Earth Science Applications of Space Based Geodesy

DES-7355

Tu-Th 9:40-11:05

Seminar Room in 3892 Central Ave. (Long building)

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678-4929

Office Hours – Wed 14:00-16:00 or if I' m in my office.

http://www.ceri.memphis.edu/people/smalley/ESCI7355/ESCI_7355_Applications_of_Space_Based_Geodesy.html

Class 12

Determining

Strain or strain rate from

Displacement or velocity field

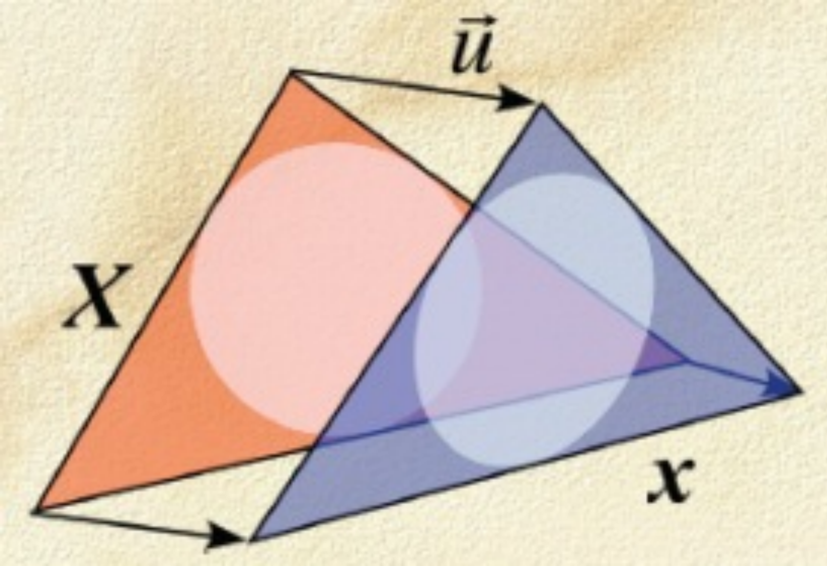
$$\boxed{u_i} = t_i + \frac{\partial u_i}{\partial X_j} X_j = \boxed{t_i + D_{ij} X_j} = t_i + \left(E_{ij} + W_{ij}\right) X_j$$

Deformation tensor

$$E_{ij} = \frac{1}{2}\left(D_{ij} + D_{ji}\right)$$

$$W_{ij} = \frac{1}{2}\left(D_{ij} - D_{ji}\right)$$

Strain (symmetric) and Rotation (anti-symmetris) tensors



Write it out

$$u_i = t_i + D_{ij} X_j$$

Deformation tensor is not symmetric, have to keep d_{xy} and d_{yx}.

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + \begin{pmatrix} d_{xx} & d_{xy} \\ d_{yx} & d_{yy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Again – this is "wrong way around"

We know
$\underline{u}$ and $\underline{x}$
and want
$\underline{t}$ and d_{ij}.

$$u_i = t_i + D_{ij} X_j$$

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + \begin{pmatrix} d_{xx} & d_{xy} \\ d_{yx} & d_{yy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

So rearrange it

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} 1 & 0 & x & y & 0 & 0 \\ 0 & 1 & 0 & 0 & x & y \end{pmatrix} \begin{pmatrix} t_x \\ t_y \\ d_{xx} \\ d_{xy} \\ d_{yx} \\ d_{yy} \end{pmatrix}$$

Now we have 6 unknowns and 2 equations

So we need at least 3 data points
That will give us 6 data

$$\begin{pmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ \vdots \\ u_{x_n} \\ u_{y_n} \end{pmatrix} = \begin{pmatrix} 1 & 0 & x_1 & y_1 & 0 & 0 \\ 0 & 1 & 0 & 0 & x_1 & y_1 \\ 1 & 0 & x_2 & y_2 & 0 & 0 \\ 0 & 1 & 0 & 0 & x_2 & y_2 \\ 1 & 0 & x_3 & y_3 & 0 & 0 \\ 0 & 1 & 0 & 0 & x_3 & y_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & x_n & y_n & 0 & 0 \\ 0 & 1 & 0 & 0 & x_n & y_n \end{pmatrix} \begin{pmatrix} t_x \\ t_y \\ d_{xx} \\ d_{xy} \\ d_{yx} \\ d_{yy} \end{pmatrix}$$

And again – the more the merrier – do least squares.

For strain rate

Take time derivative of all terms.

But be careful

Strain rate tensor

is NOT

time derivative of strain tensor.

Spatial (Eulerian) and Material (Lagrangian) Coordinates

and the

Material Derivative

Spatial description picks out a particular location in space, x.

Material description picks out a particular piece of continuum material, X.

So we can write

$$x = x(A,t) \qquad x(A,0) = A$$

x is the position now (at time t) of the section that was initially (at time zero) located at A.

or

$$A = A(x,t) \qquad A(A,0) = A$$

A was the initial position of the particle now at x

This gives by definition

$$x[A(x,t),t] = x \qquad A[x(A,t),t] = A$$

We can therefore write

$$f\left[x(A,t),t\right] = F(A,t) \qquad f(x,t) = F\left[A(x,t),t\right]$$

Next consider the derivative (use chain rule)

$$\frac{\partial}{\partial A}F(A,t) = f\left[x(A,t),t\right] = \left.\frac{\partial f}{\partial x}\right|_A \frac{\partial x}{\partial A}$$

$$\frac{\partial}{\partial t}F(A,t) = f\left[x(A,t),t\right] = \left.\frac{\partial f}{\partial x}\right|_A \frac{\partial x}{\partial t} + \left.\frac{\partial f}{\partial t}\right|_A$$

Define Material Derivative

$$\frac{\partial}{\partial t}F(A,t) = f\left[x(A,t),t\right] = \left.\frac{\partial f}{\partial x}\right|_{A}\frac{\partial x}{\partial t} + \left.\frac{\partial f}{\partial t}\right|_{A}$$

