Earth Science Applications of Space Based Geodesy

DES-7355

Tu-Th 9:40-11:05

Seminar Room in 3892 Central Ave. (Long building)

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Office Hours – Wed 14:00-16:00 or if I’ m in my office.

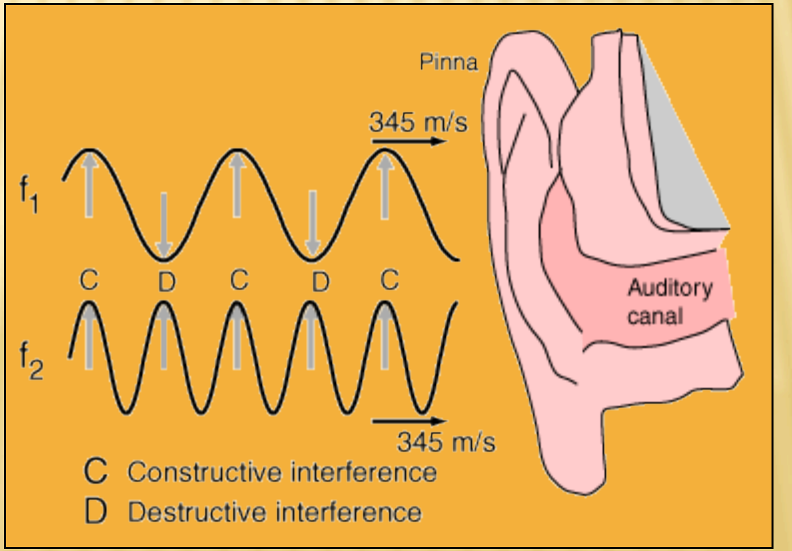
http://www.ceri.memphis.edu/people/smalley/ESCI7355/ESCI_7355_Applications_of_Space_Based_Geodesy.html

Class 9

When two sound waves of different frequency approach your ear, the alternating constructive and destructive interference causes the sound to be alternatively soft and loud

-a phenomenon which is called "beating" or producing beats.

-The beat frequency is equal to the absolute value of the difference in frequency of the two waves.



http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html

Beat Frequencies in Sound

The sound of a beat frequency or beat wave is a <u>fluctuating volume</u> caused when you add two sound waves of slightly different frequencies together.

If the frequencies of the sound waves are close enough together, you can hear a relatively slow variation in the volume of the sound.

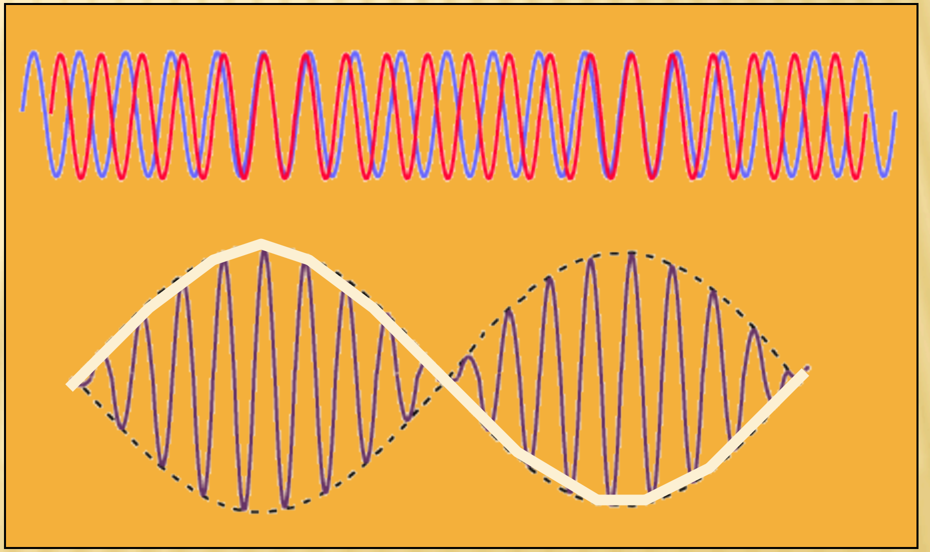
A good example of this can be heard using two tuning forks that are a few Hz apart. (or in a twin engine airplane or boat when the engines are not "synched" = you hear a "wa-wa-wa-wa-… noise)

http://www.school-for-champions.com/science/sound_beat.htm

Beats are caused by the interference of two waves at the same point in space.

$$\cos(2\pi f_1) + \cos(2\pi f_2) = 2A\cos\left(2\pi\frac{f_1 - f_2}{2}\right)\cos\left(2\pi\frac{f_1 + f_2}{2}\right)$$

$$f_{beat} = |f_1 - f_2|$$



Beat -- Frequency of minimia, which happens twice per cycle.

http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html

Note the frequencies are

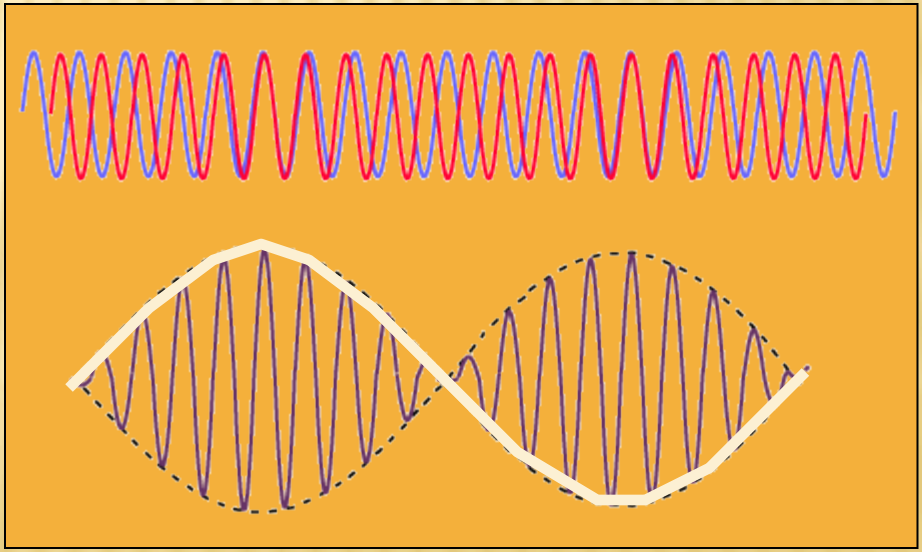
half the difference and

the average of the original frequencies.

$$\cos(2\pi f_1) + \cos(2\pi f_2) = 2A\cos\left(2\pi\frac{f_1 - f_2}{2}\right)\cos\left(2\pi\frac{f_1 + f_2}{2}\right)$$

"Different" than multiplying (mixing) the two frequencies.

(but be careful since sum on LHS is equal to product on RHS)

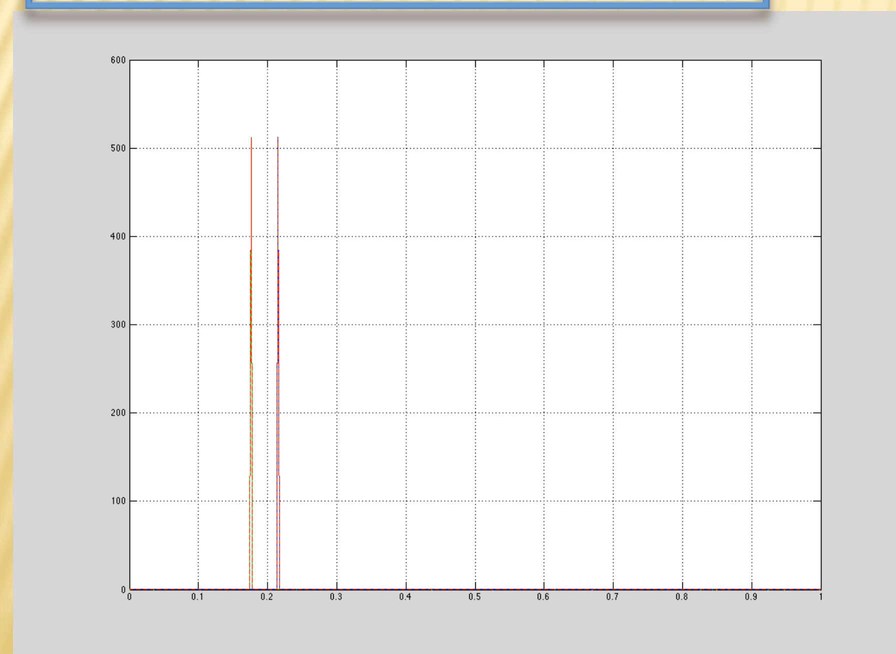


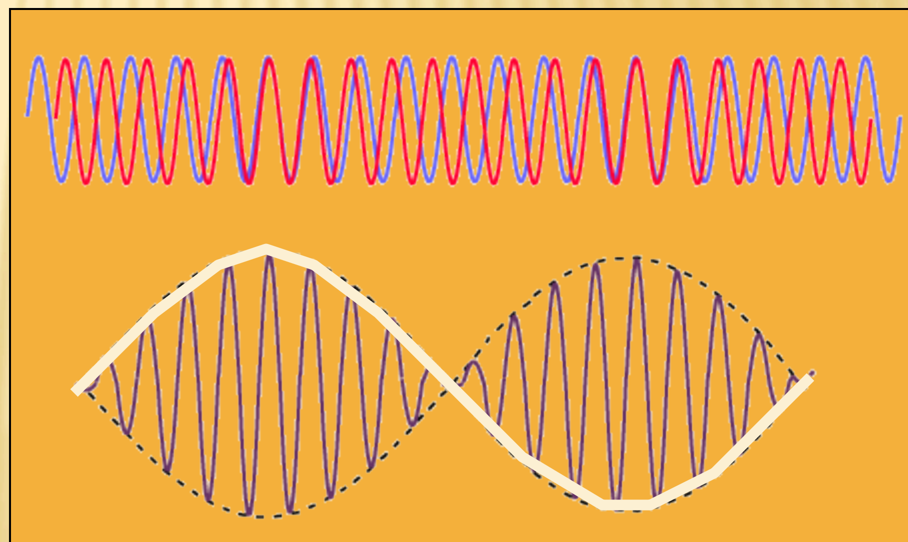
http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html

What if we look at it in the frequency domain – Adding in TD and FD is linear.

So we get same two spikes we would have gotten if we had looked at each separately.

$$\cos(2\pi f_1) + \cos(2\pi f_2) = 2A\cos\left(2\pi\frac{f_1 - f_2}{2}\right)\cos\left(2\pi\frac{f_1 + f_2}{2}\right)$$

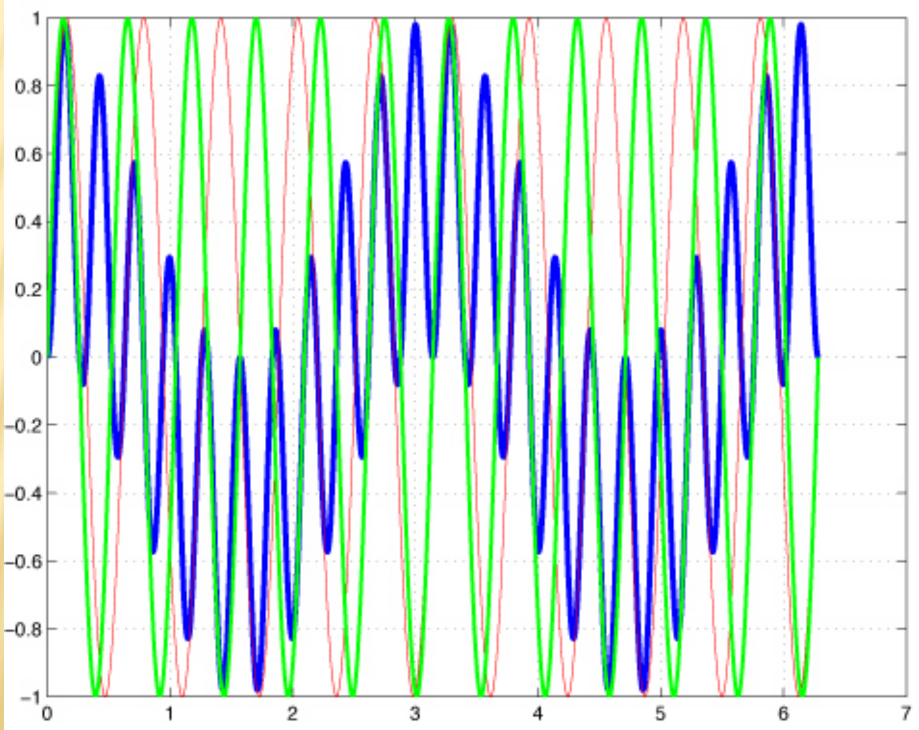




http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html

What if we take the product (multiply, or mix) – i.e. start on the RHS of the last analysis.
(get sum and difference of components, not half)

