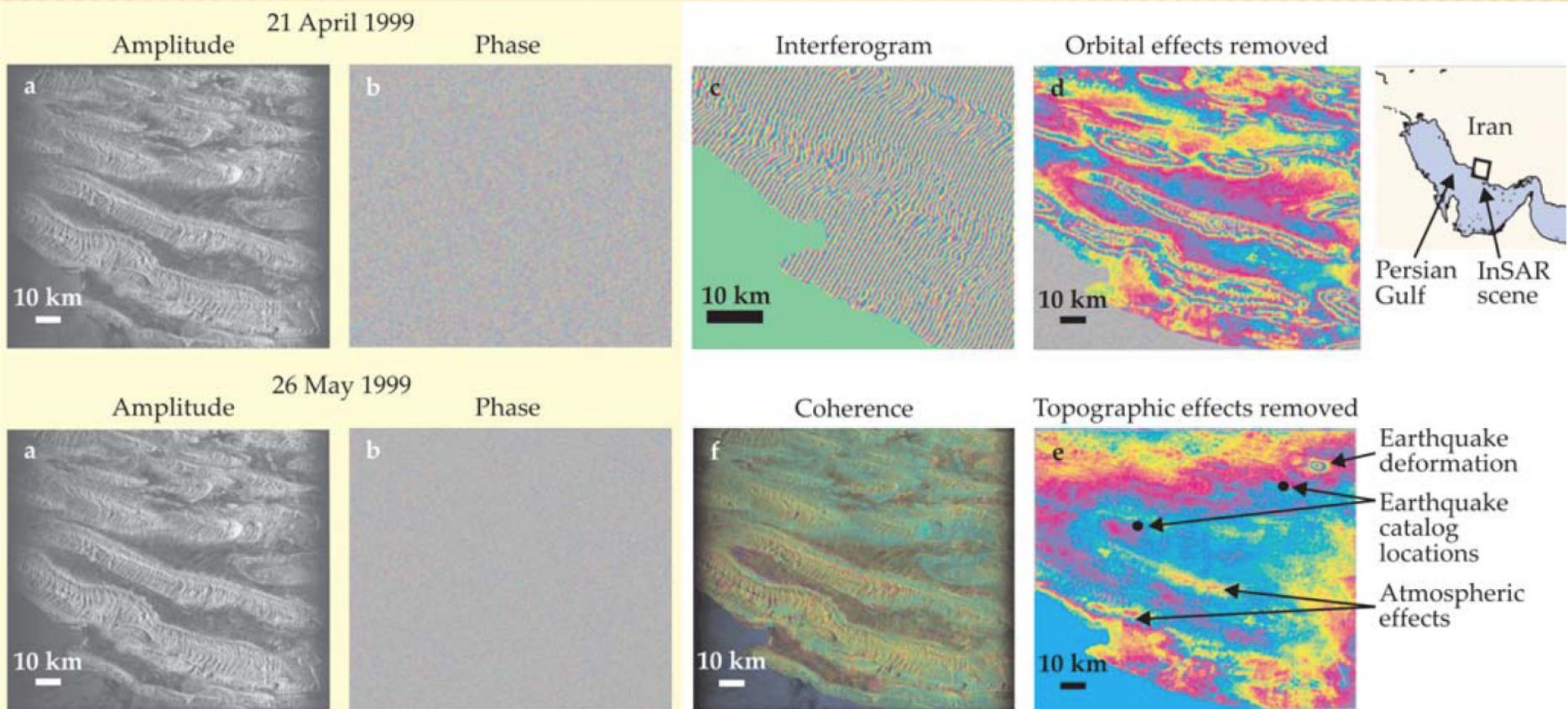
Mountainous terrain around Long's Peak, Colorado

Interferogram





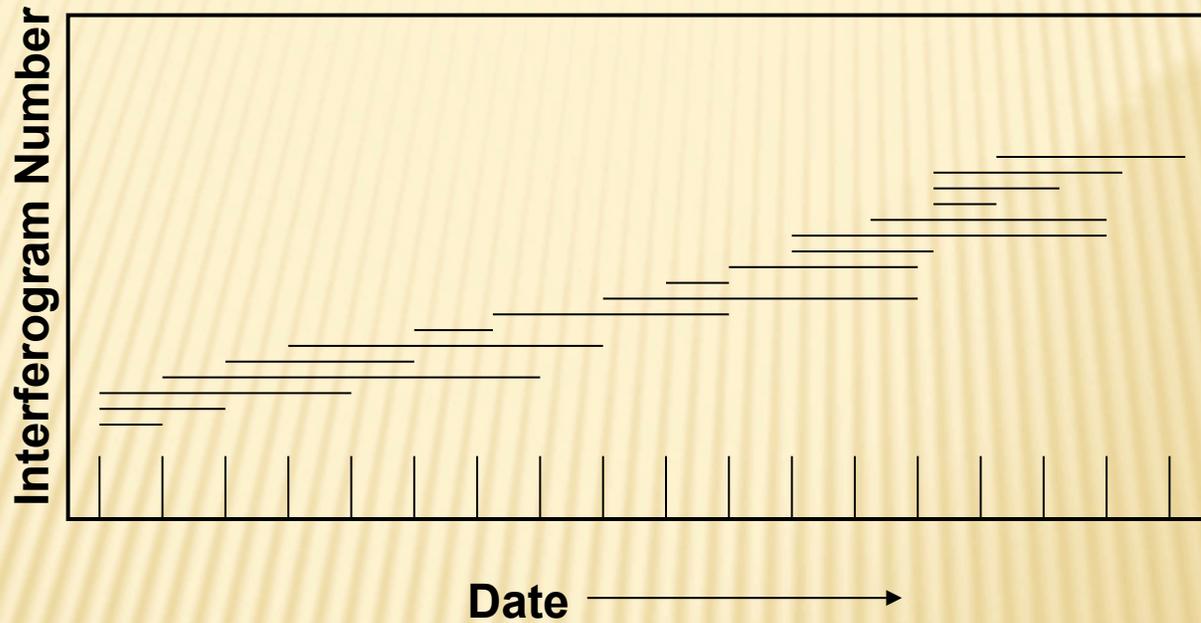




Using InSAR to improve earthquake locations (assuming it causes surface deformation).

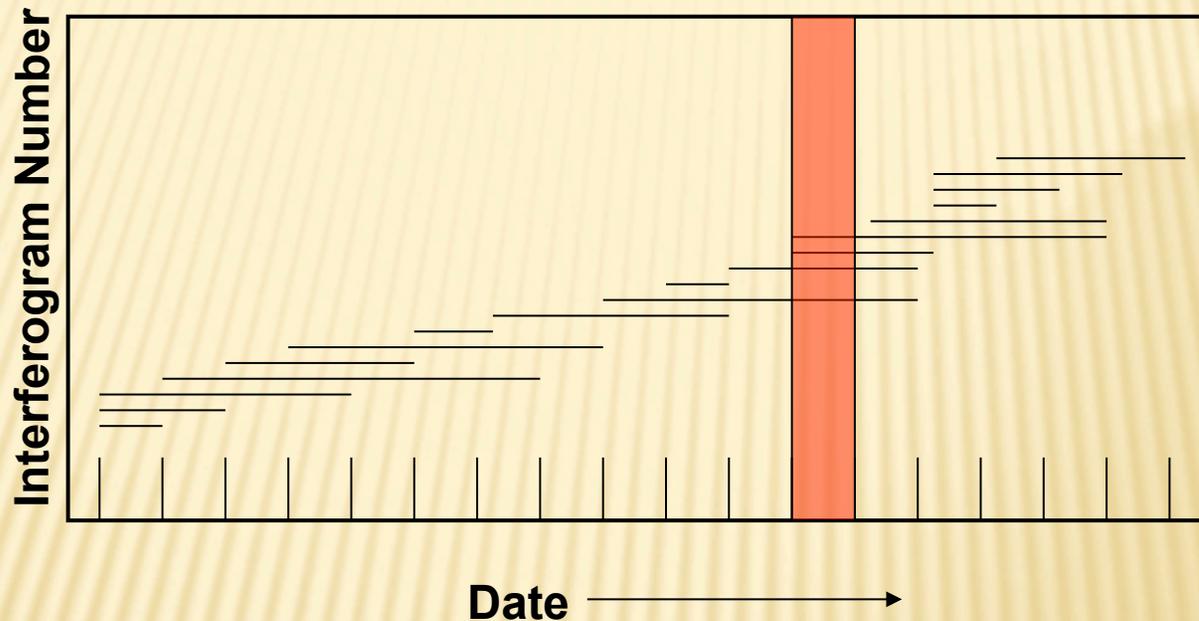
New techniques: Time series of interferograms

The Basic Idea...



New techniques: Time series of interferograms

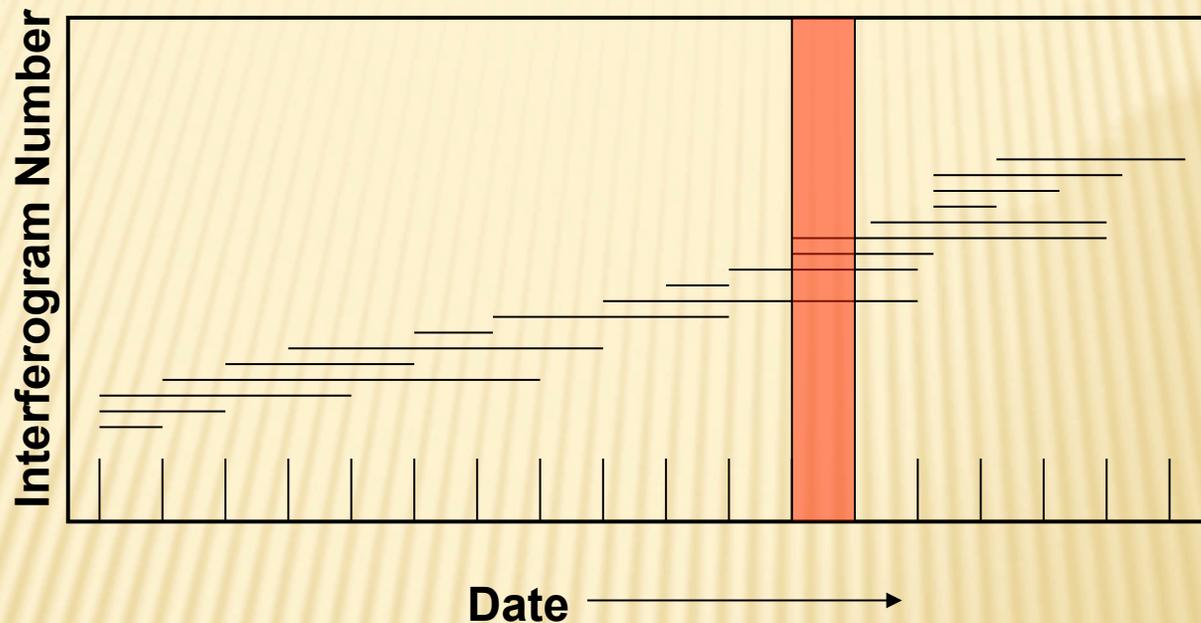
The Basic Idea...