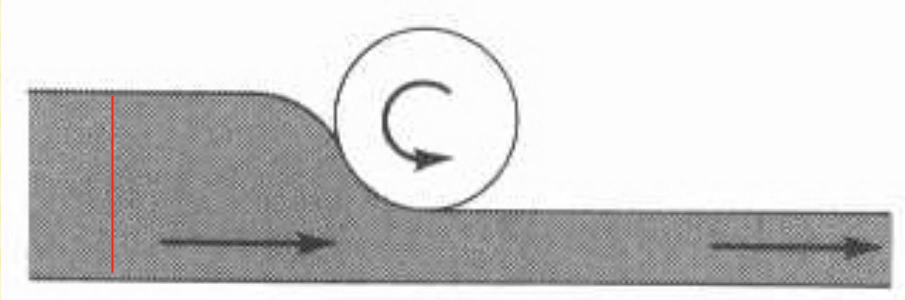
$$\frac{DF(A,t)}{Dt} = \left.\frac{\partial F(A,t)}{\partial t}\right|_{A=A(x,t)}$$

$$\boxed{\frac{Df(A,t)}{Dt} = \frac{\partial f(x,t)}{\partial t} + v(x,t)\frac{\partial f(x,t)}{\partial x}}$$

Vector version

$$\frac{D\vec{f}}{Dt} = \frac{\partial\vec{f}(x,t)}{\partial t} + \vec{v}(x,t)\bullet\nabla\vec{f}(x,t)$$