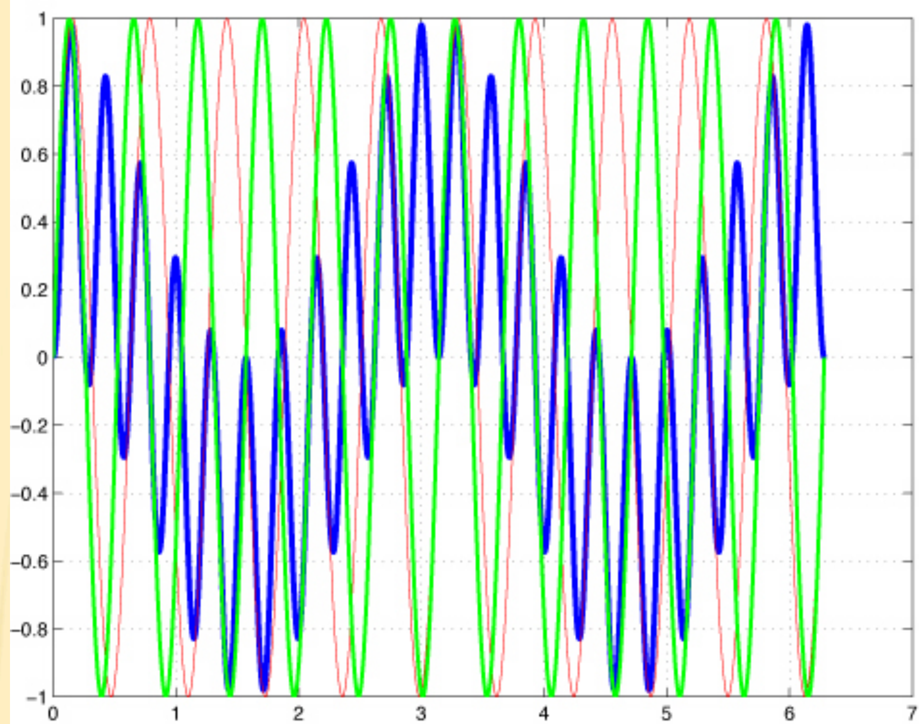
$$\cos\left(2\pi f_1\right) * \cos\left(2\pi f_2\right) = \frac{A\cos\left(2\pi\left(f_1 - f_2\right)\right) + \cos\left(2\pi\left(f_1 + f_2\right)\right)}{2}$$

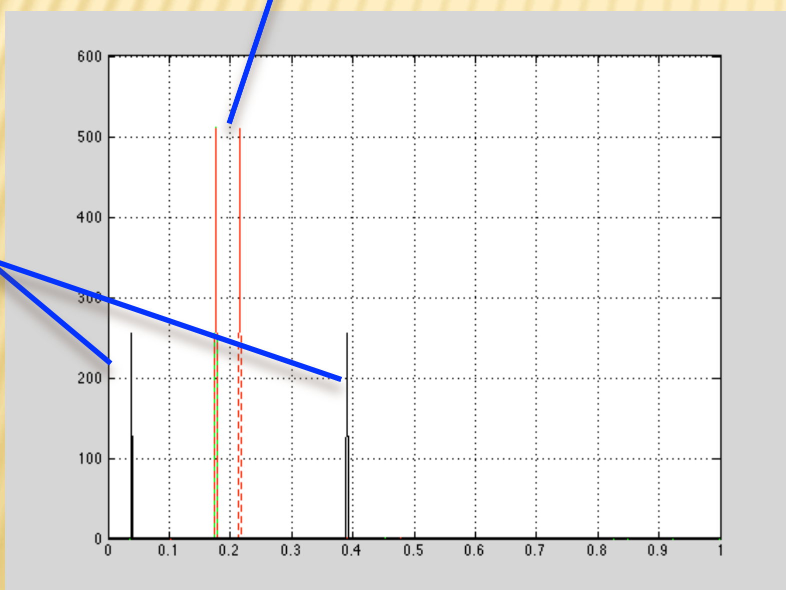


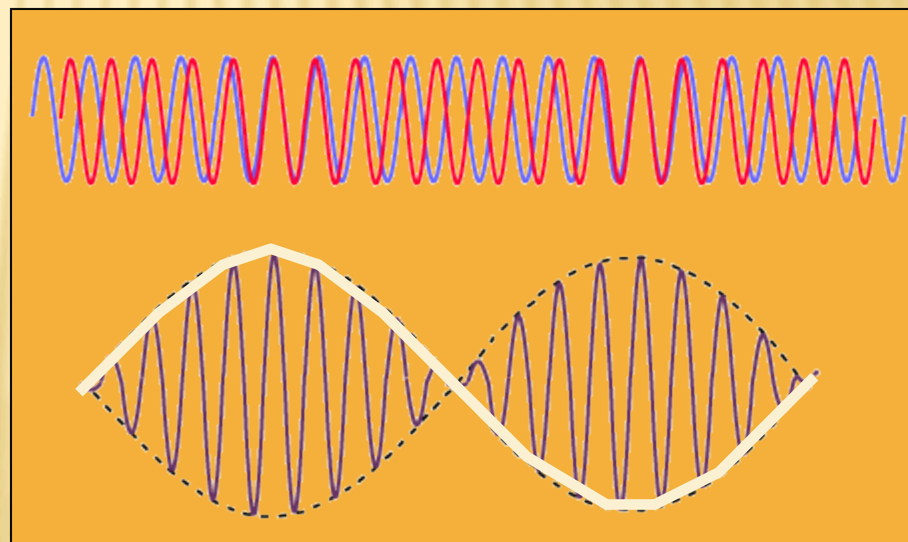
http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html

Product (multiply, or mix)

Compare FT of product to those of components and sum.







http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html

In terms of our GPS signals we get

(we are now mixing – multiplying, not adding.

G= GPS signal, R= Reference signal.)

$$R(t) \otimes G(t) = G_0 \sin\left(2\pi\phi_G(t)\right) \times R_0 \sin\left(2\pi\phi_R(t)\right)$$

$$R(t) \otimes G(t) = \frac{G_0 R_0}{2}\left(\cos\left(2\pi\left(\phi_R(t) - \phi_G(t)\right)\right)\cos\left(2\pi\left(\phi_R(t) + \phi_G(t)\right)\right)\right)$$

Note this is in terms of phase, $\phi(t)$, not frequency

(“usual” presentation; ωt, produces phase)

"Filter" to remove high frequency part

$$\left(\phi_R(t) + \phi_G(t)\right)$$

leaving beat signal

$$B(t) = \frac{G_0 R_0}{2} \cos\left(2\pi\left(\phi_R(t) - \phi_G(t)\right)\right)$$

$$B(t) = \frac{G_0 R_0}{2} \cos\left(2\pi\phi_B(t)\right)$$

if you differentiate ϕ_B

you find the

beat <u>frequency</u>

the difference between the two frequencies

(actually one wants to take the absolute value)

– as we found before

$$\frac{d\phi_B(t)}{dt} = \frac{d\phi_R(t)}{dt} - \frac{d\phi_G(t)}{dt}$$

$$f_B = f_R - f_G$$

If the receiver copy of the signal has the same code applied as the satellite signal -

This discussion continues to hold (the -1's cancel)

(one might also worry about the Doppler shift effect on the codes, but this effect is second order)

If the receiver copy of the signal does not have the code applied (e.g. – we don't know the P code)

then this discussion will not work (at least not simply)

There are essentially two means by which the carrier wave can be recovered from the incoming modulated signal:

Reconstruct the carrier wave by removing the ranging code and broadcast message modulations.

Squaring, or otherwise processing the received signal without using a knowledge of the ranging codes.

http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap3/323.htm

To reconstruct the signal, the ranging codes (C/A and/or P code)

must be known.

The extraction of the Navigation Message can then be easily performed by reversing the process by which the bi-phase shift key modulation was carried out in the satellite.

http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap3/323.htm

In the squaring method no knowledge of the ranging codes is required.

The squaring removes the effects of the -1's

(but halves the wavelength and makes the signal noisier)

More complex signal processing is required to make carrier phase measurements on the L2 signal under conditions of Anti-Spoofing (don't know P-code).

http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap3/323.htm

As mentioned earlier:

can arbitrarily add $N(2\pi)$ to phase

and get same beat signal

This is because we have no direct measure of the "total" (beat) phase

$$\Phi + N = \phi_R - \phi_G$$

(argument is $2\pi\phi$, so no 2π here)

$$\Phi + N = \phi_R - \phi_G$$

GPS receiver records ***Φ***

total number of (beat) cycles since lock on satellite

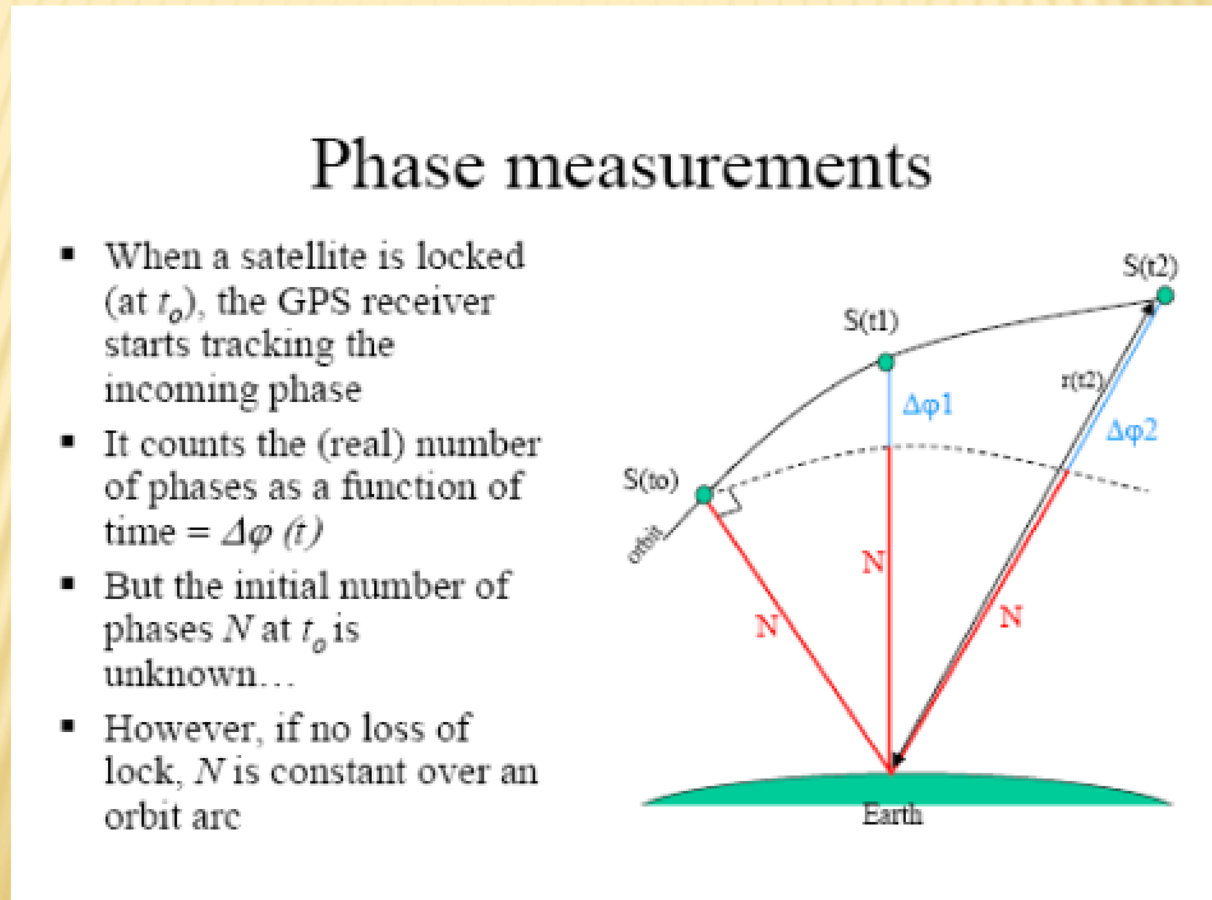
N is fixed (as long as lock on satellite is maintained)

N is called the "ambiguity" (or "integer ambiguity")

It is an integer (theoretically)

If loose lock – cycle slip, have to estimate new N.