A stack of interferograms provides multiple constraints on a given time interval

New techniques: Time series of interferograms

The Basic Idea...



Goal: Solve for the deformation history that, in a least-squared sense, fits the set of observations (i.e., interferograms),

Many different methods (e.g., Lundgren et al. (2001), Schmidt & Burgmann, 2003), but SBAS (Berardino et al. (2002)) is perhaps most common one

Persistent scatterers (PS or PSInSAR)

- **Select pixels with stable scattering behavior over time**
- **Only focus on “good” pixels**

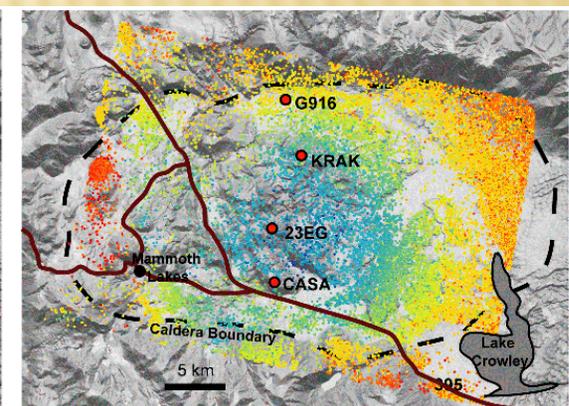
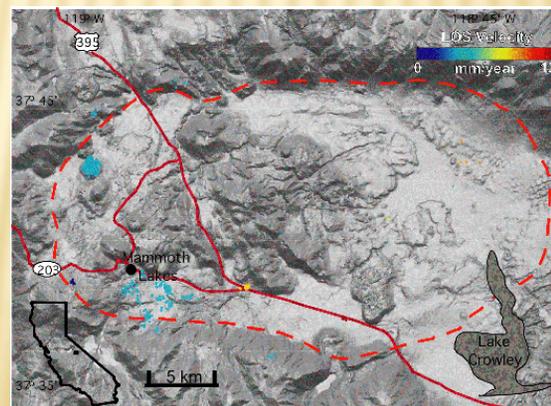
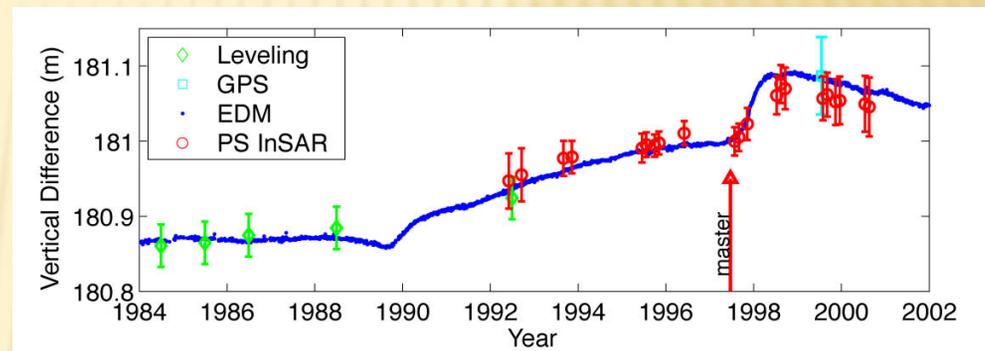
InSAR

- **Spatial coherence @ 1 time**
- **Need neighborhoods of good pts**

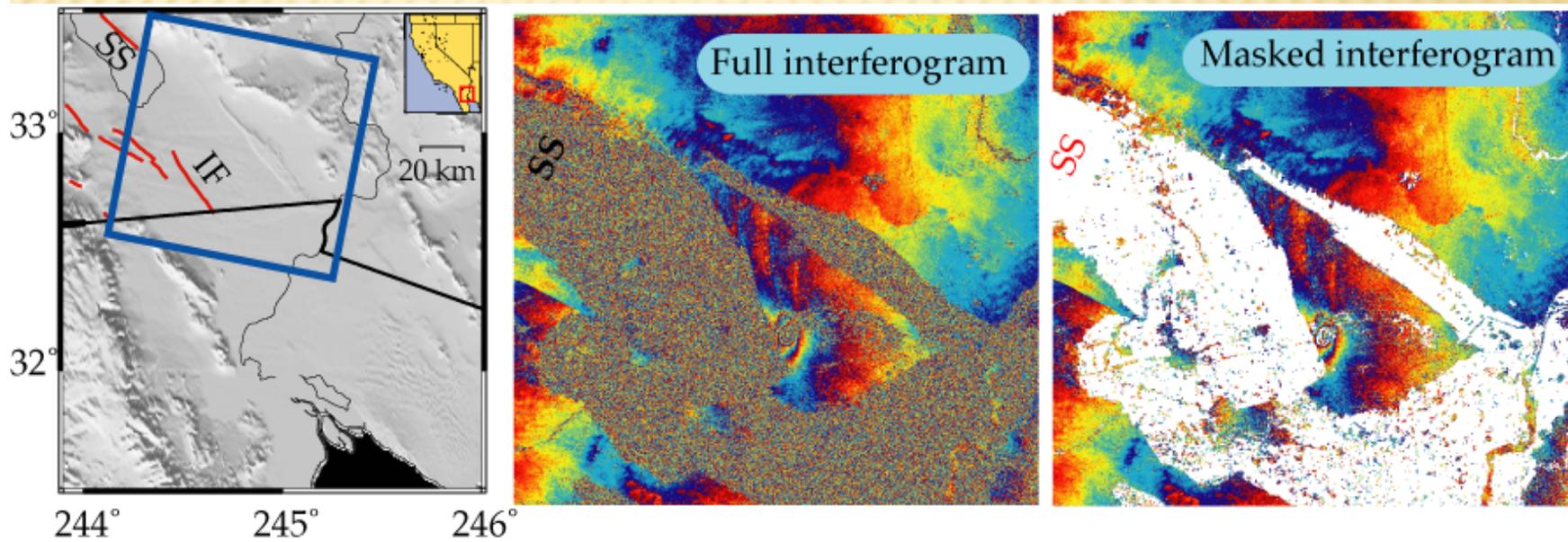
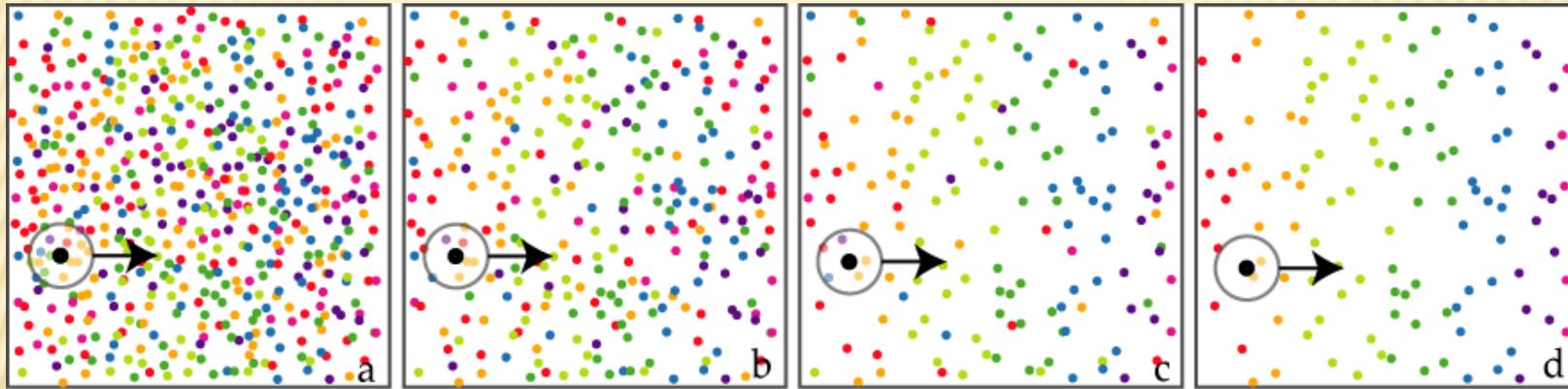
PS

- **Coherence @ 1 point**
- **Need > 15-20 scenes**
- **Added bonus:
DEM errors!**

Long Valley Caldera, Hooper et al. 2004



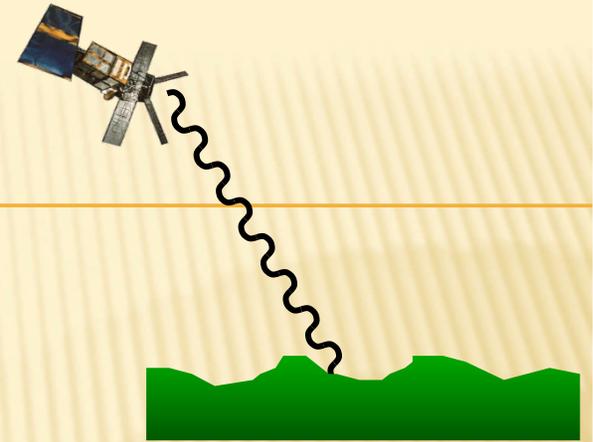
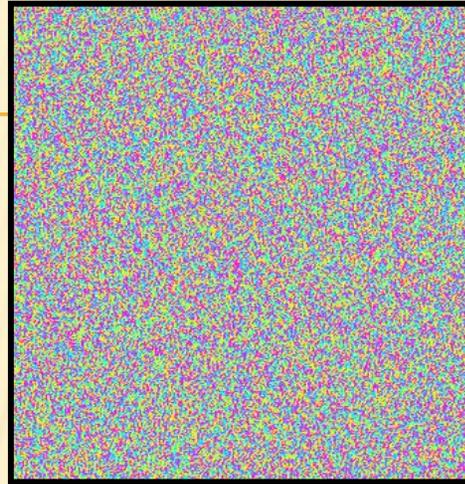
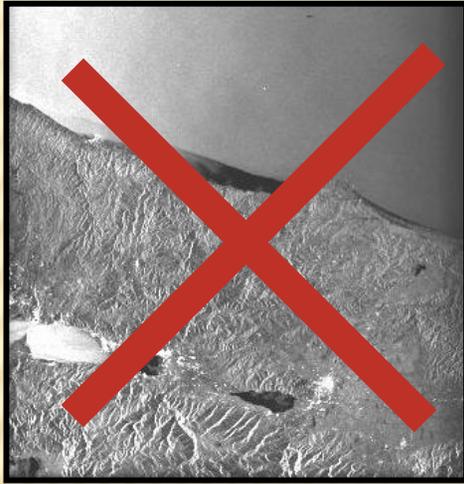
STAMPS METHOD (HOOPER ET AL., 2004)