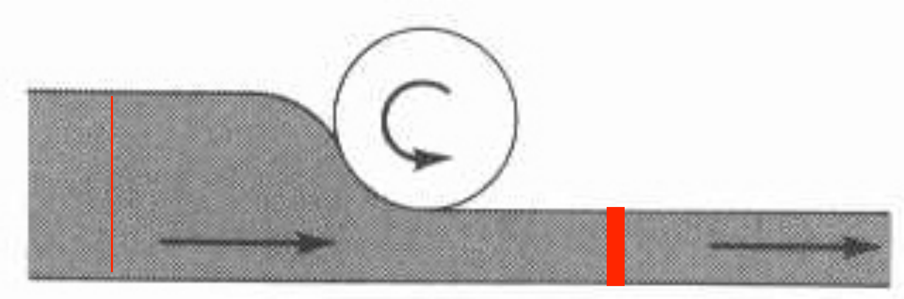
Example



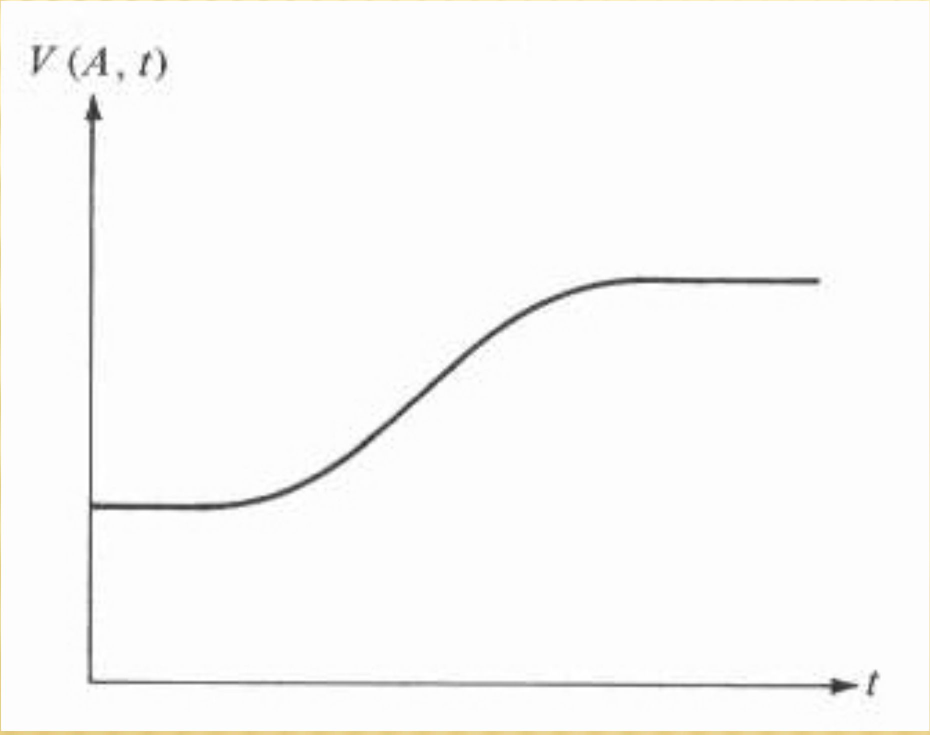
A

Consider bar steadily moving through a roller that thins the bar

Examine velocity as a function of time of cross section A



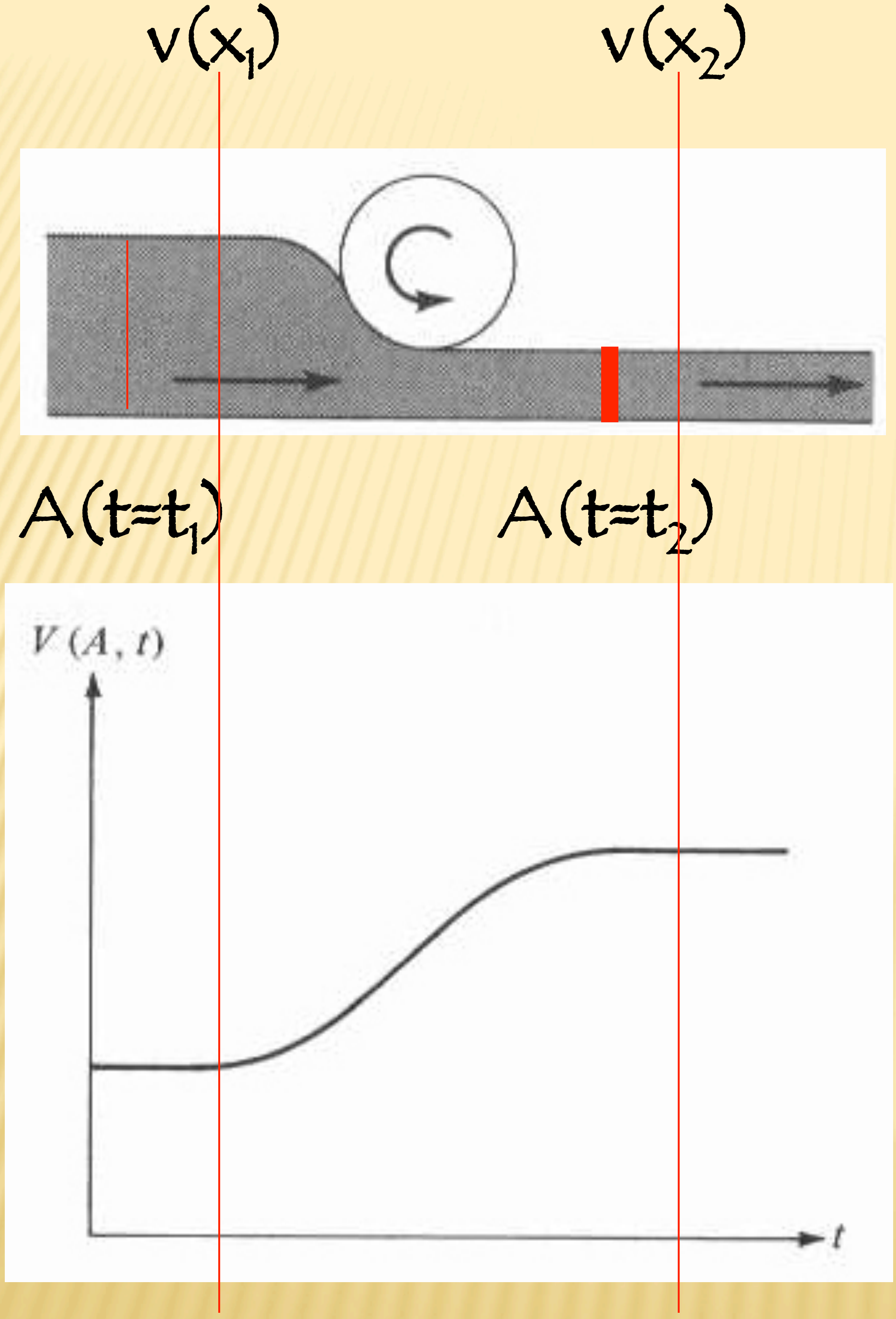
$A(t=t_1)$ $A(t=t_2)$



The velocity will be constant until the material in A reaches the roller

At which point it will speed up (and get a little fatter/ wider, but ignore that as second order)

After passing through the roller, its velocity will again be constant



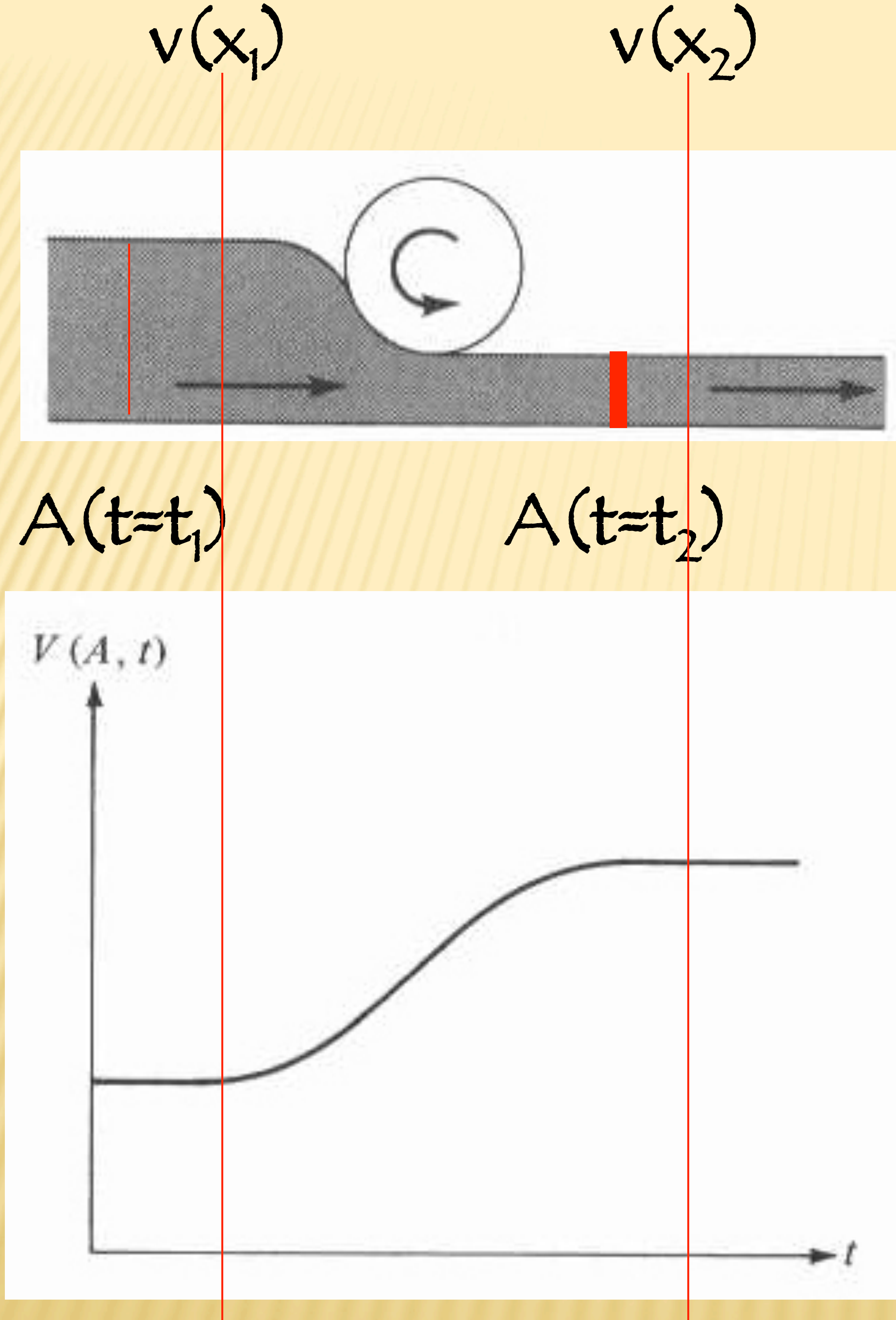
If one looks at a particular position, x, however the velocity is constant in time.