Making a few reasonable assumptions we can interpret N geometrically to be the number of <u>carrier wavelengths</u> between the receiver (when it makes the first observation) and the satellite (when it transmitted the signal)



Blewitt, Basics of GPS in "Geodetic Applications of GPS"

From E. Calais

How to use (beat) phase to measure distance?

phase -> clock time -> distance

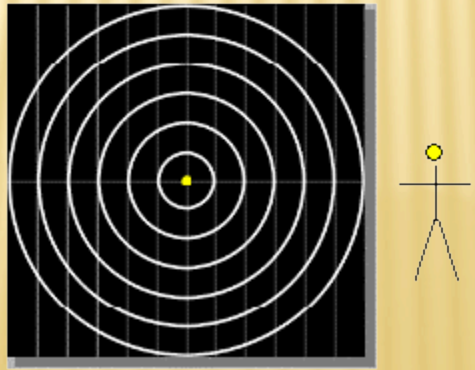
Phase to velocity and position

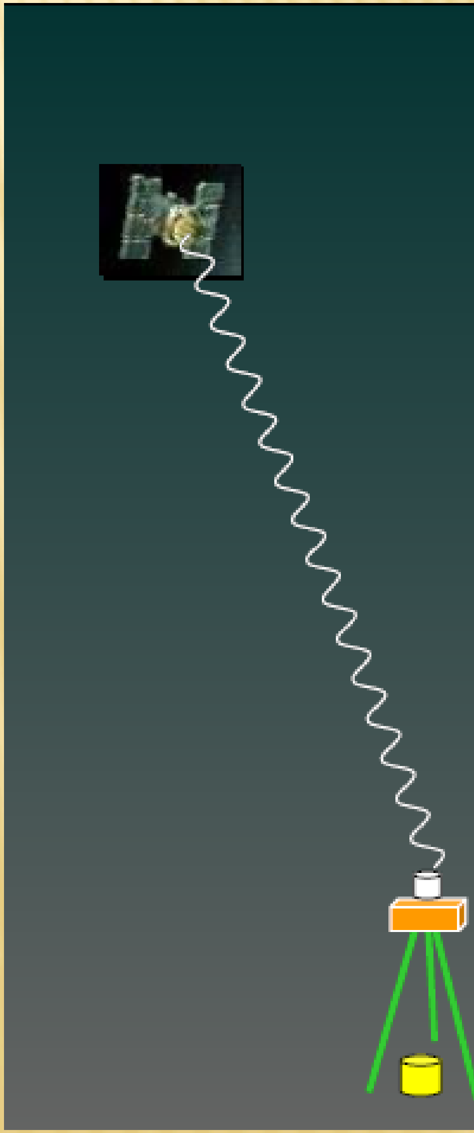
Consider a fixed transmitter and a fixed receiver

Receiver sees constant rate of change of phase (fixed frequency) equal to that of the transmitter

$$\Phi(t) = \phi_0 t + (N)$$

Integrated phase increases linearly with time





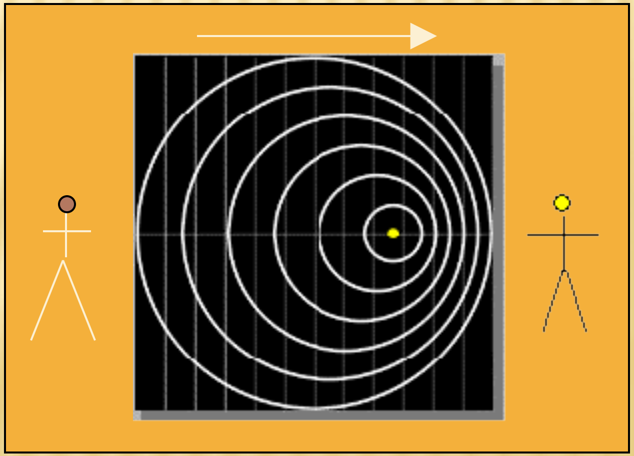
http://www.npwrc.usgs.gov/perm/cranemov/location.htm
http://electron9.phys.utk.edu/phys135d/modules/m10/doppler.htm

Next consider a transmitter moving on a line through a fixed receiver

Receiver again sees a constant rate of change of phase (frequency) – but it is no longer equal to that of the transmitter

$$\Phi(t) = \phi' t + (N)$$

See lower frequency when XTR moving away



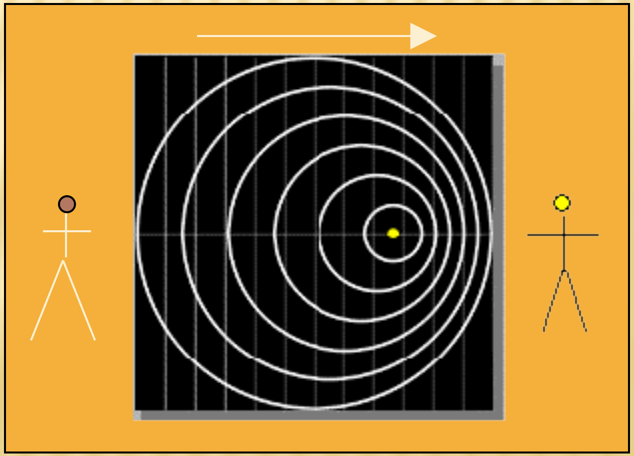
See higher frequency when XTR moving towards

http://electron9.phys.utk.edu/phys135d/modules/m10/doppler.htm

The change in the rate of phase change (fixed change in frequency) observed at receiver, with respect to stationary transmitter, is proportional to velocity of moving transmitter.

$$f\left(\vec{x},t\right) = f_0 - \frac{f_0}{c}v$$

c is speed of waves in medium, v is velocity of transmitter



(this is classical, not relativistic)

http://electron9.phys.utk.edu/phys135d/modules/m10/doppler.htm

If you knew the frequency transmitted by the moving transmitter.

You can use the

beat frequency

produced by combining the received signal with a receiver generated signal that is at the transmitted frequency

to determine the speed.

http://electron9.phys.utk.edu/phys135d/modules/m10/doppler.htm

But we can do more.

We can

count the (beat) cycles
or measure the (beat) phase

of the <u>beat</u> signal as a function of time.

This will give us the change in distance.
(as will velocity times time)

So we can write

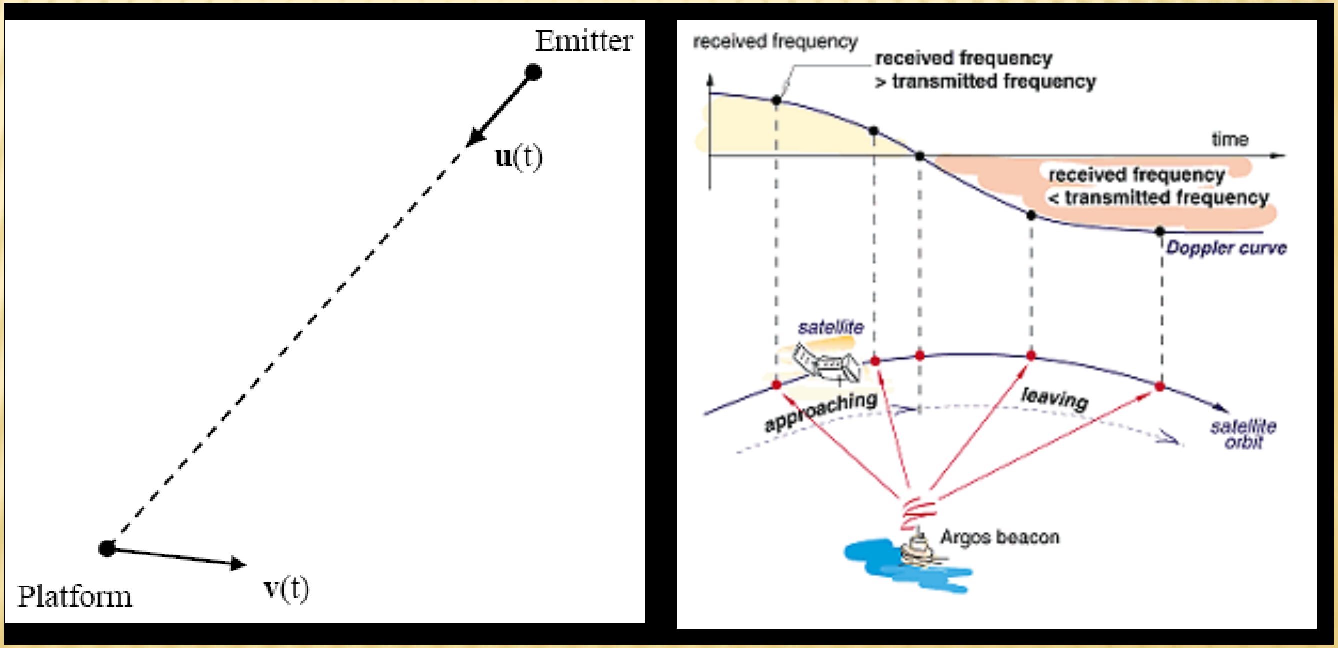
Beat phase (t) = change in distance to transmitter + constant

Beat phase (at t = t_{fixed}) = distance to transmitter + constant

Note the arbitrary constant –
can redo measurements from another position
(along trajectory of moving transmitter)
and get same result
(initial phase measurement will be different, but that will not change the frequency or distance estimation)

Next – move the receiver off the path of the transmitter (and can also let the transmitter path be arbitrary, now have to deal with vectors.)

$$f(\vec{x},t) = f_0 - \frac{f_0}{c}\vec{v}(t) \bullet \vec{u}(t)$$



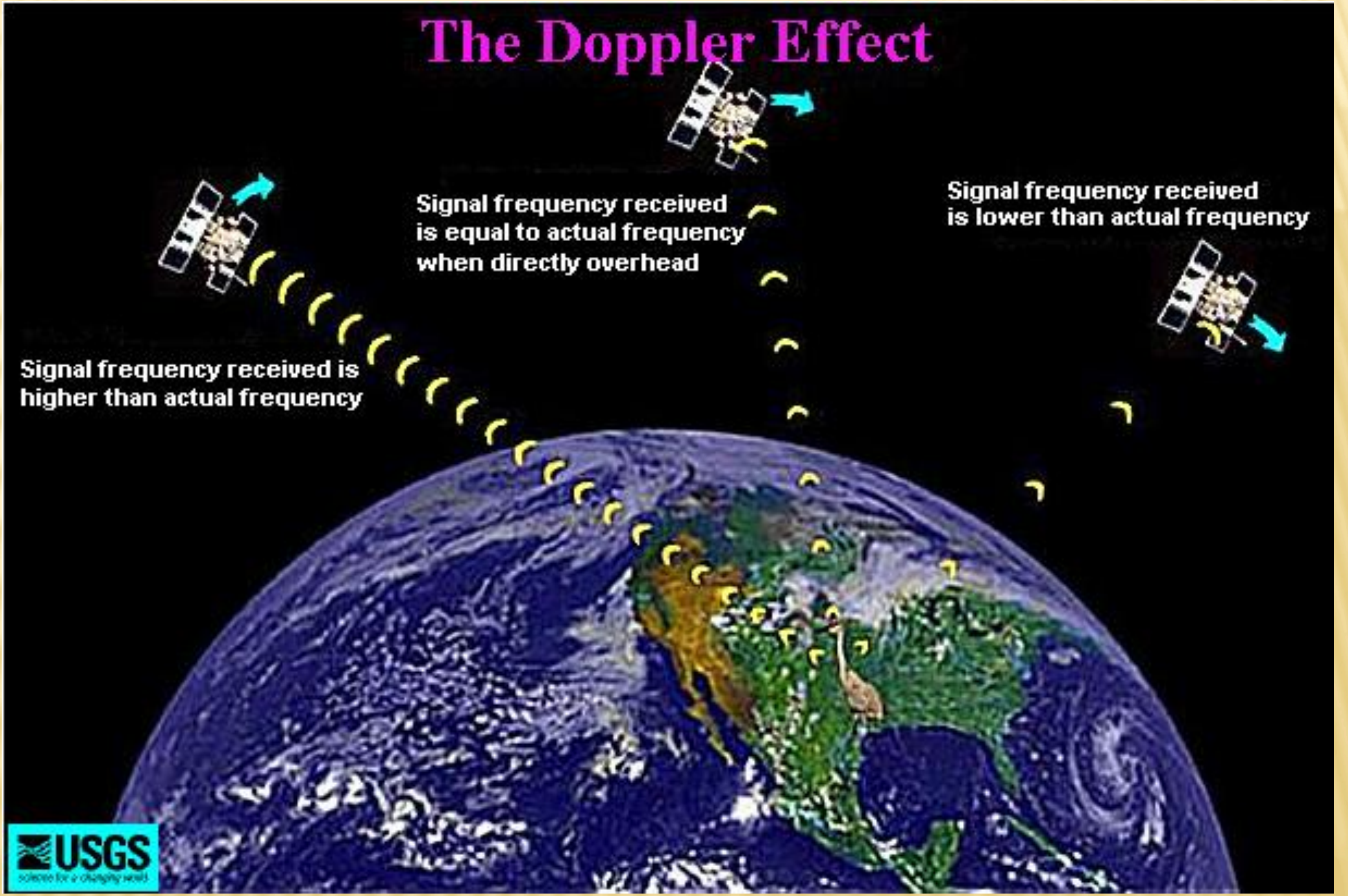
www.ws.binghamton.edu/fowler/fowler personal page/ EE522_files/CRLB for Dopp_Loc Notes.pdf
http://www.cls.fr/html/argos/general/doppler_gps_en.html