Example: Imperial Valley, CA From: Rowena Lohman

- InSAR can resolve surface displacements with \sim cm precision, 10s m spatial resolution, and monthly temporal resolution using remote satellites
- Global coverage; day/night, all-weather imaging capabilities

Image A - 12 August 1999



SAR image

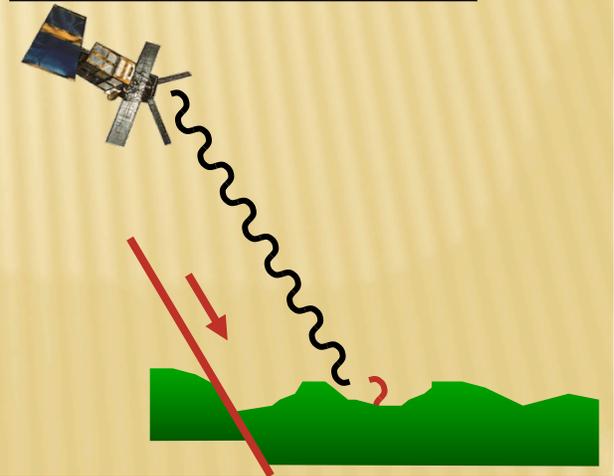
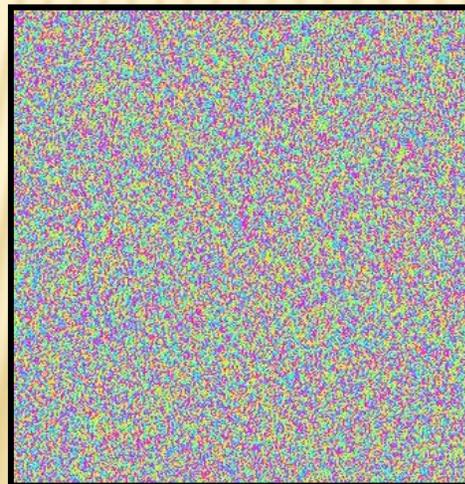
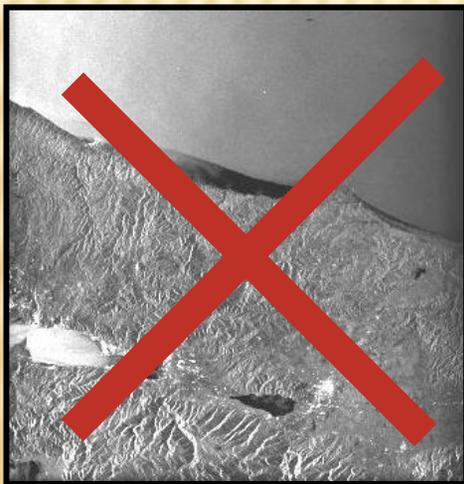
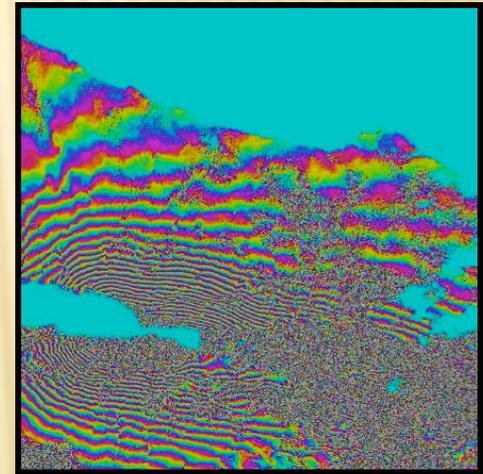
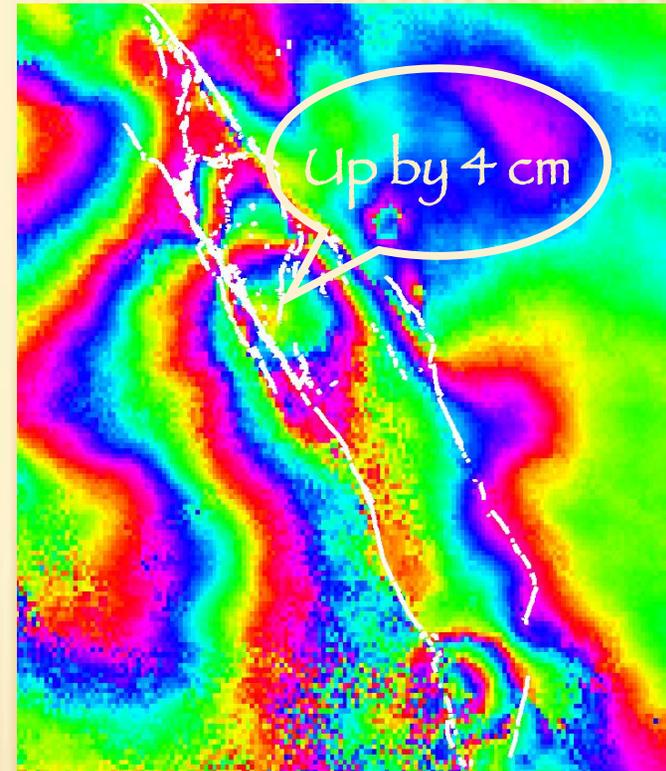
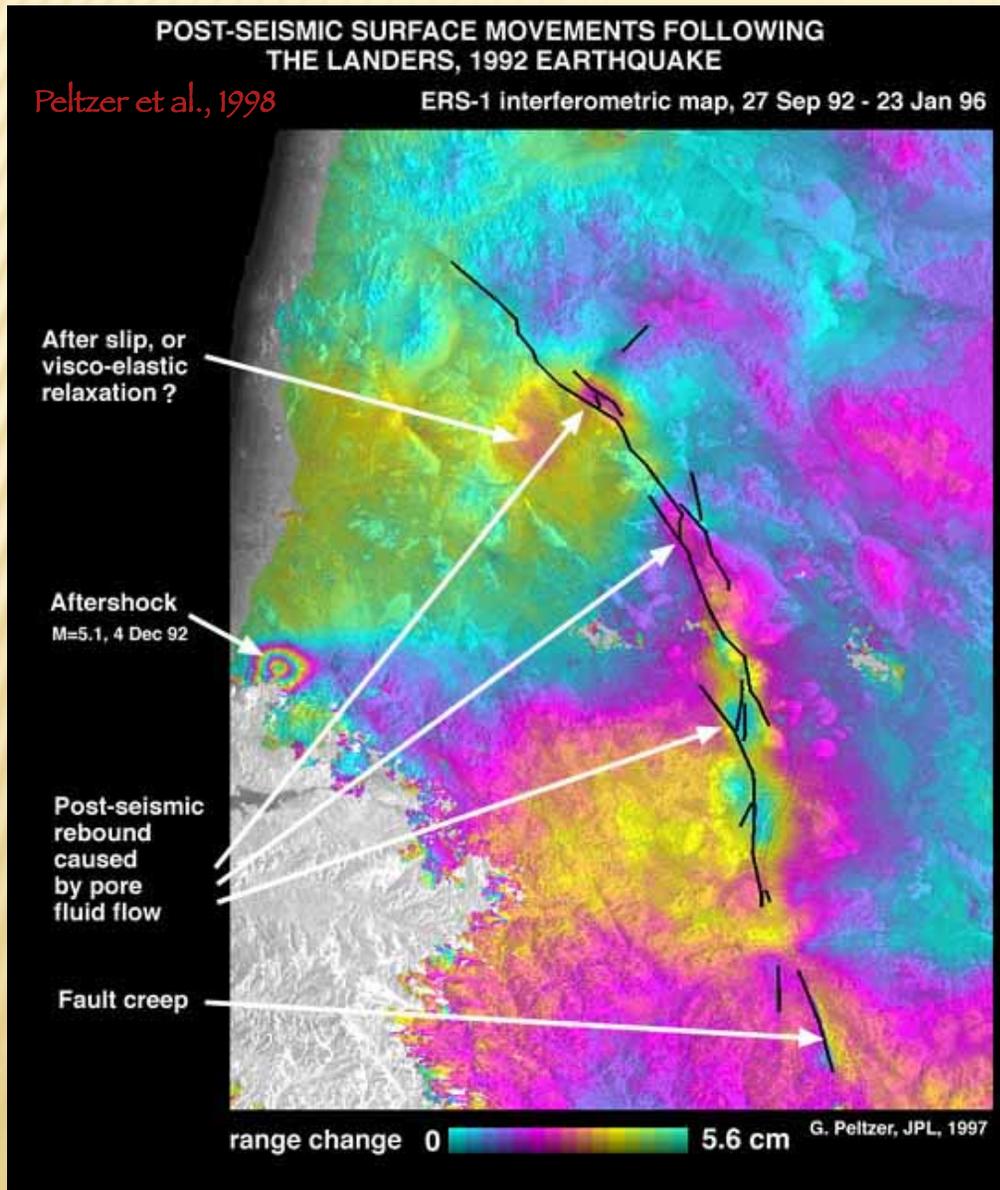