So for any fixed point in space

$$\frac{\partial v(x,t)}{\partial t} = 0$$

So the acceleration seems to be zero

(which we know it is not)



The problem is that we need to compute the time rate of change of the

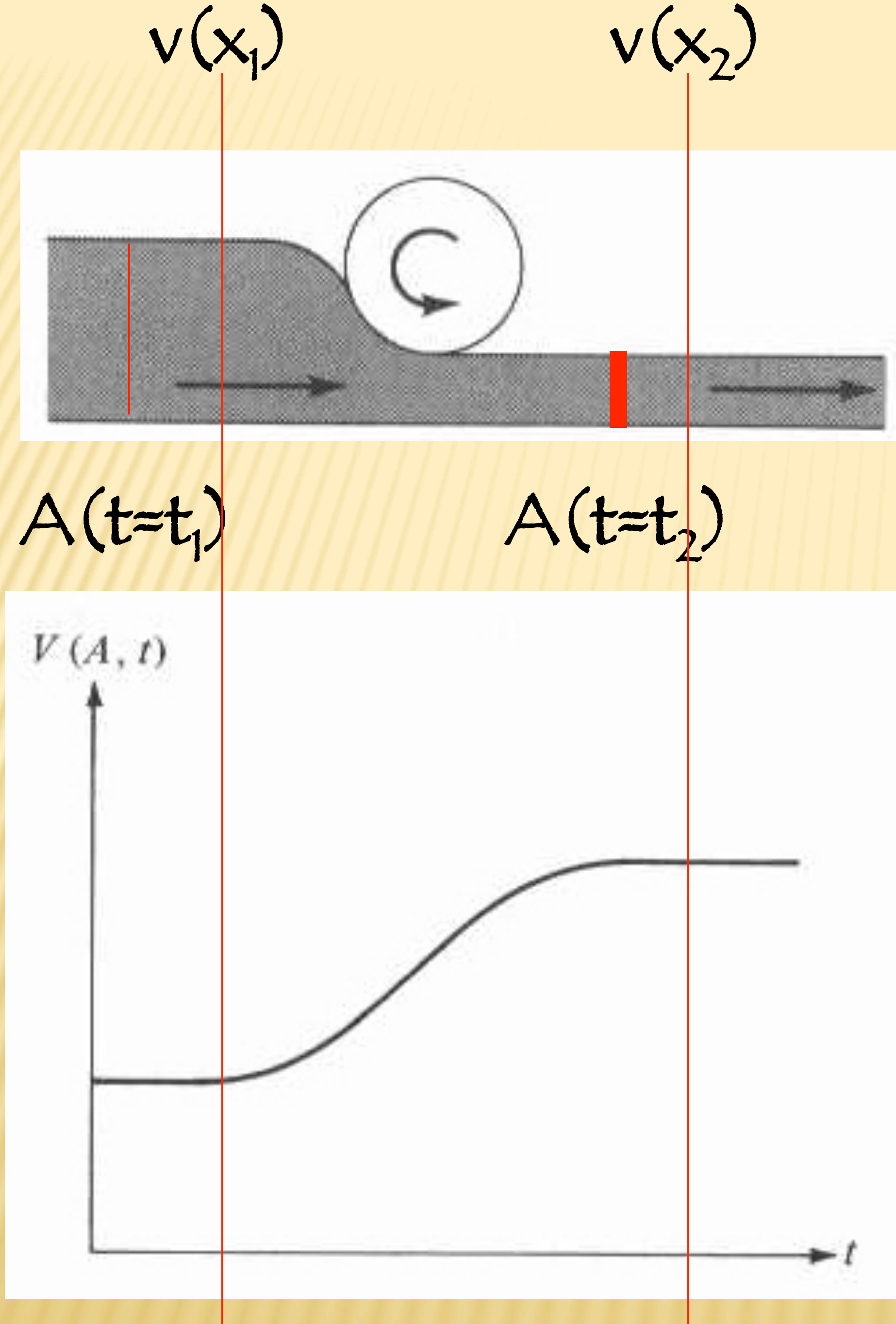
material

which is moving through space

and deforming

(not rigid body)

(we want/need our reference frame to be with respect to the material, not the coordinate system.



We know acceleration of material is not zero.

$$\frac{Df(A,t)}{Dt} = \frac{\partial f(x,t)}{\partial t} + \boxed{v(x,t)\frac{\partial f(x,t)}{\partial x}}$$

Term gives acceleration as one

<u>follows the material</u>

through space

(have to consider same material at t_1 and t_2)