Can solve this for

Location of stationary transmitter from a moving receiver (if you know x and v of receiver – how SARSAT, ELT, EPIRB's [Emergency Position Indicating Radio Beacon's] work [or used to work – now also transmit location from GPS])

Location of moving transmitter
(solve for x and v of transmitter)
from a stationary receiver
(if you know x of receiver)

(Doppler shift, change in frequency, more useful for estimating velocity than position.
Integrate Doppler phase to get position.)



http://www.npwrc.usgs.gov/perm/cranemov/location.htm

Apply this to GPS
So far we have

Satellite carrier signal
Mixed with copy in receiver
After "low pass filter" – left with beat signal

Phase of beat signal equals reference phase minus received phase plus unknown integer number full cycles

From here on we will follow convention and call
- Carrier beat phase -
-Carrier phase –
(remember it is NOT the phase of the incoming signal)

Consider the observation of satellite S

We can write the observed carrier (beat) phase as

$$\Phi^{S}(T) = \phi(T) - \phi^{S}(T) - N^{S}$$

Receiver replica of signal

Incoming signal received from satellite S

Receiver clock time

Now assume that the phase from the satellite received at time T is equal to what it was when it was transmitted from the satellite

(we will eventually need to be able to model the travel time)

$$\phi^S(x,y,z,T) = \phi^S_{transmit}\left(x^S, y^S, z^S, T^S_{transmit}\right)$$

Blewitt, Basics of GPS in "Geodetic Applications of GPS"

$$\Phi^S(T) = \phi(T) - \phi^S_{transmit}\left(T^S_{transmit}\right) - N^S$$

Use from before for receiver time $T(t) = \dfrac{\left(\phi(t) - \phi_0\right)}{f_0}$

$$\phi(T) = f_0 T + \phi_0$$

$$\phi^S_{transmit}\left(T^S_{transmit}\right) = f_0 T^S_{transmit} + \phi^S_0$$

So the carrier phase observable becomes

$$\Phi^S(T) = f_0 T + \phi_0 - f_0 T^S_{transmit} - \phi^S_0 - N^S$$

$$\Phi^S(T) = f_0\left(T - T^S_{transmit}\right) + \phi_0 - \phi^S_0 - N^S$$

Blewitt, Basics of GPS in "Geodetic Applications of GPS"

$$\Phi^S(T) = f_0\left(T - T^S_{transmit}\right) + \phi_0 - \phi_0^S - N^S$$

Terms with S are for each satellite
All other terms are equal for all observed satellites

(receiver ϕ_0 should be same for all satellites– no interchannel bias, and receiver should sample all satellites at same time – or interpolate measurements to same time)

T^S and N^S will be different for each satellite
Last three terms cannot be separated (and will not be an integer) – call them "carrier phase bias"

Now we will convert carrier phase to range

(and let the superscript *S*-> satellite number, *j*, to handle more than one satellite, and

add a subscript for multiple receivers, *A*, to handle more than one receiver.)

$$\Phi_A^j\left(T_A\right) = f_0\left(T_{A,received} - T^{j,transmited}\right) + \left(\phi_0\right)_A - \left(\phi_0\right)^j - N_A^j$$

We will also drop the “received” and “transmitted” reminders.

Times with superscripts will be for the transmission time by the satellite.

Times with subscripts will be for the reception time by the receiver.

$$\Phi_A^j(T_A) = f_0\left(T_A - T^j\right) + \phi_{0_A} - \phi_0^j - N_A^j$$

If we are using multiple receivers, they should all sample at

exactly the same time
(same value for receiver clock time).

Values of clock times of sample – epoch.

With multiple receivers the clocks are not perfectly synchronized, so the true measurement times will vary slightly.

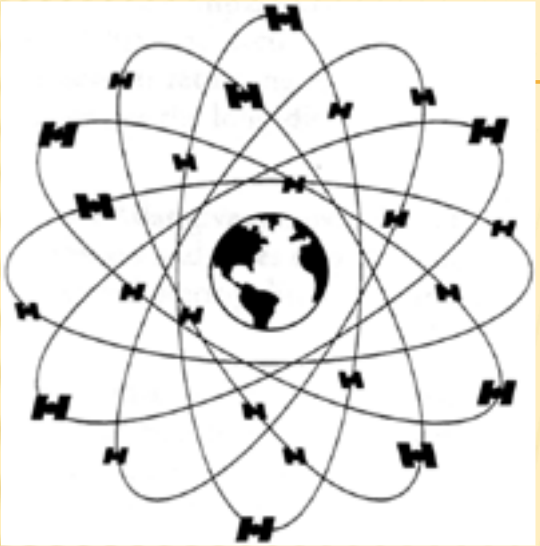
Also note – each receiver-satellite pair has its own carrier phase ambiguity.

Aside

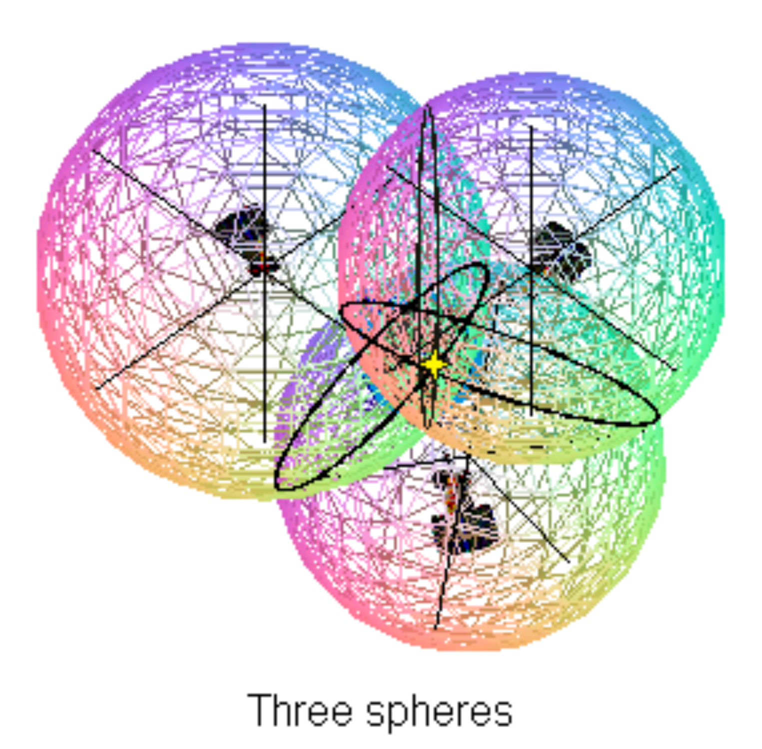
Synchronizing clocks brings up

<u>Relativity</u>

GPS



24 satellites in 6 orbital planes. Carry highly stable atomic clocks to generate GPS signals (GPS really measures time, not distance)



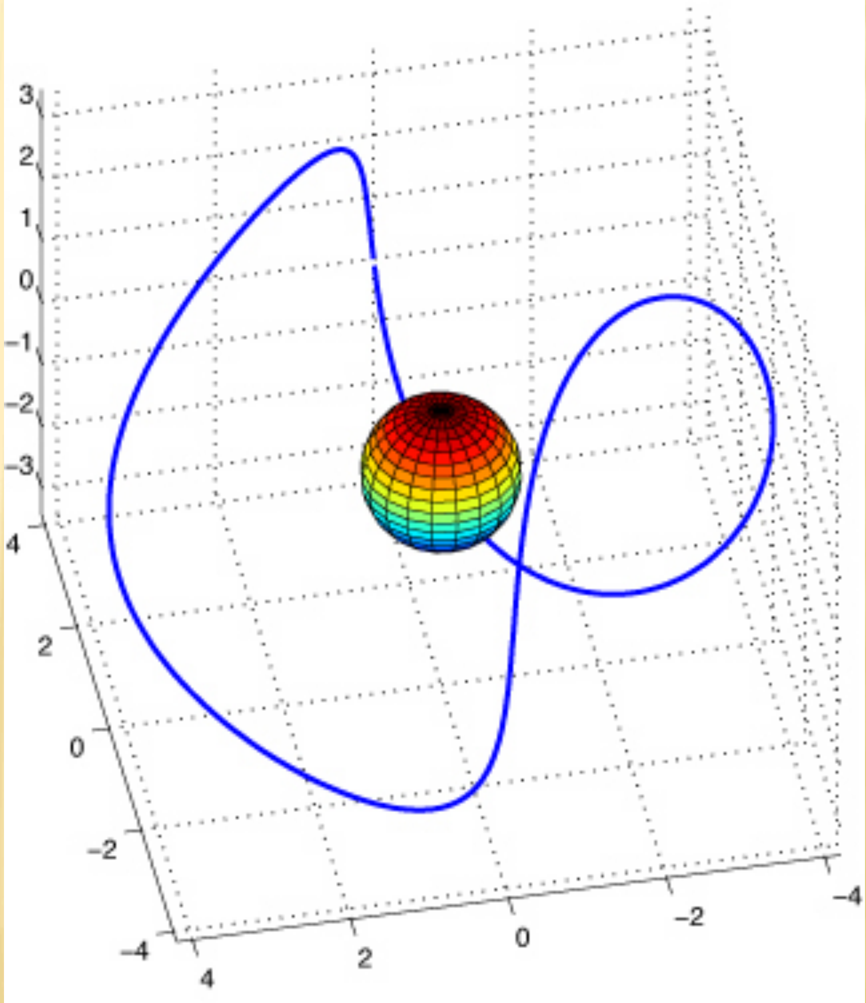
Measure distance (really time of flight, and use speed of light to get distance) to each satellite in view and use trilateration (not triangulation) to determine position.

GPS and Relativity

Measurement of time in different places (4+ satellites and receiver) brings up question of simultaneity.

This brings in special relativity.

Variation of gravity between surface of earth and orbits brings in general relativity.



Special and General Theories of Relativity

- Special relativity

Published 1905
kinematics, mechanics, and electromagnetism

- General relativity

Published 1915
gravitation
(includes special relativity)

Relativistic effects must be considered when i) measuring time with moving clocks and b) in the propagation of electromagnetic signals.

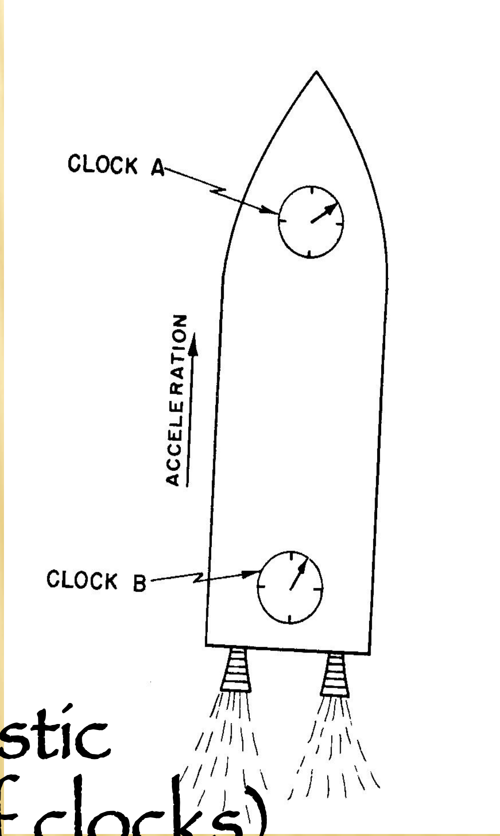
After: **Robert A. Nelson, Satellite Engineering Research Corporation, Bethesda, MD**

Relativistic Effects

Three (this number varies with author) effects

- Velocity (time dilation)
  Moving (satellite) clock runs slow
  Function of speed only

- Gravitational potential (red shift)
  clock in lower G (satellite) runs faster
  Function of altitude only



- Sagnac effect (can be classical or relativistic
  Interferometry (synch rotating system of clocks)
  Depends on direction and path traveled

After: **Robert A. Nelson, Satellite Engineering Research Corporation, Bethesda, MD, and N. Ashby U. Colorado**

- Gravitational redshift (blueshift)
Orbital altitude 20,183 km
Clock runs fast by 45.7 μs per day

- Time dilation
Satellite velocity 3.874 km/s
Clock runs slow by 7.1 μs per day

Net secular effect (satellite clock runs fast)
Clock runs fast by 38.6 μs per day

- Residual periodic effect
Orbital eccentricity 0.02
Amplitude of periodic effect 46 ns

- Sagnac effect
Maximum value 133 ns for a stationary receiver on the earth

After: **Robert A. Nelson, Satellite Engineering Research Corporation, Bethesda, MD, and N. Ashby U. Colorado**

Net secular relativistic effect is 38.6 μs per day

- Nominal clock rate is 10.23 MHz
Satellite clocks are offset by – 4.464733 parts in 10^{10} to compensate effect

- Resulting (proper) frequency in orbit is 10229999.9954326 Hz

- Observed (on earth) average rate of satellite clock is same as clock on earth.