Image B - 16 September 1999

POSTSEISMIC - 1992 LANDERS



Massonnet & Feigl, 1998

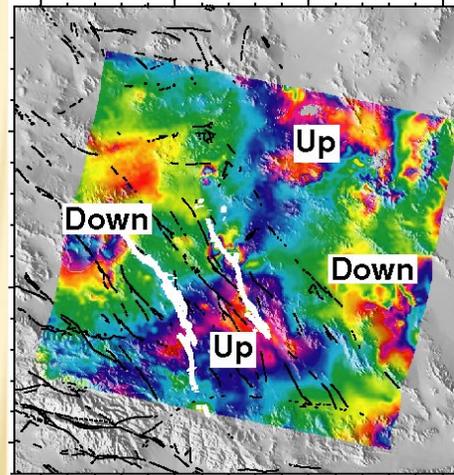
- Releasing stepovers (pull apart) rebound

POSTSEISMIC - 1999 HECTOR MINE

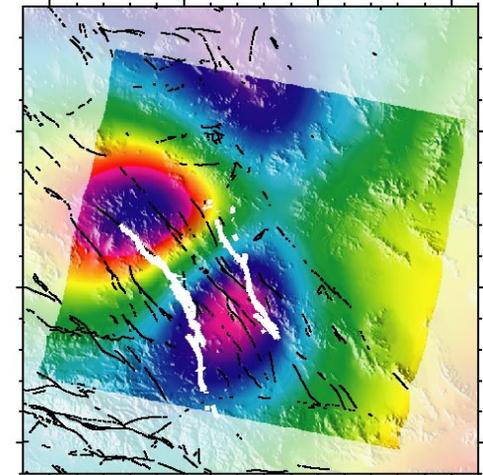
Near field: Shallow afterslip and fault zone collapse?

Jacobs, A., Sandwell, D., Fialko, Y. and L. Sichoix, BSSA, 2002.

Observed Range Change
10/20/99-6/21/00



Upper Mantle Flow Model



0 28.3 mm
0 Range Change 2π

Far field: Upper mantle relaxation?

Pollitz et al., Science, 2001

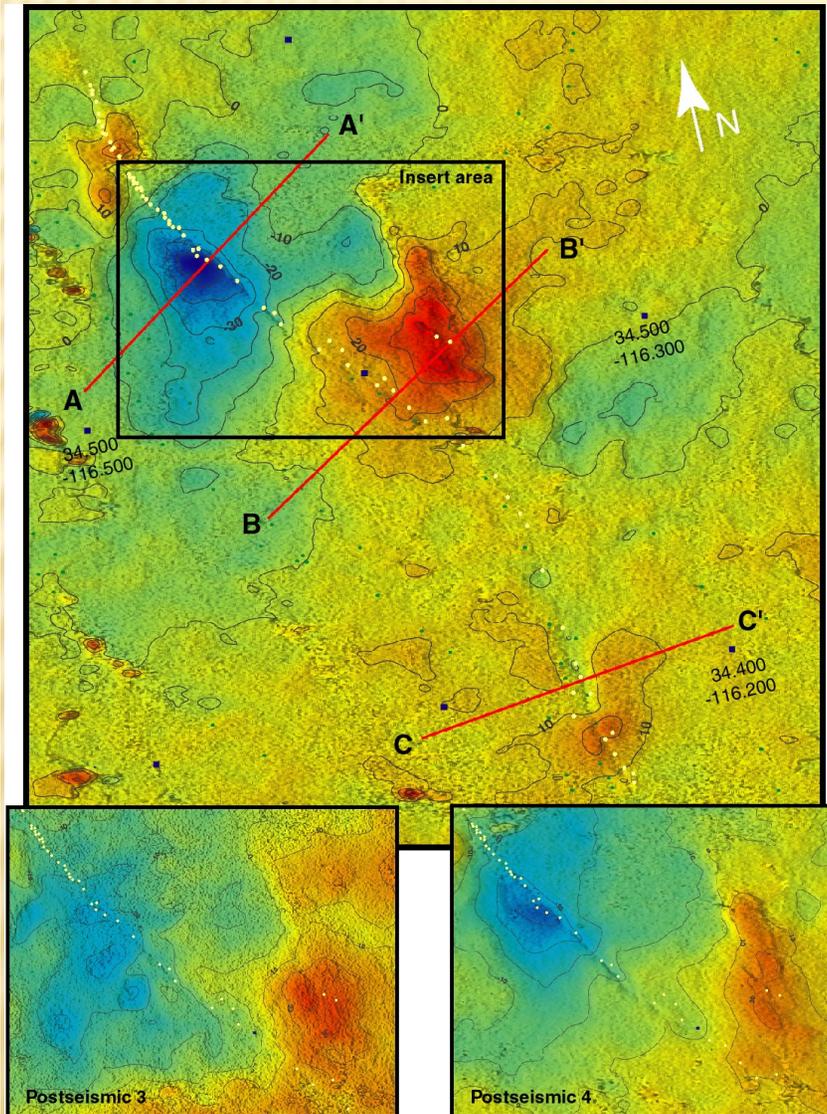
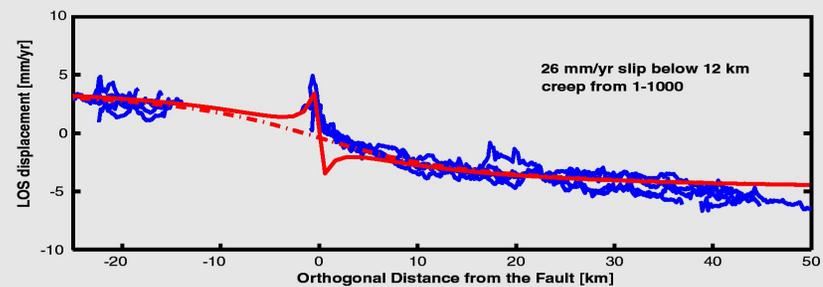
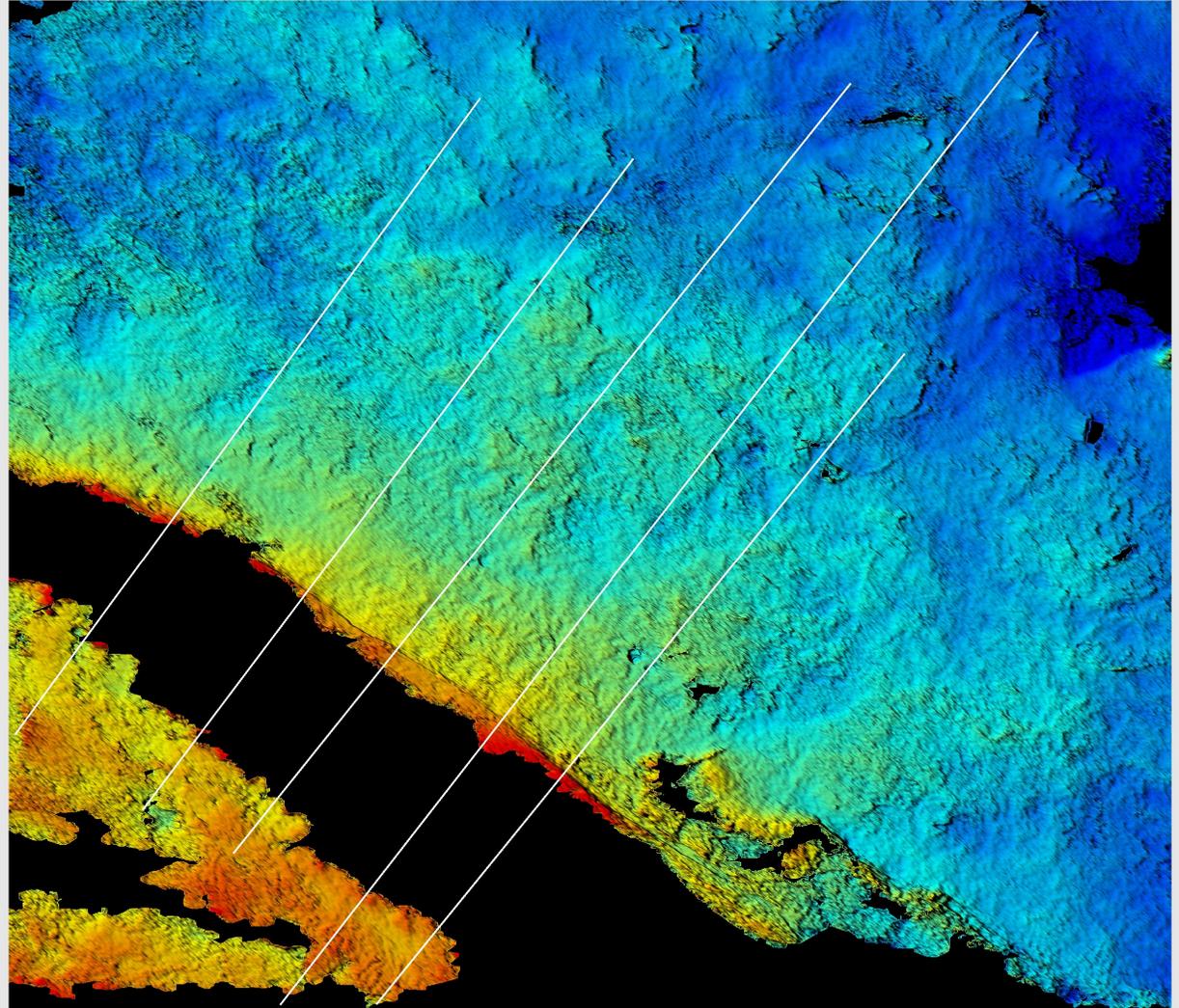
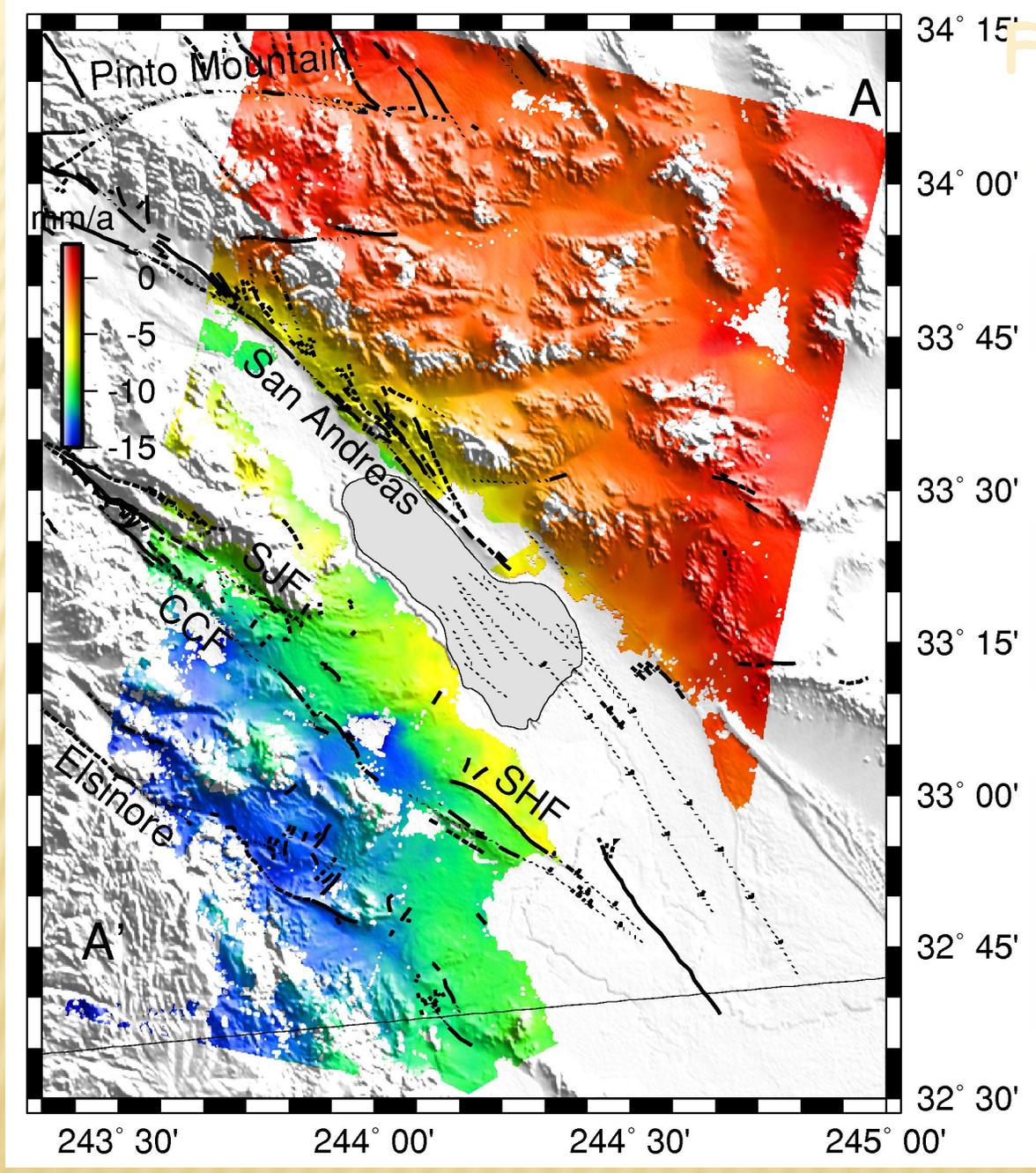