Various names for this derivative

Substantive derivative

Lagrangian derivative

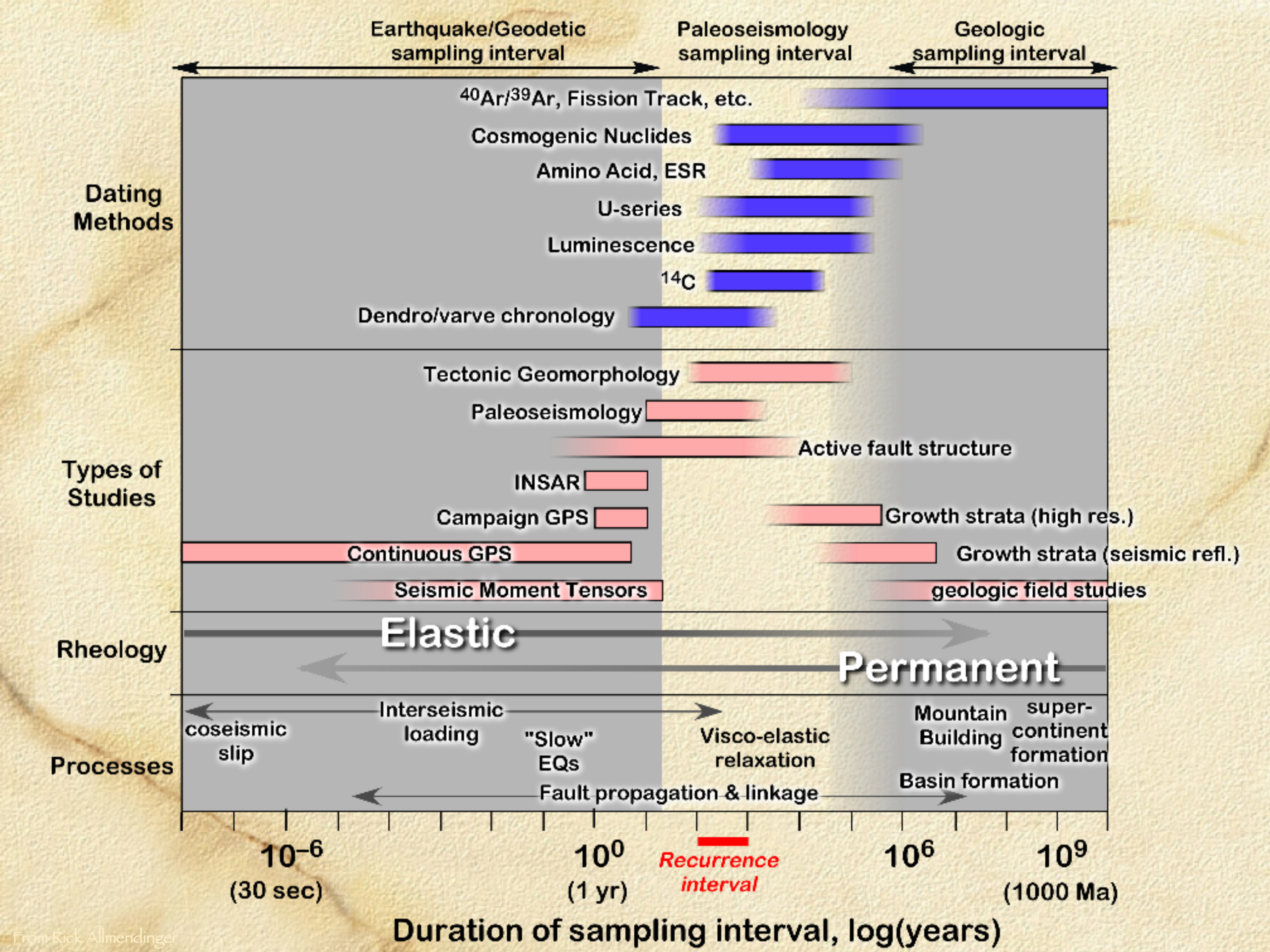
Material derivative

Advective derivative

Total derivative

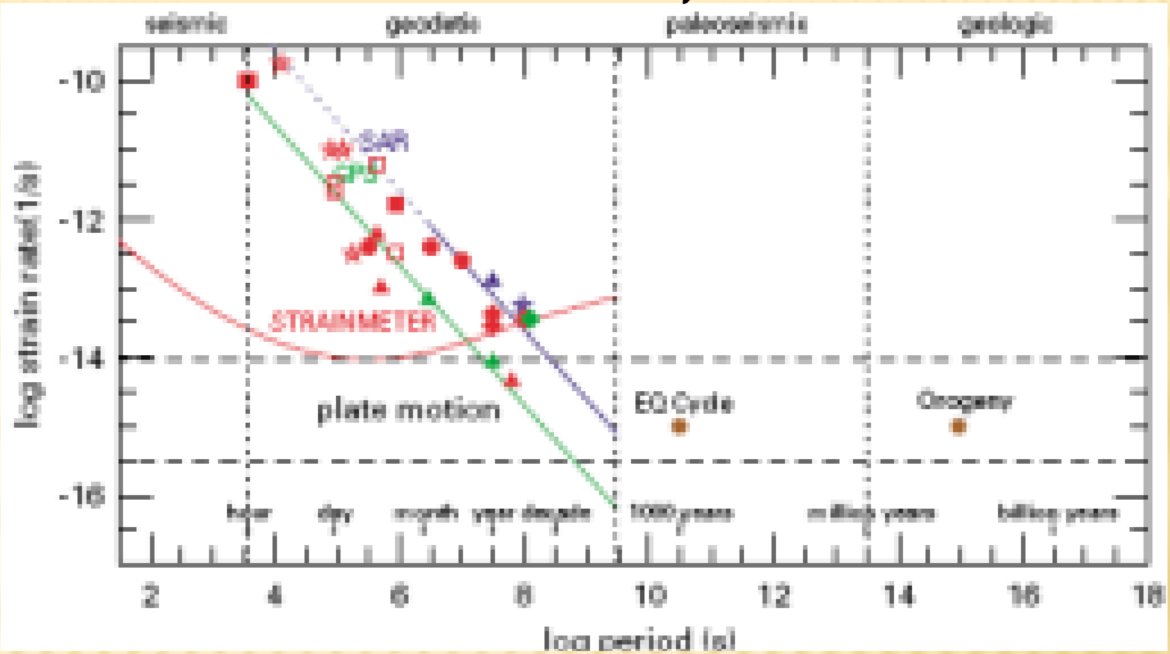
GPS and deformation

Now we examine relative movement between sites



Earthquake/Geodetic sampling interval
Paleoseismology sampling interval
Geologic sampling interval
Dating Methods
$^{40}Ar/^{39}Ar$, Fission Track, etc.
Cosmogenic Nuclides
Amino Acid, ESR
U-series
Luminescence
^{14}C
Dendro/varve chronology
Types of Studies
Tectonic Geomorphology
Paleoseismology
Active fault structure
INSAR
Campaign GPS
Growth strata (high res.)
Continuous GPS
Growth strata (seismic refl.)
Seismic Moment Tensors
geologic field studies
Rheology
Elastic
Permanent
Processes
coseismic slip
Interseismic loading
"Slow" EQs
Visco-elastic relaxation
Mountain Building
super-continent formation
Basin formation
Fault propagation & linkage
10^{-6} (30 sec)
10^{0} (1 yr)
Recurrence interval
10^{6}
10^{9} (1000 Ma)
Duration of sampling interval, log(years)
From Rick Allmendinger

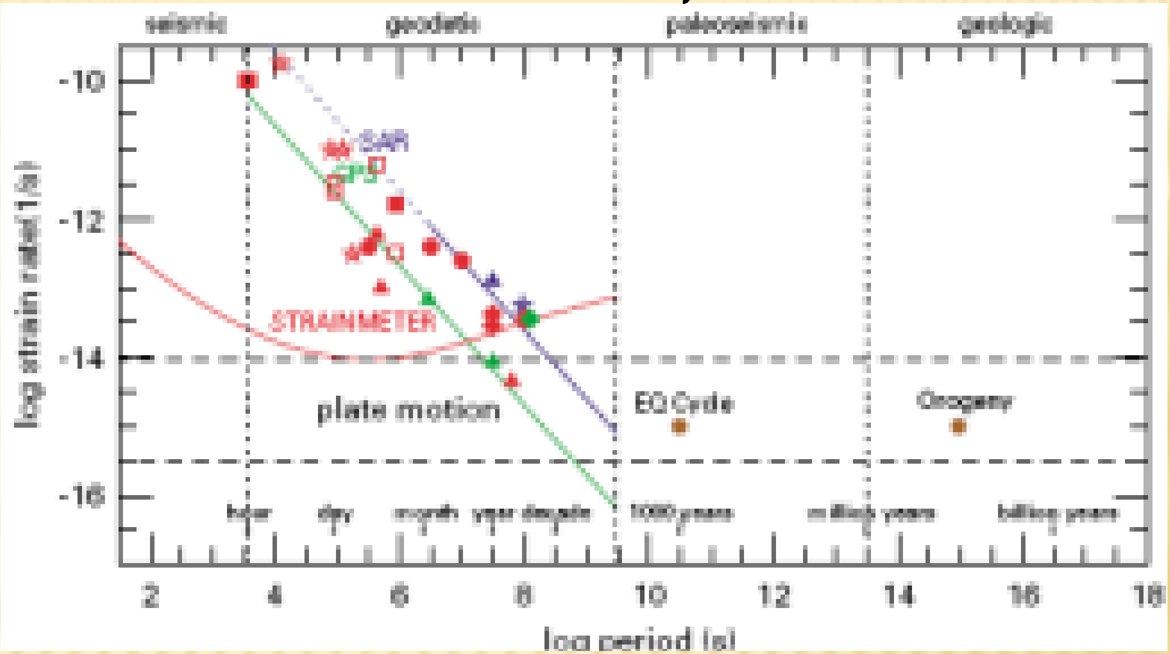
Strain-rate sensitivity thresholds (schematic) as functions of period