After: **Robert A. Nelson, Satellite Engineering Research Corporation, Bethesda, MD, and N. Ashby U. Colorado**

- Residual periodic effect

Maximum amplitude 46 ns

Correction applied in receiver

- Sagnac effect

Maximum value 133 ns

Correction applied in receiver

After: Robert A. Nelson, Satellite Engineering Research Corporation, Bethesda, MD, and N. Ashby U. Colorado

Return to carrier phase processing

carrier phase to range

Multiply phase (in cycles, not radians) by wavelength to get "distance"

$$L_A^j(T_A) = \lambda_0 \Phi_A^j(T_A)$$

$$L_A^j(T_A) = \lambda_0\left(f_0\left(T_A - T^j\right) + \phi_{0_A} - \phi_0^j - N_A^j\right)$$

$$L_A^j(T_k) = c\left(T_A - T^j\right) + \lambda_0\left(\phi_{0_A} - \phi_0^j - N_A^j\right)$$

$$L_A^j(T_A) = c\left(T_A - T^j\right) + B_A^j$$

$L_A^j(T_A)$ is in units of meters

B_A^j is "carrier phase bias" (in meters) (is not an integer)

Blewitt, Basics of GPS in "Geodetic Applications of GPS"

$$L_A^j(T_A) = c\,(T_A - T^j) + B_A^j$$

a distance

This equation looks exactly like the equation for pseudo-range

$$P_R^S = \rho_R^S(t_R, t^S) + (\tau_R - \tau^S)\,c = \rho_R^S(t_R, t^S) + c\delta t$$

That we saw before

pseudo-range constant

$$L_A^j(T_A) = c\left(T_A - T^j\right) + B_A^j$$

This equation also holds for both

L1 and L2

Clock biases same for L1 and L2, but ambiguity values different (different wavelengths).

Now that we have things expressed as "distance" (range)

Follow pseudo range development

$$L_A^j(T_A) = c\left(T_A - T^j\right) + B_A^j$$

$$L_A^j(T_A) = \rho_A^j\left(t_A, t^j\right) + c\tau_A - c\tau^j + Z_A^j - I_A^j + B_A^j$$

Added a few things related to propagation of waves
Delay in signal due to

Troposphere – Z_A^j
Ionosphere – $-I_A^j$

(ionospheric term has "-" since phase velocity increases)

Blewitt, Basics of GPS in "Geodetic Applications of GPS"

Can include these effects in pseudo range development also

$$P_A^j(T_k) = c\left(T_A - T^j\right)$$

$$P_A^j(T_A) = \rho_A^j\left(t_A, t^j\right) + c\tau_A - c\tau^j + Z_A^j + I_A^j$$

Delay in signal due to

Troposphere + Z_A^j
Ionosphere + $-I_A^j$

(ionospheric term now has "+" since group velocity to first order is same magnitude but opposite sign as phase velocity)

Blewitt, Basics of GPS in "Geodetic Applications of GPS"

Another aside

http://webphysics.ph.msstate.edu/jc/library/15-11/index.html

Set up to see phase vel and group vel opposite sign

(package goes one way, waves inside go other)

λ = 24 and 22, v = 5 and 3 respectively

http://www.geneseo.edu/~freeman/animations/phase_versus_group_velocity.htm

One thing to keep in mind about the

phase velocity

is that it is an entirely mathematical construct.

Pure sine waves do not exist,
as a monochromatic wave train is infinitely long.

They are merely a tool to construct wave packets,
which have a group velocity,
and that is what we are measuring in experiments.

http://www.everything2.com: Source: R. U. Sexl and H. K. Urbantke, Relativität, Gruppen, Teilchen, chap. 2, 24, 3rd edn., Springer, Wien (1992)

In fact, it may very well be that the phase velocity comes out
higher than c,
(e.g. in wave guides!)

This puzzles people, and some use that fact to claim that the theory of relativity is wrong.

However, even if *you* had a pure sine wave, *you* couldn't use it to transmit any information,

because it is unmodulated,

so there is no contradiction.

http://www.everything2.com: Source: R. U. Sexl and H. K. Urbantke, Relativität, Gruppen, Teilchen, chap. 2, 24, 3rd edn., Springer, Wien (1992)

But it turns out that

even the group velocity may be higher than c,

namely in the case of anomalous dispersion

http://www.everything2.com: Source: R. U. Sexl and H. K. Urbantke, Relativität, Gruppen, Teilchen, chap. 2, 24, 3rd edn., Springer, Wien (1992)

Now how do we get around this?

Well, this kind of dispersion is so bad that the definition of our wave packet loses its meaning because it just disintegrates, and again we cannot use it to transmit information.

The only way would be to switch the signal on and off - these discontinuities propagate with the wavefront velocity

$$v_F = \lim_{k \to \infty} (\omega(k)/k)$$

And again, relativity is saved!

http://www.everything2.com: Source: R. U. Sexl and H. K. Urbantke, Relativität, Gruppen, Teilchen, chap. 2, 24, 3rd edn., Springer, Wien (1992), or http://en.wikipedia.org/wiki/Faster-than-light

Back to phase processing

Now we have to fix the time

So far our expression has receiver and satellite clock time

-Not true time

Remember that the true time is the clock time adjusted by the clock bias

$$t_A = T_A - \tau_A$$

We know T_A exactly

(it is the receiver clock time which is written into the observation file – called a "time tag")

$$t_A = T_A - \tau_A$$

But we don't know τ_A

(we need it to an accuracy of *1 μsec*)

How to estimate τ_A

- Use estimate of τ_A from pseudo range point positioning (if have receiver that uses the codes)

- LS iteration of code and phase data simultaneously

- If know satellite position and receiver location well enough (300 m for receiver – *1 µsec* of distance) can estimate it
(this is how GPS is used for time transfer, once initialized can get time with only one satellite visible [if don't loose lock])

- Modeling shortcut – linearize (Taylor series)

Big trick

Eliminating clock biases using differencing

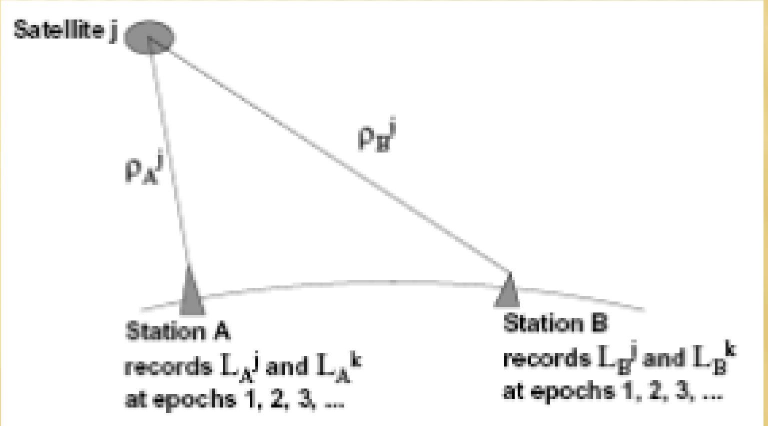
Return to our model for the phase observable

$$L_A^j(T_A) = \rho_A^j(t_A, t^j) + c\tau_A - c\tau^j + Z_A^j - I_A^j + B_A^j$$

clock error - receiver

clock error - satellite

What do we get if we combine measurements made by two receivers at the same epoch?



Blewitt, Basics of GPS in "Geodetic Applications of GPS"

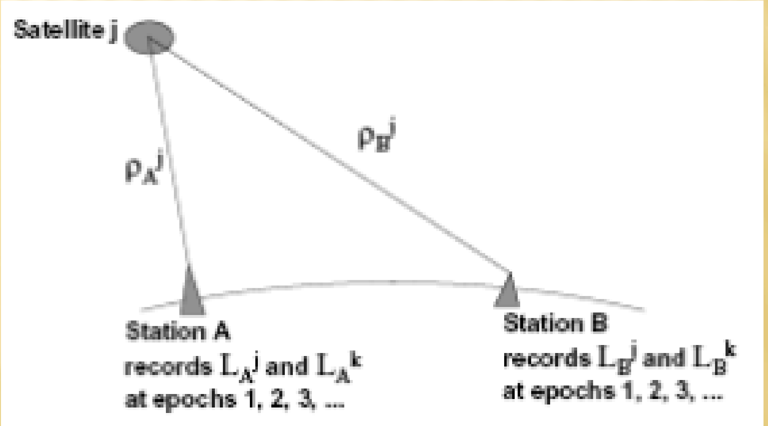
Define the single difference

$$L_A^j\left(T_A\right) = \rho_A^j\left(t_A, t^j\right) + c\tau_A - c\tau^j + Z_A^j - I_A^j + B_A^j$$

$$L_B^j\left(T_B\right) = \rho_B^j\left(t_B, t^j\right) + c\tau_B - c\tau^j + Z_B^j - I_B^j + B_B^j$$

$$\Delta L_{AB}^j = L_A^j\left(T_A\right) - L_B^j\left(T_B\right)$$

Use triangle to remember is difference between satellite (top) and two receivers (bottom)



Blewitt, Basics of GPS in "Geodetic Applications of GPS"

$$\Delta L_{AB}^{j} = L_{A}^{j}\left(T_{A}\right) - L_{B}^{j}\left(T_{B}\right)$$

$$\Delta L_{AB}^{j} = \rho_{A}^{j} - \rho_{B}^{j} + c\tau_{A} - c\tau_{B} \cancel{- c\tau^{j} + c\tau^{j}}$$
$$+ Z_{A}^{j} - Z_{B}^{j} - I_{A}^{j} + I_{B}^{j} + B_{A}^{j} - B_{B}^{j}$$

$$\Delta L_{AB}^{j} = \Delta\rho_{AB}^{j} + \Delta c\tau_{AB} + \Delta Z_{AB}^{j} - \Delta I_{AB}^{j} + \Delta B_{AB}^{j}$$

Satellite time errors cancel
(assume transmission times are same – probably not unless range to both receivers from satellite the same)

If the two receivers are close together the tropospheric and ionospheric terms also (approximately) cancel.

Blewitt, Basics of GPS in "Geodetic Applications of GPS"

How about we do this trick again

This time using two single differences to two satellites (all at same epoch)

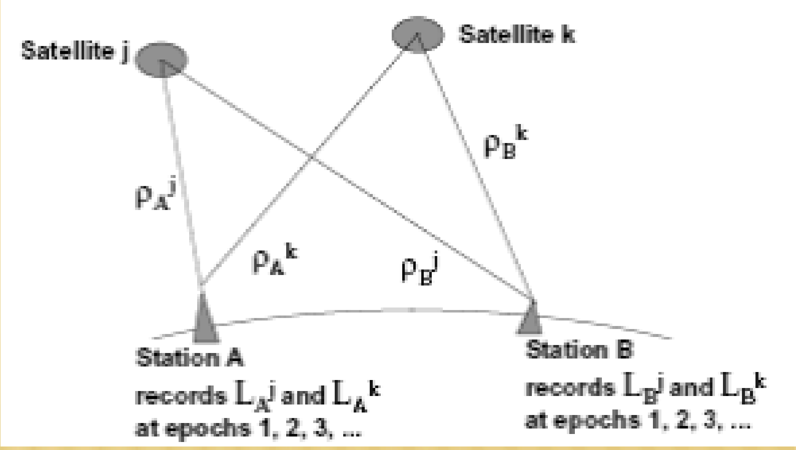
Define the double difference

$$\Delta L_{AB}^{j} = \Delta\rho_{AB}^{j} + \Delta c\tau_{AB} + \Delta Z_{AB}^{j} - \Delta I_{AB}^{j} + \Delta B_{AB}^{j}$$

$$\Delta L_{AB}^{k} = \Delta\rho_{AB}^{k} + \Delta c\tau_{AB} + \Delta Z_{AB}^{k} - \Delta I_{AB}^{k} + \Delta B_{AB}^{k}$$

$$\nabla\Delta L_{AB}^{jk} = \Delta L_{AB}^{j} - \Delta L_{AB}^{k}$$

Use inverted triangle to remember is difference between two satellites (top) and one receiver (bottom)



Blewitt, Basics of GPS in "Geodetic Applications of GPS"

$$\nabla\Delta L_{AB}^{jk} = \Delta\rho_{AB}^{j} - \Delta\rho_{AB}^{k} + \Delta c\tau_{AB} - \Delta c\tau_{AB}$$

$$+ \Delta Z_{AB}^{j} - \Delta Z_{AB}^{k} + \Delta I_{AB}^{j} - \Delta I_{AB}^{k} + \Delta B_{AB}^{j} - \Delta B_{AB}^{k}$$

$$\nabla\Delta L_{AB}^{jk} = \nabla\Delta\rho_{AB}^{jk} + \nabla\Delta Z_{AB}^{jk} - \nabla\Delta I_{AB}^{jk} + \nabla\Delta B_{AB}^{jk}$$

Now we have gotten rid of the receiver clock bias terms (again to first order – and results better for short baselines)

Double differencing
- removes (large) clock bias errors
-approximately doubles (smaller) random errors due to atmosphere, ionosphere, etc. (no free lunch)
- have to see both satellites from both receivers.