Figure 3

Interseismic ~ Southern SAF

(Lyons, S. and Sandwell, JGR, 2003)

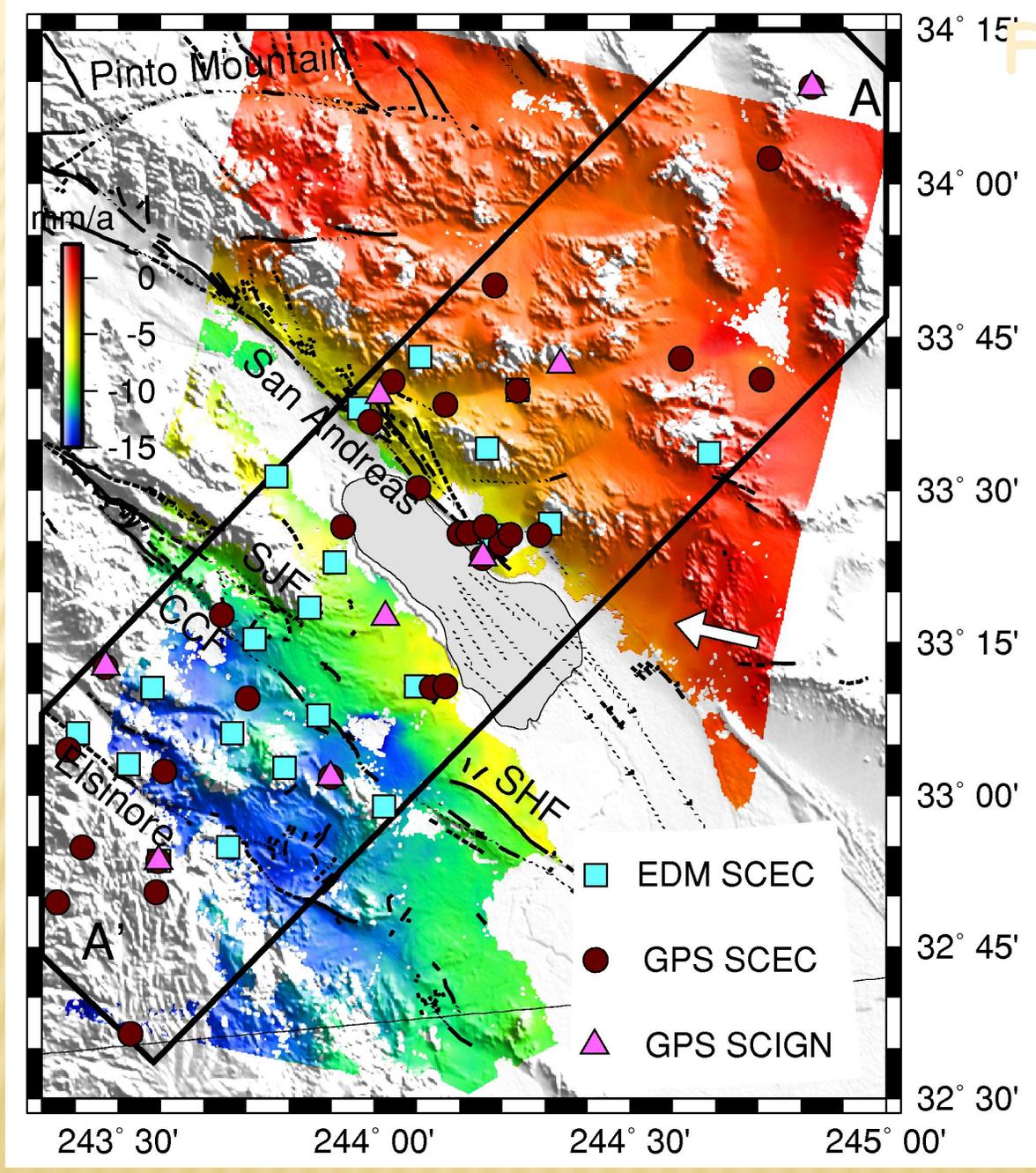




Line of sight
velocities from
stacked InSAR
data

35 interferograms

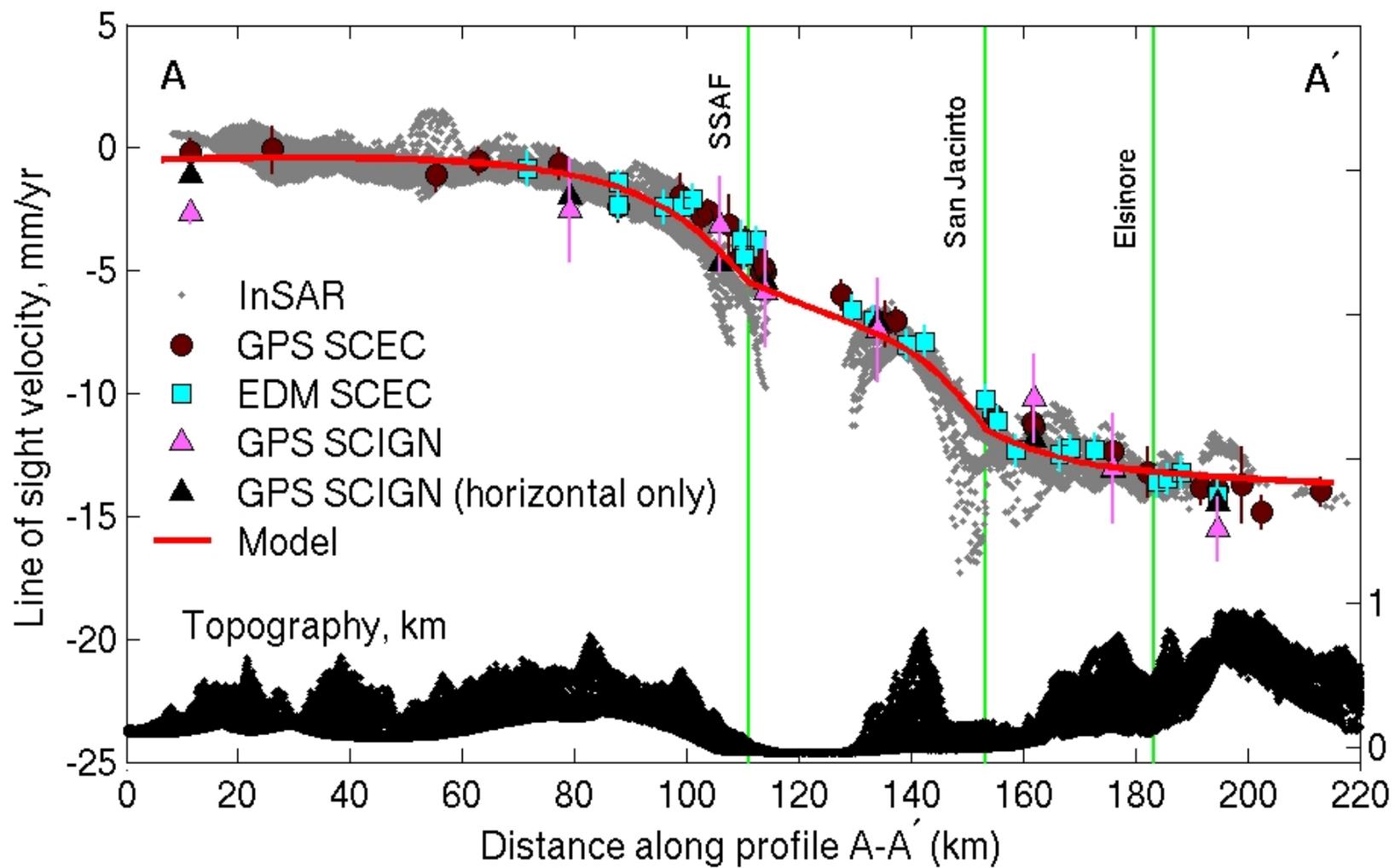
Epoch:
1992-2000



Line of sight
velocities from
stacked InSAR
data

35 interferograms

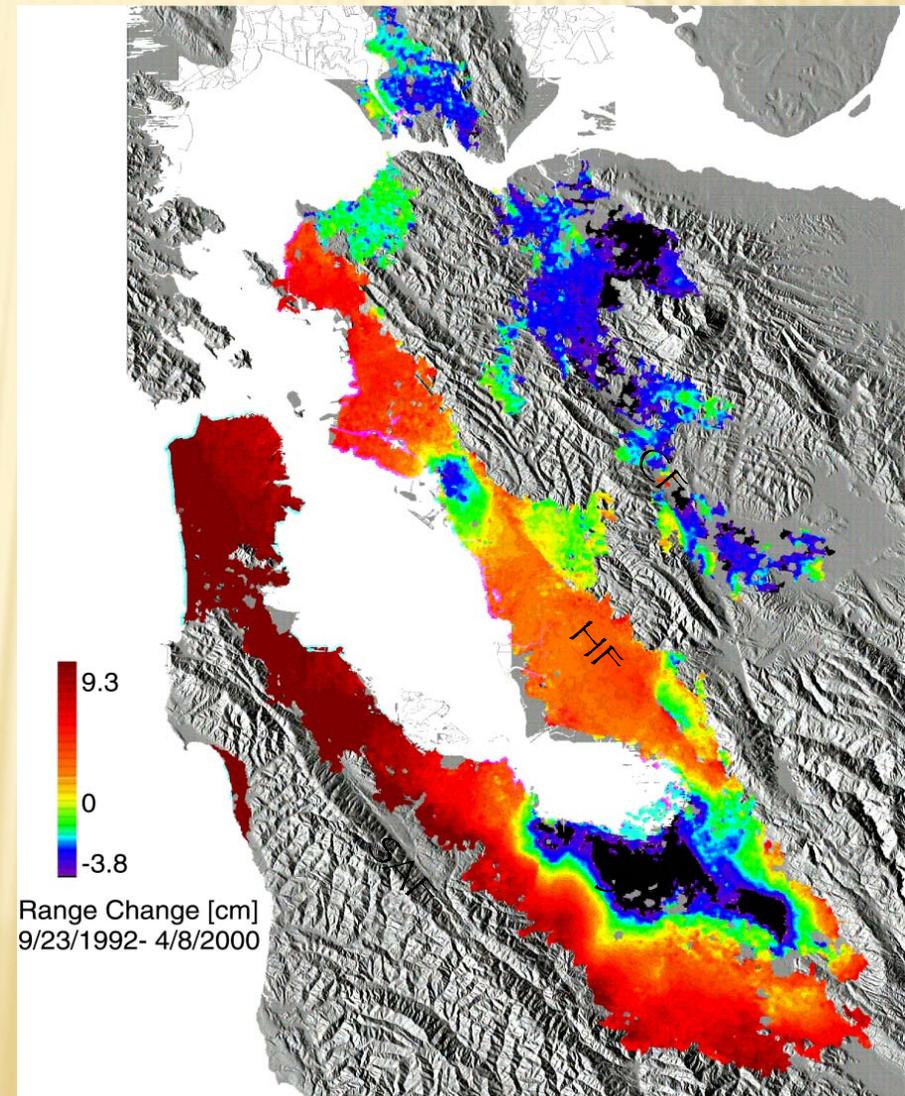
Epoch:
1992-2000



INTERSEISMIC - SAN FRANCISCO BAY AREA

(Burgmann et al., Science, 2000)

- Eight-year interferogram showing:
 - Elastic strain accumulation across San Andreas fault system
 - Creep along Hayward fault
 - Rebound of Santa Clara Valley from aquifer recharge

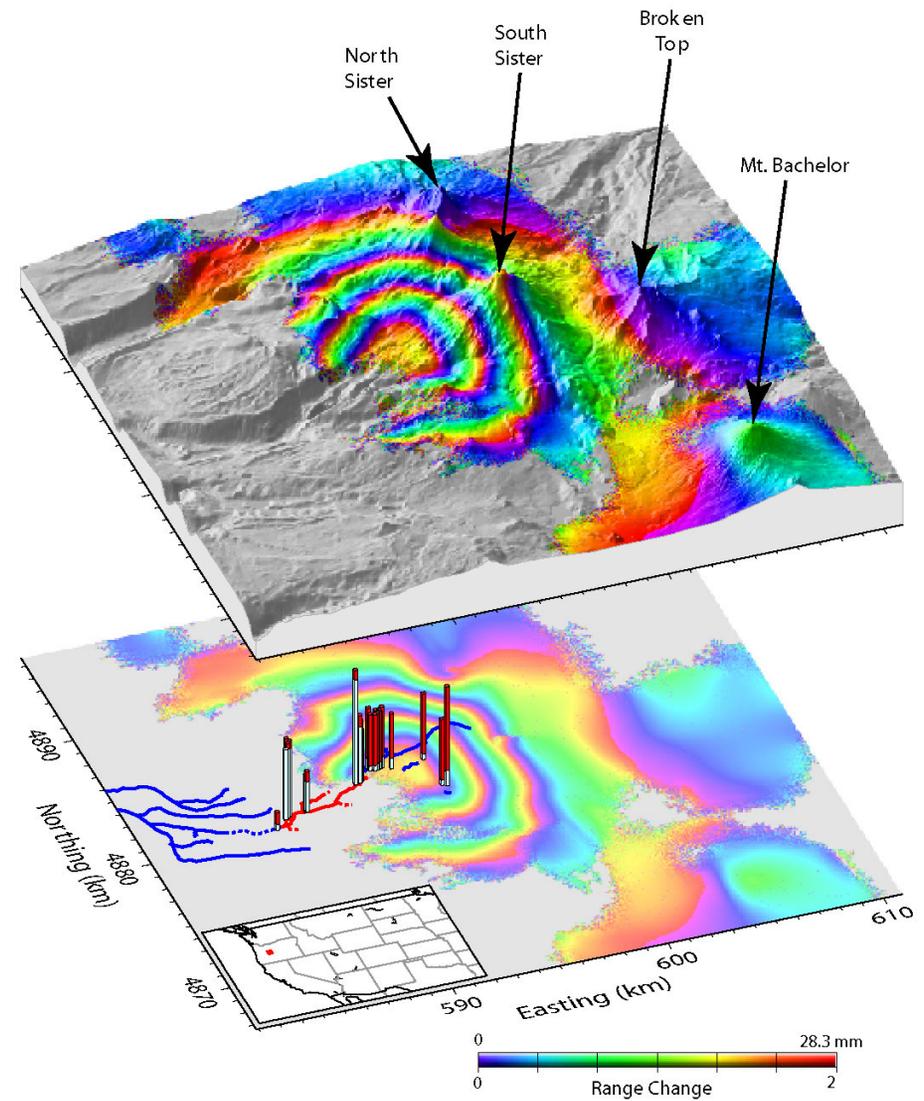


MAGMATIC ACTIVITY: DETECTION AND INTERPRETATION

- ✘ Location and geometry of magma reservoirs
- ✘ Dynamics of magma supply
- ✘ Precursory phenomena
 - + Intrusion
 - + Eruption

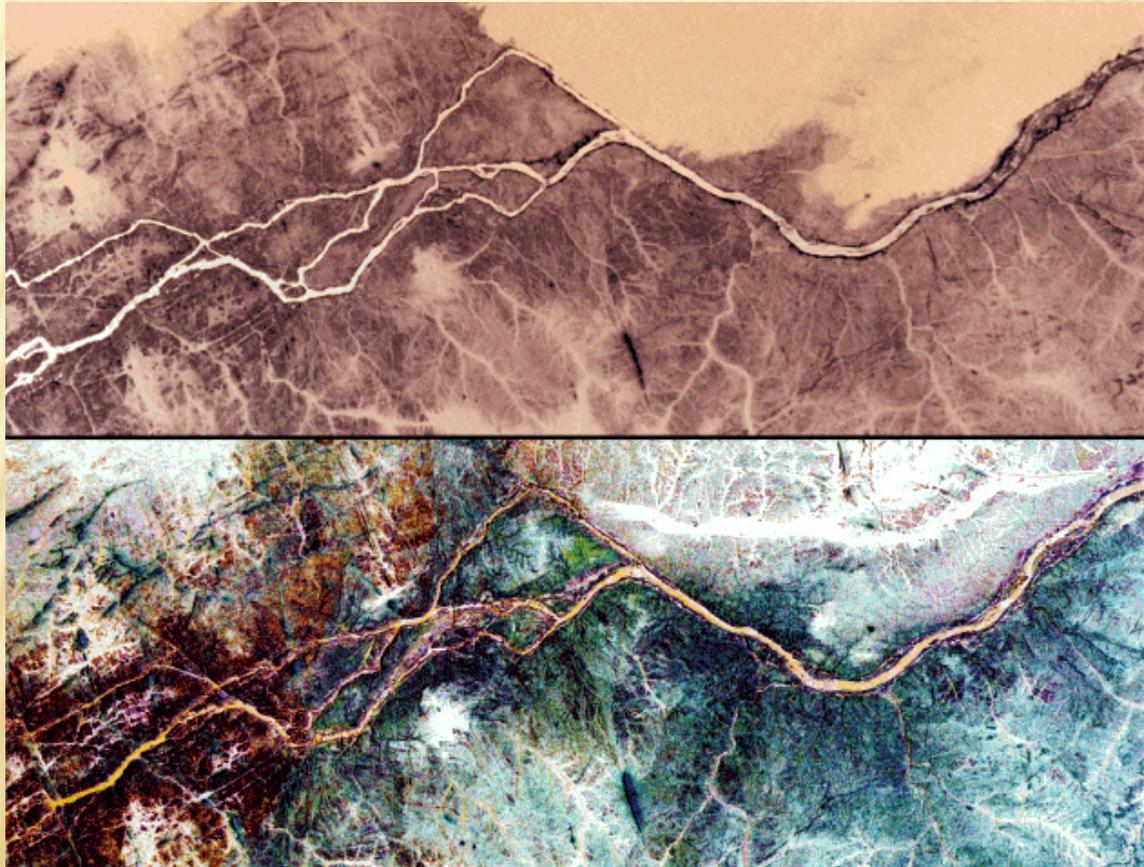
VOLCANIC INFLATION THREE SISTERS, CASCADE RANGE

(Wicks et al., Science, 2002)