GPS and INSAR detection thresholds for 10-km baselines, assuming 2-mm and 2-cm displacement resolution for GPS and INSAR, respectively (horizontal only).

http://www.iris.iris.edu/USArray/EllenMaterial/assets/es_proj_plan_lo.pdf, http://www.iris.edu/news/IRISnewsletter/EE.Fall98.web/plate.html

Strain-rate sensitivity thresholds (schematic) as functions of period



Post-seismic deformation (triangles), slow earthquakes (squares), long-term aseismic deformation (diamonds), preseismic transients (circles), and volcanic strain transients (stars).

http://www.iris.iris.edu/USArray/EllenMaterial/assets/es_proj_plan_lo.pdf, http://www.iris.edu/news/IRISnewsletter/EE.Fall98.web/plate.html

Study deformation at two levels

-Kinematics –

describe motions

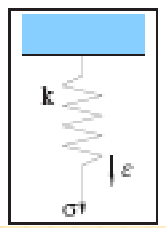
(Have to do this first)

-Dynamics –

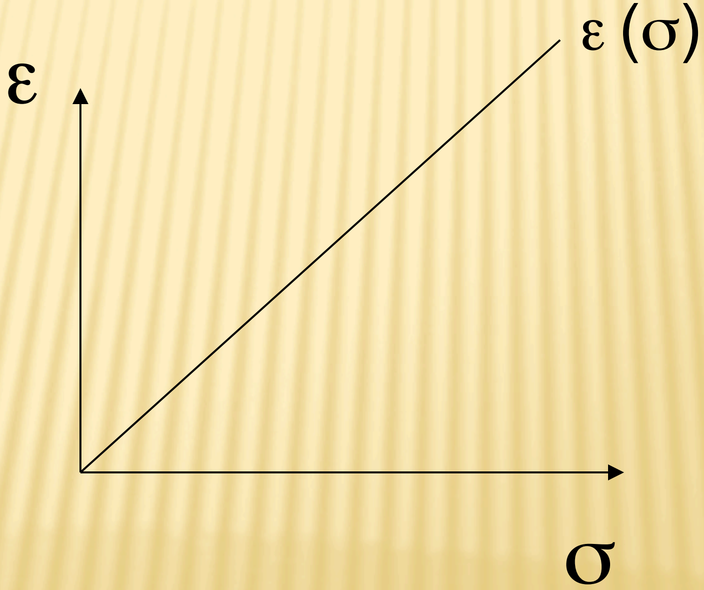
relate motions (kinematics) to forces (physics)

(Do through rheology/constitutive relationship/model. Phenomenological, no first principle prediction)

Simple rheological models

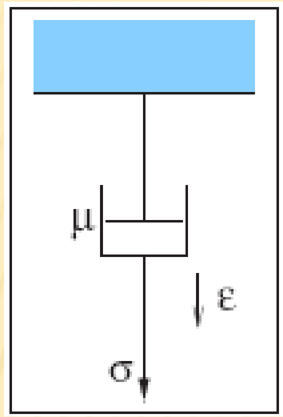


elastic $\sigma = K\varepsilon$

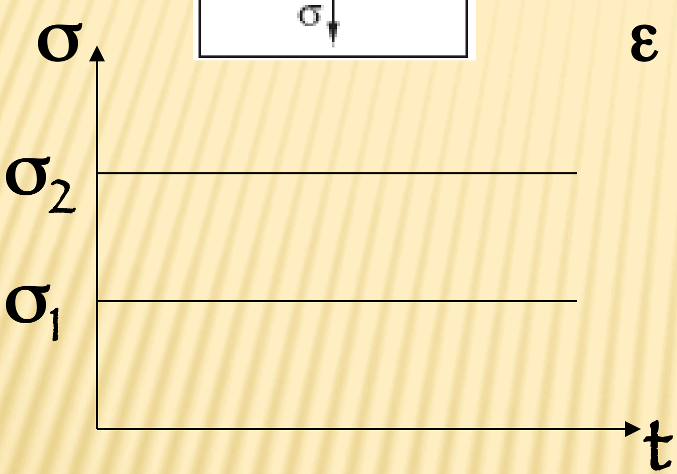


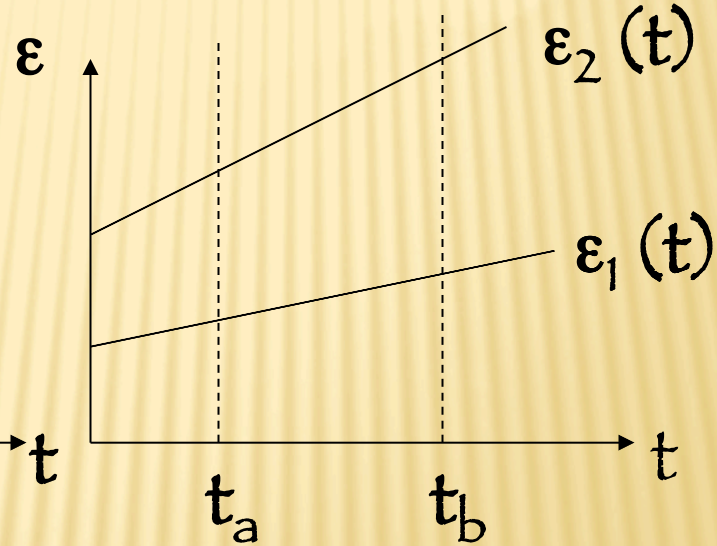
http://hcgl.eng.ohio-state.edu/~ce552/3rdMat06_handout.pdf