Next – what is the ambiguity term after double difference (remembering definition of B_A^j)

$$\nabla\Delta B_{AB}^{jk} = \Delta B_{AB}^{j} - \Delta B_{AB}^{k}$$

$$\nabla\Delta B_{AB}^{jk} = \left(B_A^j - B_B^j\right) - \left(B_A^k - B_A^k\right)$$

$$\nabla\Delta B_{AB}^{jk} = \lambda_0\left(\phi_{0_A} - \phi_0^j - N_A^j\right) - \lambda_0\left(\phi_{0_B} - \phi_0^j - N_B^j\right) +$$

$$- \lambda_0\left(\phi_{0_A} - \phi_0^k - N_A^k\right) + \lambda_0\left(\phi_{0_B} - \phi_0^k - N_B^k\right)$$

$$\nabla\Delta B_{AB}^{jk} = -\lambda_0\left(N_A^j - N_B^j - N_A^k + N_B^k\right)$$

$$\nabla\Delta B_{AB}^{jk} = -\lambda_0 N_{AB}^{jk}$$

The ambiguity term reduces to an integer

So our final

Double difference observation

is

$$\nabla\Delta L_{AB}^{jk} = \nabla\Delta\rho_{AB}^{jk} + \nabla\Delta Z_{AB}^{jk} - \nabla\Delta I_{AB}^{jk} - \lambda_0 \nabla\Delta N_{AB}^{jk}$$

One can do the differencing in either order

The sign on the ambiguity term is arbitrary

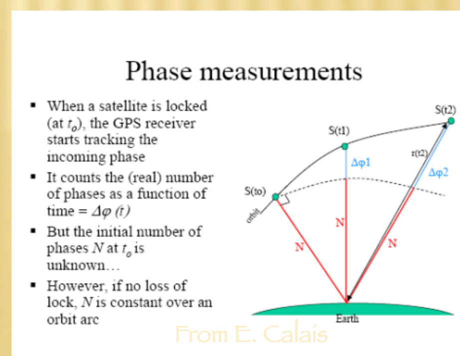
We seem to be on a roll here, so let' s do it again. This time (take the difference of double differences) between two epochs

$$\nabla\Delta L_{AB}^{jk}(i) = \nabla\Delta\rho_{AB}^{jk}(i) + \nabla\Delta Z_{AB}^{jk}(i) - \nabla\Delta I_{AB}^{jk}(i) - \nabla\Delta N_{AB}^{jk}(i)$$

$$\nabla\Delta L_{AB}^{jk}(i+1) = \nabla\Delta\rho_{AB}^{jk}(i+1) + \nabla\Delta Z_{AB}^{jk}(i+1) - \nabla\Delta I_{AB}^{jk}(i+1) - \nabla\Delta N_{AB}^{jk}(i+1)$$

Equal if no loss of lock (no cycle slip)

$$\delta(i,i+1)\nabla\Delta L_{AB}^{jk} = \nabla\Delta L_{AB}^{jk}(i+1) - \nabla\Delta L_{AB}^{jk}(i)$$



Blewitt, Basics of GPS in "Geodetic Applications of GPS"

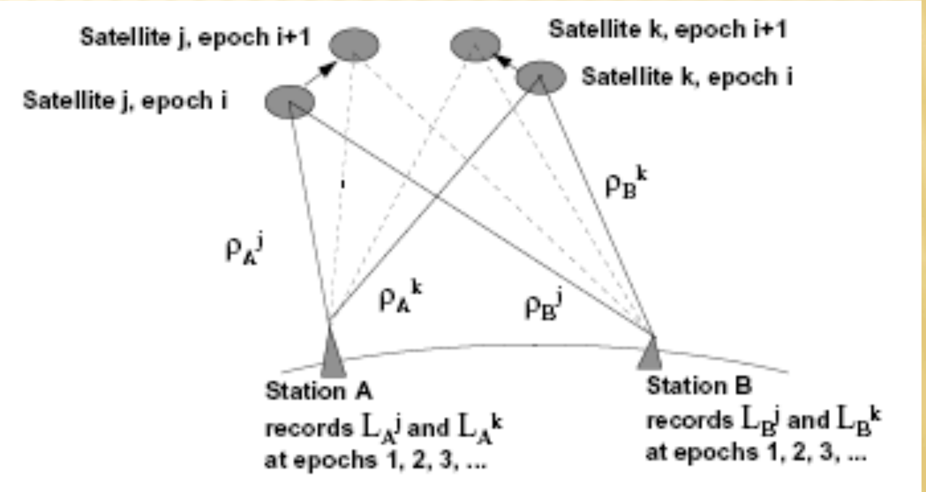
So now we have gotten rid of the integer ambiguity

$$\delta(i,i+1)\Delta L_{AB}^{jk} = \nabla\Delta L_{AB}^{jk}(i+1) - \nabla\Delta L_{AB}^{jk}(i)$$

$$\delta(i,i+1)\Delta L_{AB}^{jk} = \delta(i,i+1)\nabla\Delta\rho_{AB}^{jk}(i) + \delta(i,i+1)\nabla\Delta Z_{AB}^{jk}(i) - \delta(i,i+1)\nabla\Delta I_{AB}^{jk}(i)$$

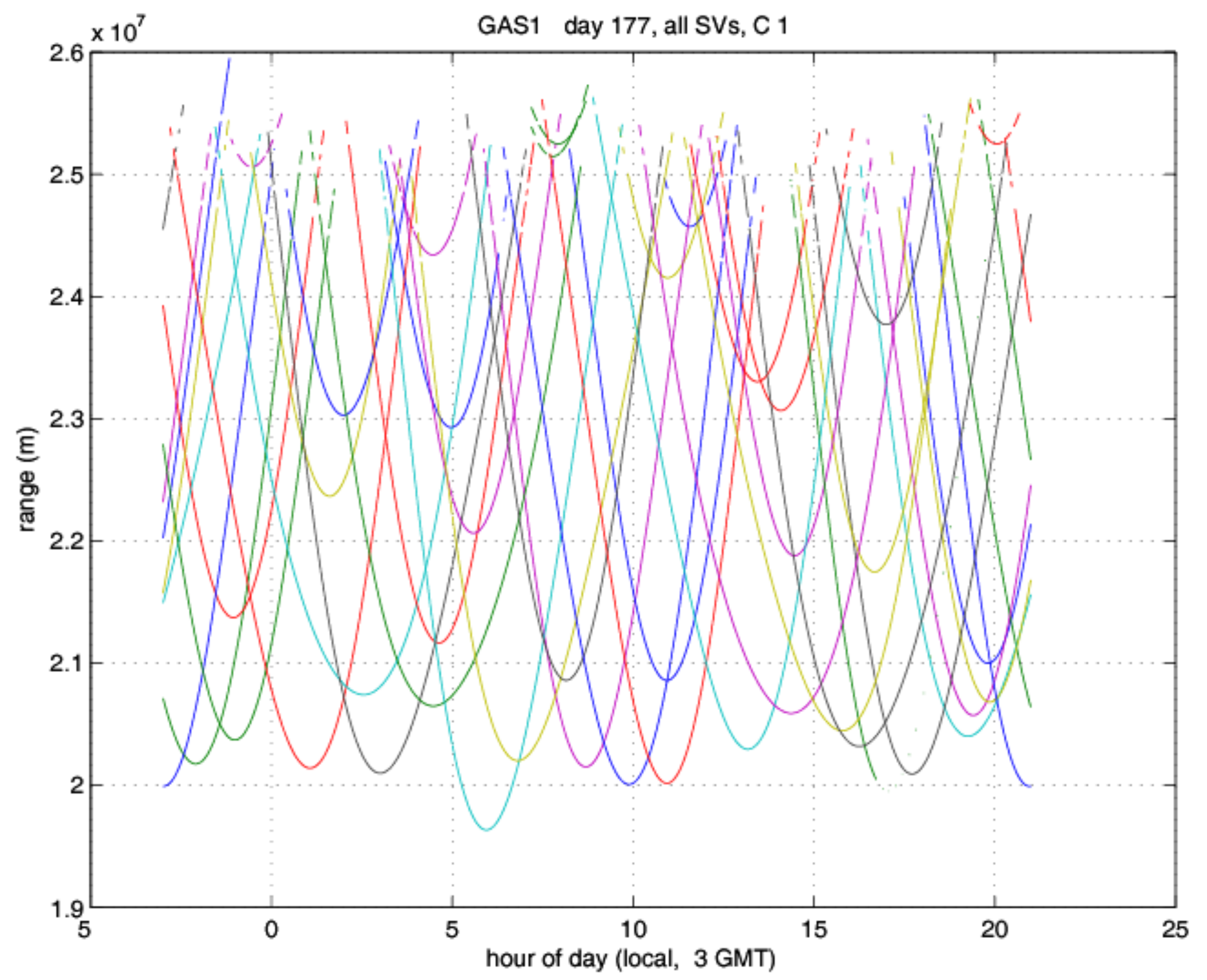
If no cycle slip – ambiguities removed.

If there is a cycle slip – get a spike in the triple difference.



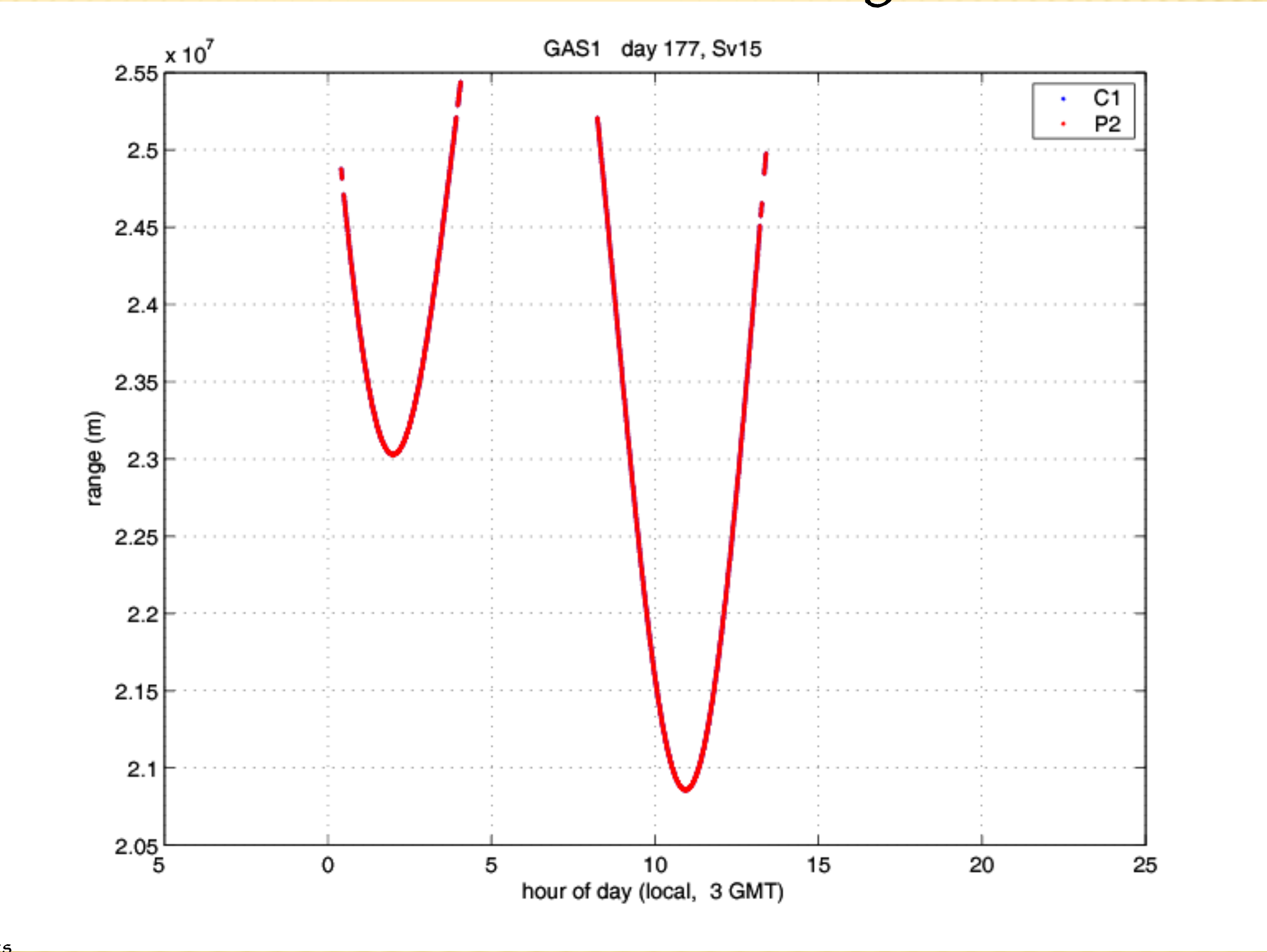
Blewitt, Basics of GPS in "Geodetic Applications of GPS"