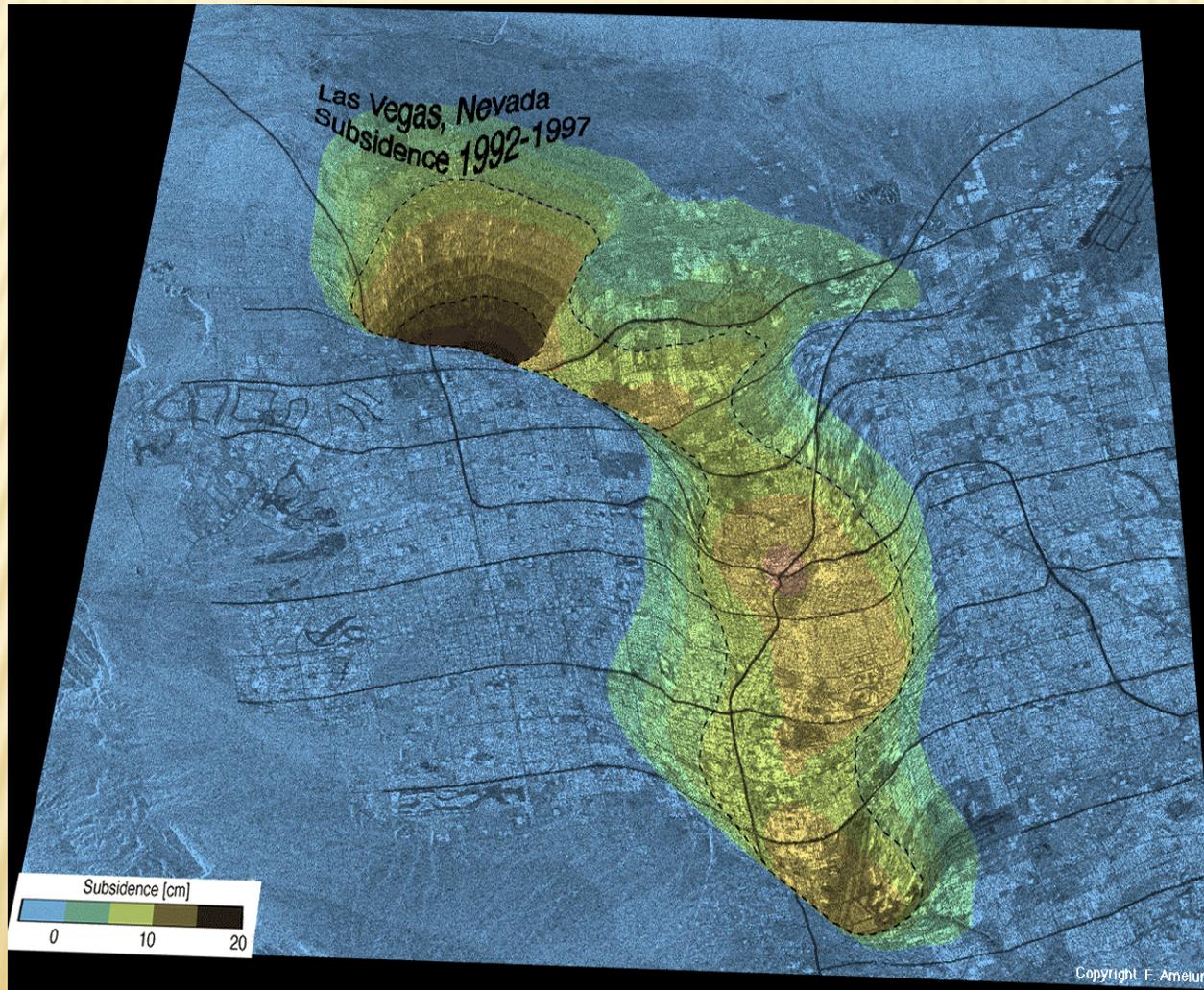


SIR-C image of Nile Paleochannel, Sudan

- The top image is a photograph taken with color infrared. The radar image at the bottom is a SIR-C/X-SAR image. The thick, white band in the top right of the radar image is an ancient channel of the Nile that is now buried under layers of sand. This channel cannot be seen in the photograph and its existence was not known before this radar image was processed.



SUBSIDENCE DUE TO GROUND WATER WITHDRAWAL. (PRESENTATION HAS EXTREME VERTICAL EXAGGERATION)



REVIEW: WILL INSAR WORK FOR YOU?

- × What is the local rate of deformation?
 - + Sensitivity of single igram $\sim 1\text{cm}$
 - + How many years to get signal this big and will it be overcome by noise?
 - + Can you stack several igrams together?
- × What is the scale of deformation?
 - + Pixel size $\sim 10\text{m}$, but generally need to average many together
 - + Image size is $\sim 100\text{ km}$, but if too broad worry about precision of orbits
- × What is the local noise?
 - + How much vegetation/precipitation/water vapor/human cultivation?
 - + Can you only make igrams with data from the same seasons?
 - + Can you get L-band data and find persistent scatterers?
- × What data is available?
- × Is there data from multiple satellites and/or imaging geometries?
- × Is a digital elevation model available?
- × Do you need rapid response for hazard assessment?

REVIEW: HOW TO SET UP INSAR CAPABILITY?

1) Establish access to data

- Main sources: see next slide

- How? Can be purchased commercially. Lower cost/no-cost data available with restrictions. In Europe, through ESA. In U.S., through ASF and UNAVCO. Some foreign access is allowed to UNAVCO

Can useful interferograms be made with available data? Worry about ground conditions, radar wavelength, frequency of observations, perpendicular baseline, availability of advanced processing techniques

2) Purchase/Install software to process and visualize data

- Open source: ROI_PAC, DORIS, RAT and IDIOT (TU Berlin)
- Commercial: Gamma, TR Europa, Vexcel/Atlantis, DIAPASON, SARscape

3) Download/create DEM (SRTM is only +/- 60 degrees latitude, but ASTER G-DEM in 2009)

4) Download precise orbital information & instrument files (Only ERS & Envisat)

5) Interpret results, create stacks, time series, persistent scatterers. May need to buy/download/create new software

6) Publish new discoveries and software tools!

FOR MORE INFORMATION:

- Good overview of classical & space based geodesy (but no InSAR): John Wahr's online textbook <http://samizdat.mines.edu/geodesy>
- Introductions to InSAR:
 - 2 page overview from Physics Today http://www.geo.cornell.edu/eas/PeoplePlaces/Faculty/matt/vol59no7p68_69.pdf
 - Overviews of applications: Massonnet & Feigl, Rev. Geophys., 1998; Burgmann et al., AREPS, 2000.
- More advanced InSAR:
 - The definitive SAR book: Curlander & McDonough, 1990
- More technical reviews: Rosen et al., IEEE 2000; Hanssen's Radar Interferometry book, 2001; Simons & Rosen, Treatise on Geophysics, 2007;
 - Time series analysis: Berardino et al., IEEE, 2002; Schmidt & Burgmann, JGR, 2003
- Persistent scatterers: Ferretti IEEE, 2001; Hooper et al., GRL, 2004; Kampes' Persistent Scatterers book, 2006

Absolute Phase Estimation

Problem: Given a set of observations $(\cos \phi_p, \sin \phi_p) \equiv e^{j\phi_p}$, determine ϕ_p (up to a constant), for $p \in \mathcal{V} \equiv \{1, \dots, n\}$

Given that $e^{j\phi}$ is 2π -periodic, absolute phase estimation is a nonlinear ill-posed problem

Continuous/discrete flavor: $\phi = W[\phi] + 2k\pi \quad W : \mathbb{R} \rightarrow [\pi, \pi[$

Phase Unwrapping (PU)