Simple rheological models



viscous $\quad \sigma = \mu \frac{d\varepsilon}{dt} = \mu\dot{\varepsilon}$

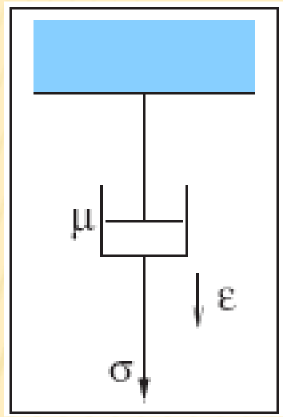




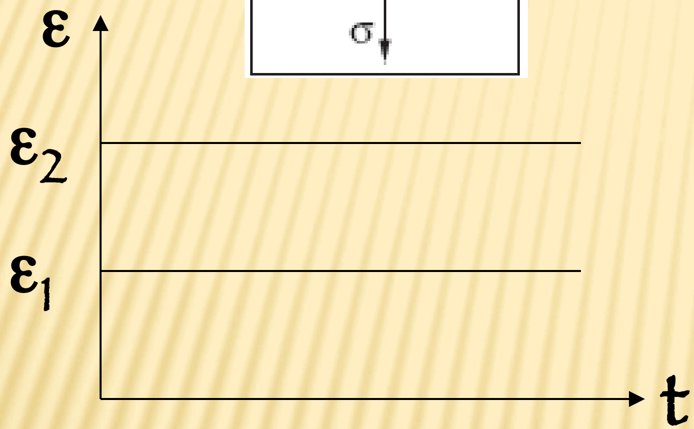
Apply constant stress, σ, to a viscoelastic material.
Record deformation (strain, ε) as a function of time.
ε increases with time.

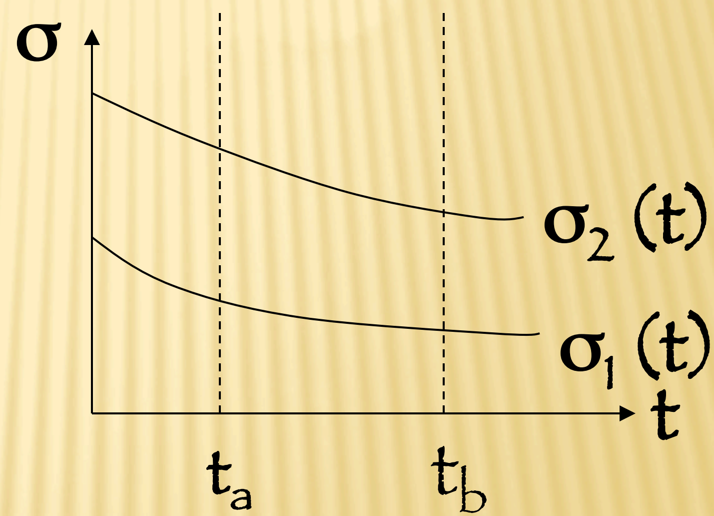
http://hcgl.eng.ohio-state.edu/~ce552/3rdMat06_handout.pdf

Simple rheological models



viscous $\sigma = \mu \dfrac{d\varepsilon}{dt} = \mu\dot{\varepsilon}$

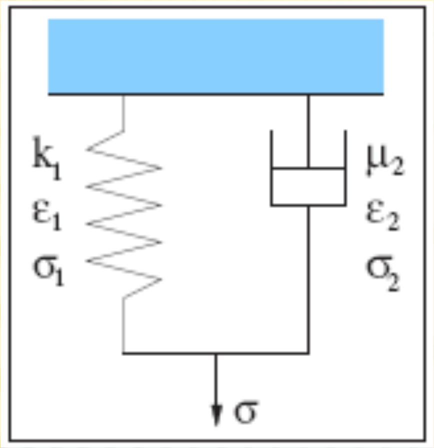




Maintain constant strain, record load stress needed.
Decreases with time.
Called relaxation.