Raw Data from RINEX file: RANGE
Plot of C1 (range in meters)
For all satellites for full day of data



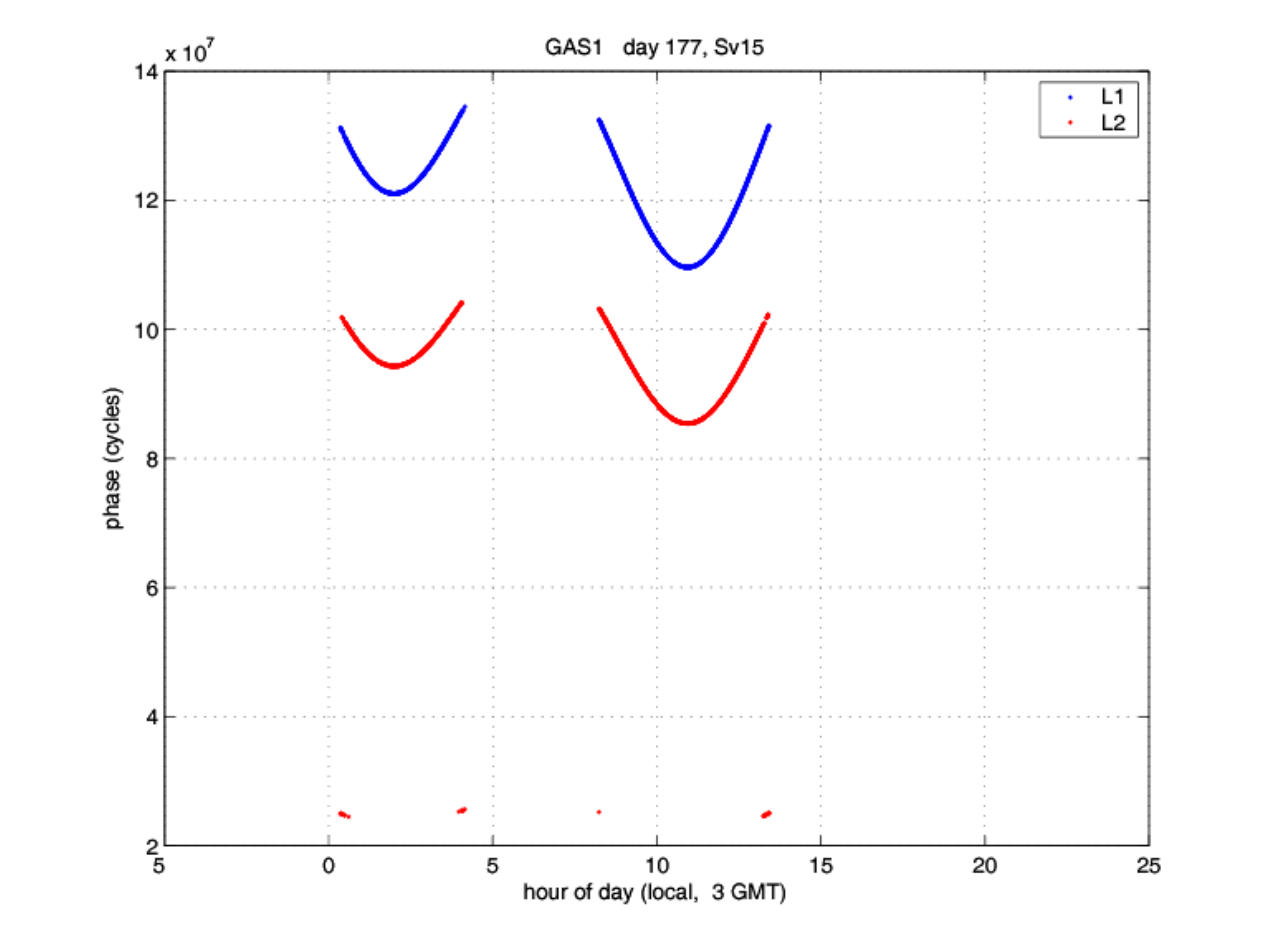
From Ben Brooks

Raw Data from RINEX file: RANGE
Plot of P1 (range in meters)
For one satellite for full day of data