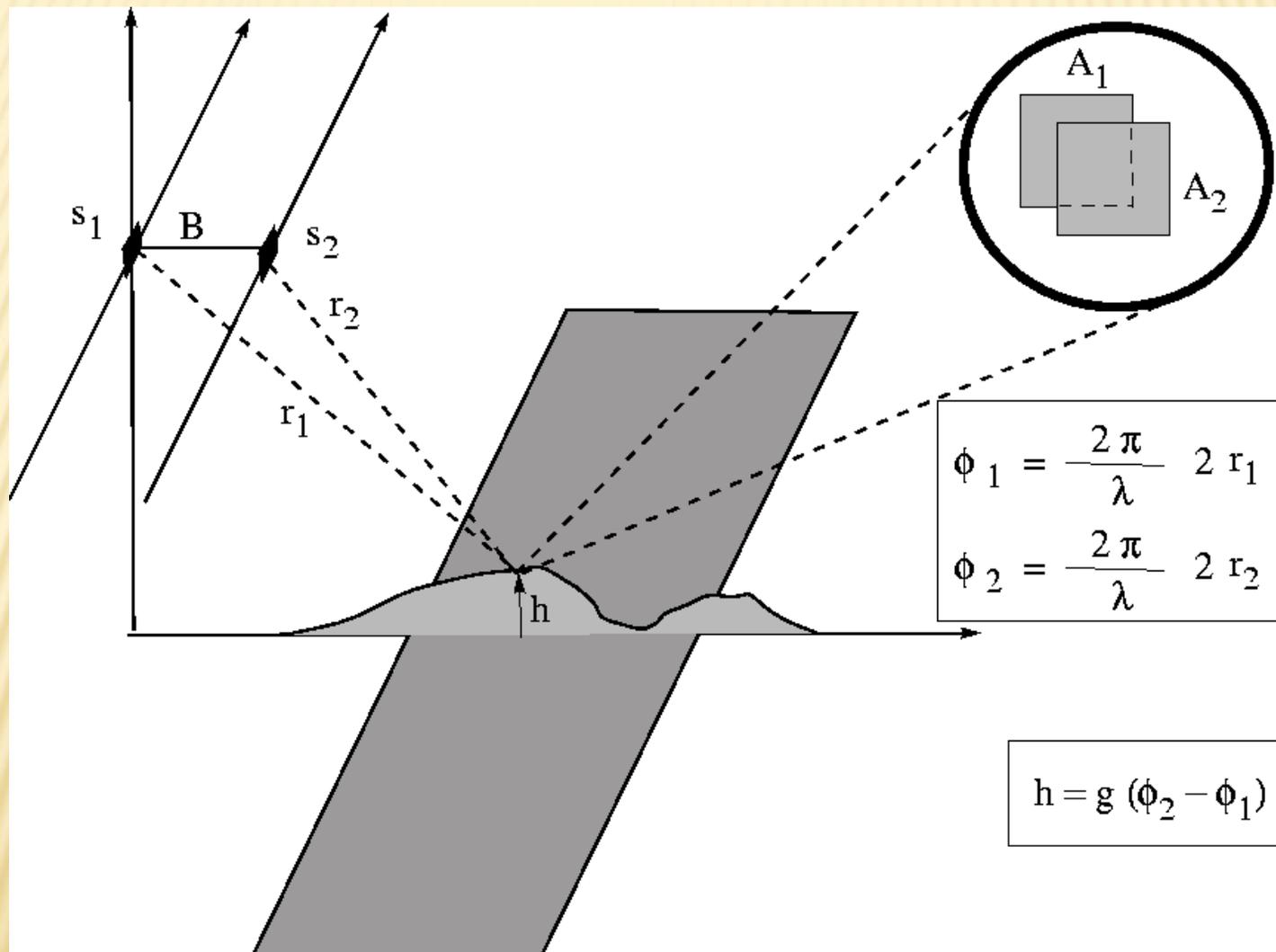
Estimation of $k \in \mathbb{Z}$

Phase Denoising (PD)



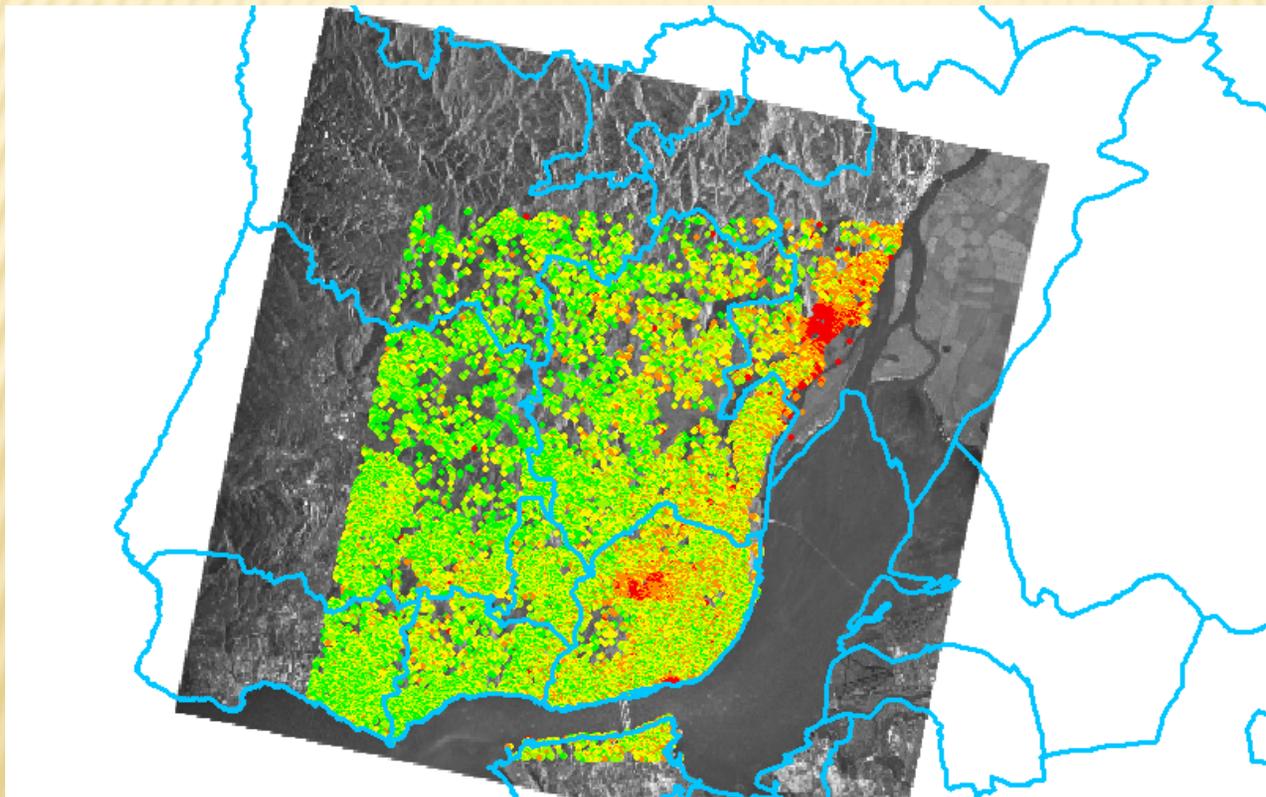
**Estimation of $W(\phi) \in [\pi, \pi[$
(wrapped phase)**

Absolute Phase Estimation in InSAR (Interferometric SAR)



InSAR Problem: Estimate $\phi_2 - \phi_1$ from signals read by s_1 and s_2

Differential Interferometry

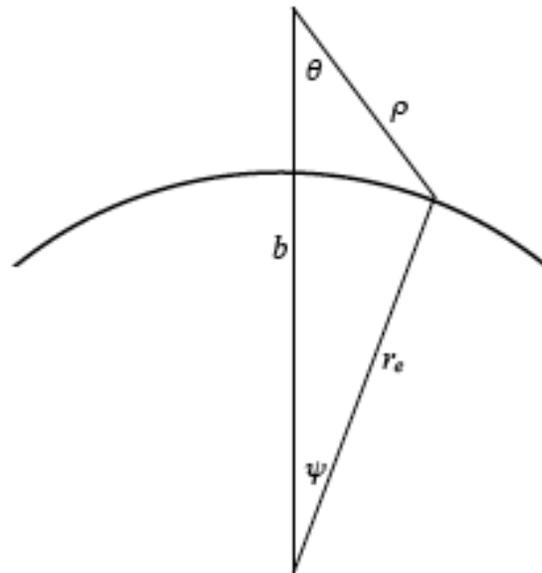


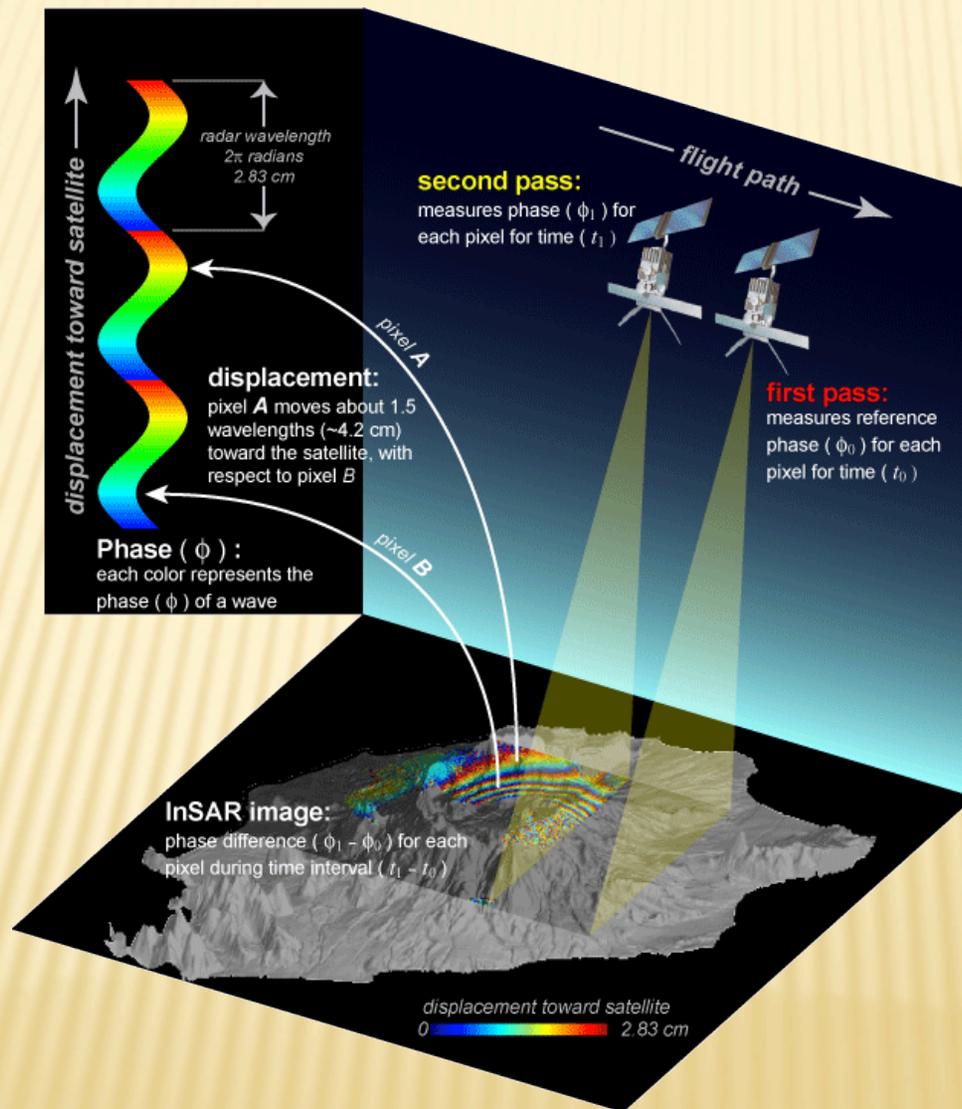
Height variation

7 mm/year

-17 mm/year

The look angle of the radar depends only on the local earth radius r_e , the range to the sphere ρ , and the height of the spacecraft above the center of the Earth b .





Look
 Direction