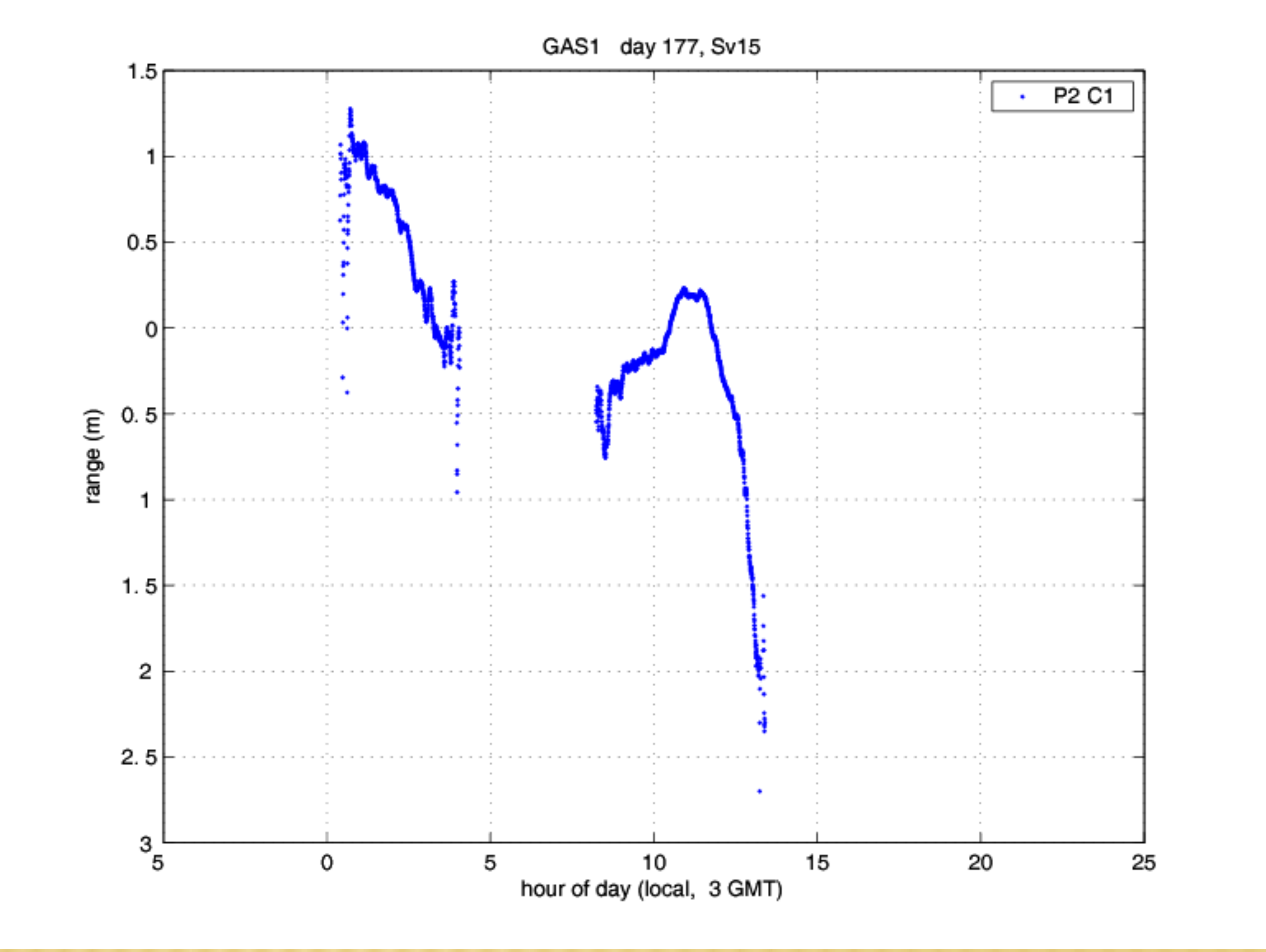
From Ben Brooks

Raw Data from RINEX file: PHASE

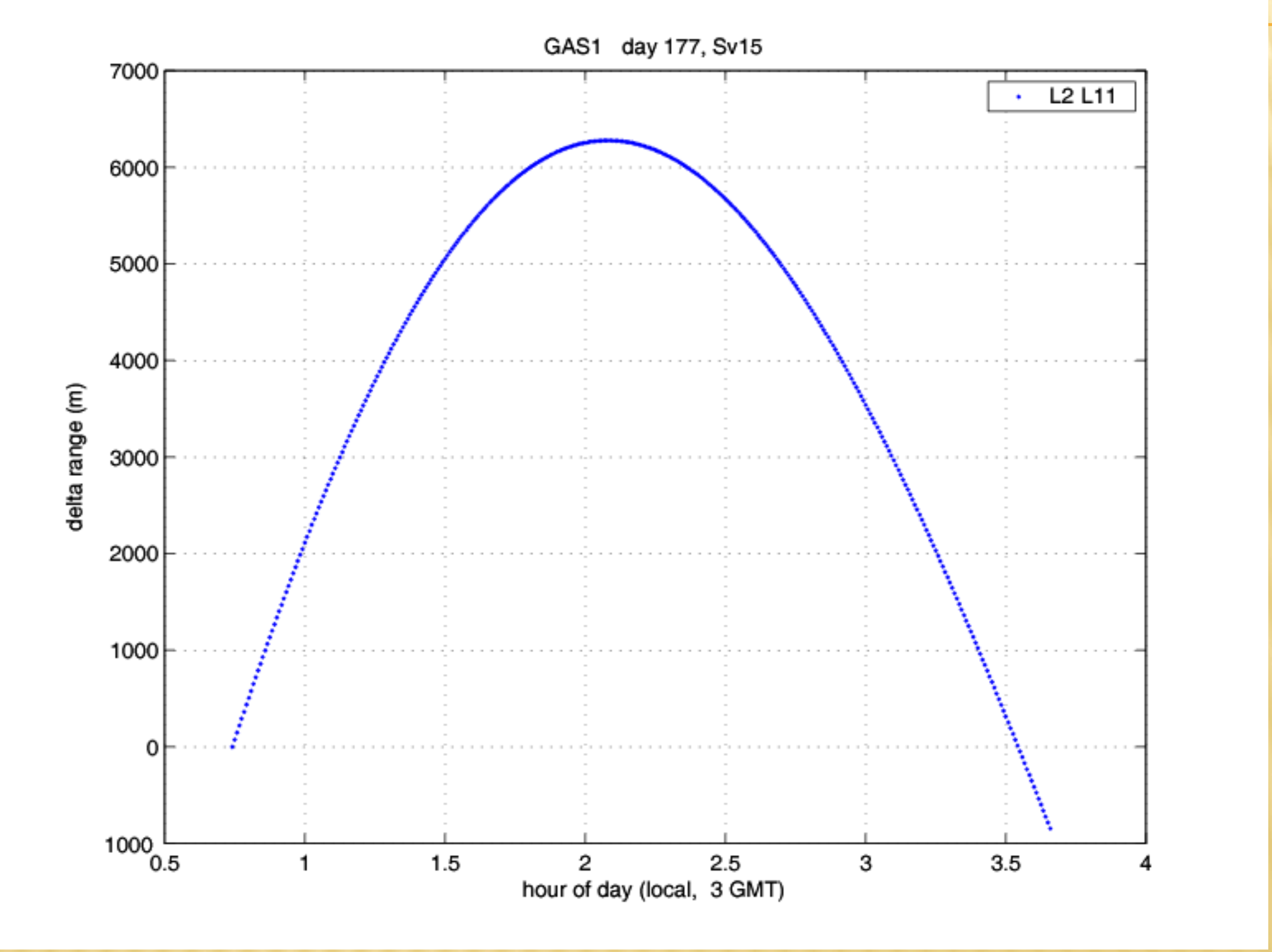


From Ben Brooks

Raw Data from RINEX file: RANGE DIFFERENCE

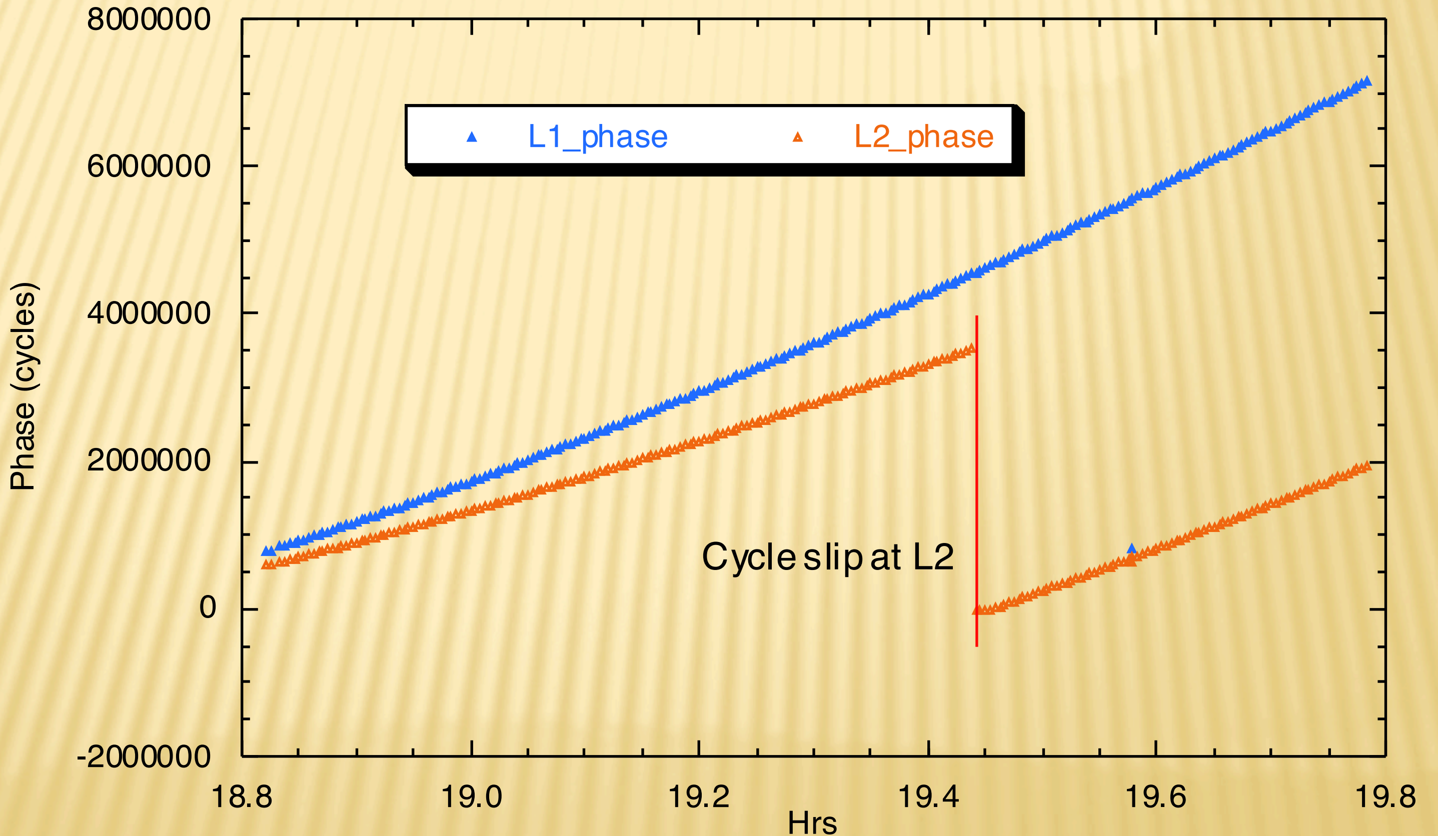


Raw Data from RINEX file: PHASE DIFFERENCE



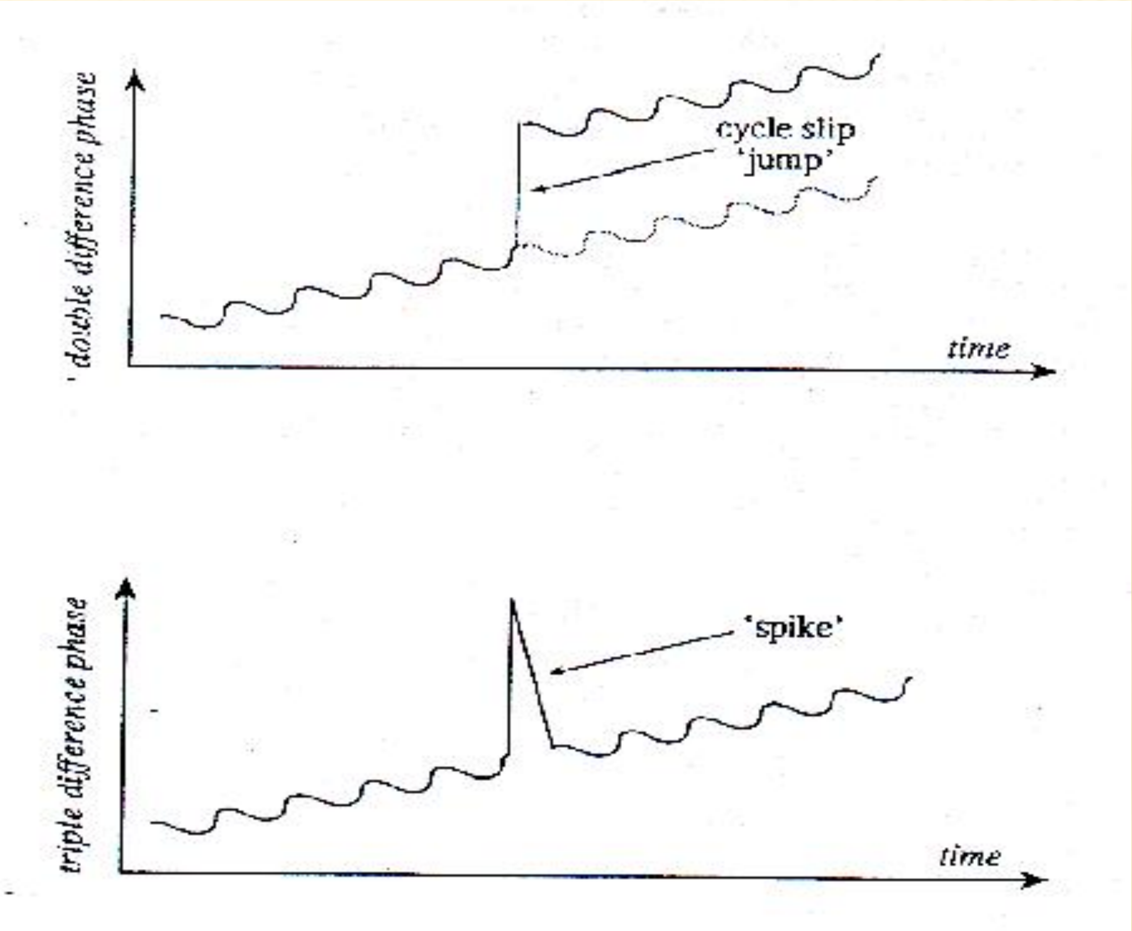
From Ben Brooks

Zoom in on phase observable
Without an (L1) and with an (L2) cycle slip



http://www-gpsg.mit.edu/~tah/12.540/

Cycle slip shows up as spike in triple difference (so can identify and fix)



Have to do this for "all" pairs of receiver-satellite pairs.

http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap7/735.htm

Effects of triple differences on estimation

Further increase in noise

Additional effect – introduces correlation between observations in time

This effect substantial

So triple differences limited to identifying and fixing cycle slips.

Using double difference phase observations for relative positioning

First notice that if we make all double differences - even ignoring the obvious duplications

$$\nabla\Delta L_{AB}^{jk} = \nabla\Delta L_{AB}^{kj} = \nabla\Delta L_{BA}^{kj} = \nabla\Delta L_{BA}^{jk}$$

We get a lot more double differences than original data.

This can't be (can't create information).

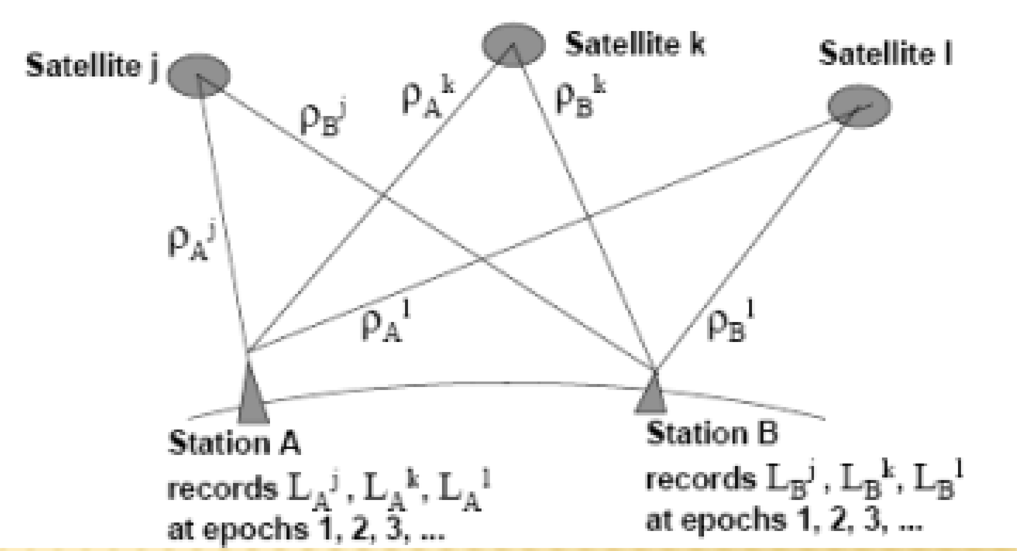
Consider the case of 3 satellites observed by 2 receivers.

Form the (non trivial) double differences

$$L_{AB}^{jk} = \left(L_A^j - L_B^j\right) - \left(L_A^k - L_B^k\right)$$

$$L_{AB}^{jl} = \left(L_A^j - L_B^j\right) - \left(L_A^l - L_B^l\right)$$

$$L_{AB}^{lk} = \left(L_A^l - L_B^l\right) - \left(L_A^k - L_B^k\right)$$



Note that we can form any one from a linear combination of the other two

(linearly dependent)

$$L_{AB}^{jk} = L_{AB}^{jl} - L_{AB}^{lk}$$

$$L_{AB}^{jl} = L_{AB}^{jk} - L_{AB}^{lk}$$

$$L_{AB}^{lk} = L_{AB}^{jk} - L_{AB}^{jl}$$

We need a linearly independent set for Least Squares.

From the linearly dependent set

$$\left\{ L_{AB}^{jk}, L_{AB}^{jl}, L_{AB}^{lk} \right\}$$

We can form a number of linearly independent subsets

$$\left\{ L_{AB}^{jk}, L_{AB}^{jl} \right\} = \Lambda^{j} = \left\{ L_{AB}^{ab} \middle| a = j; b \neq j \right\}$$

$$\left\{ L_{AB}^{kj}, L_{AB}^{kl} \right\} = \Lambda^{k} = \left\{ L_{AB}^{ab} \middle| a = k; b \neq k \right\}$$

$$\left\{ L_{AB}^{lj}, L_{AB}^{lk} \right\} = \Lambda^{l} = \left\{ L_{AB}^{ab} \middle| a = l; b \neq l \right\}$$

Which we can then use for our Least Squares estimation.