viscoelastic

Kelvin rheology



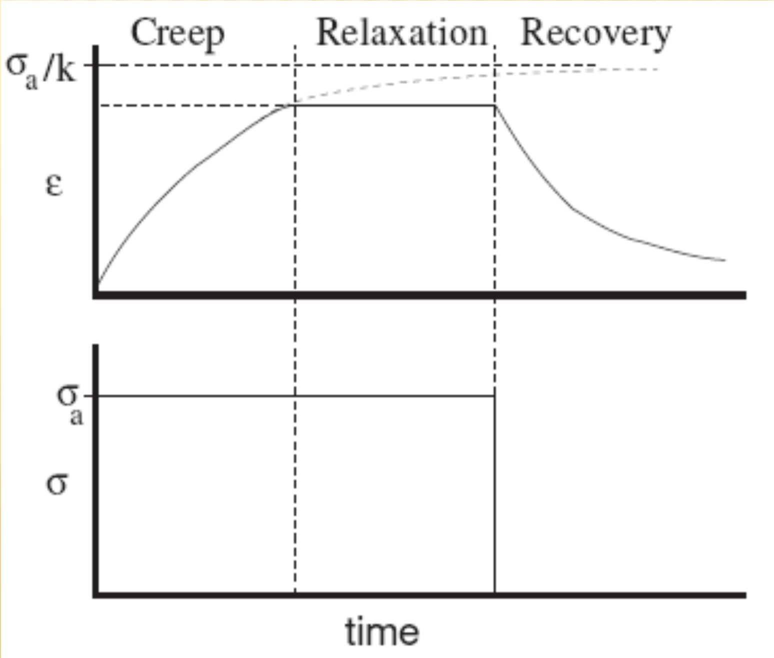
$$\sigma = \sigma_1 + \sigma_2$$

$$\varepsilon = \varepsilon_1 = \varepsilon_2$$

$$\sigma = K\varepsilon + \mu\dot{\varepsilon}$$

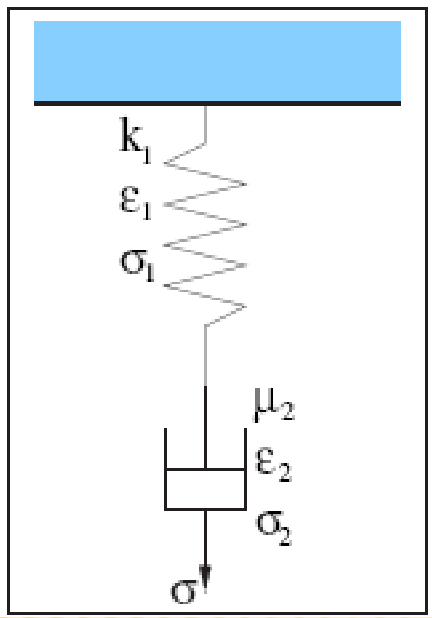
Handles creep and recovery fairly well

Does not account for relaxation



http://hcgl.eng.ohio-state.edu/~ce552/3rdMat06_handout.pdf

viscoelastic



Maxwell rheology

$$\sigma = \sigma_1 = \sigma_2$$

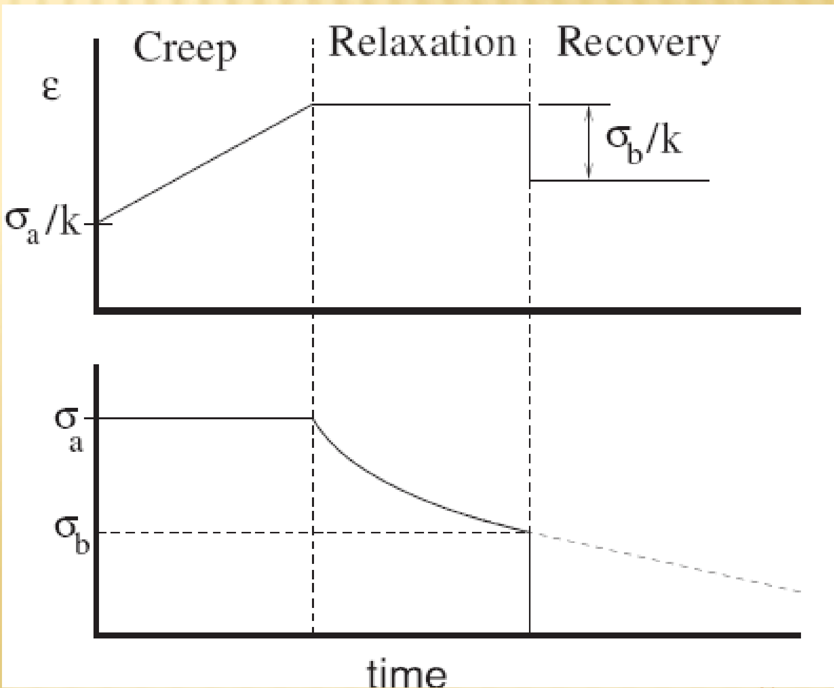
$$\varepsilon = \varepsilon_1 + \varepsilon_2$$

$$\dot{\varepsilon} = \frac{\sigma}{\mu} + \frac{\dot{\sigma}}{k}$$

Handles creep badly (unbounded)

Handles recovery badly (elastic only, instantaneous)

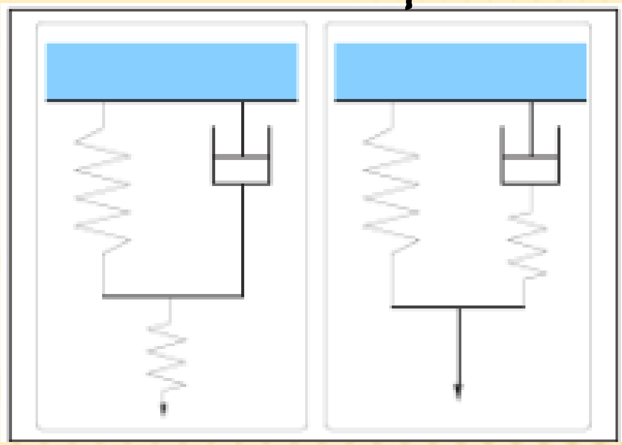
Accounts for relaxation fairly well



http://hcgl.eng.ohio-state.edu/~ce552/3rdMat06_handout.pdf

viscoelastic

Standard linear/Zener (not unique)



Spring in series with Kelvin

Spring in parallel with Maxwell

Stress – equal among components in series

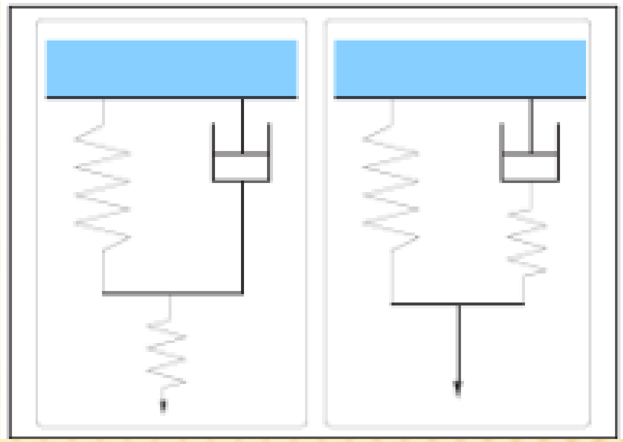
Total strain – sum all components in series

Strain – equal among components in parallel

Total stress – total of all components in parallel

http://hcgl.eng.ohio-state.edu/~ce552/3rdMat06_handout.pdf
www.mse.mtu.edu/~wangh/my4600/chapter4.ppt

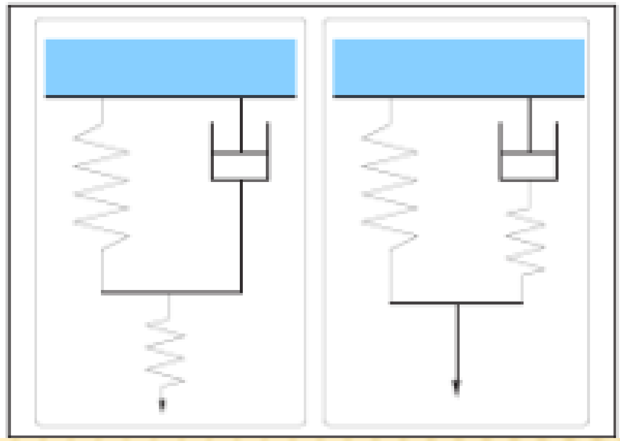
viscoelastic
Standard linear/Zener



Instantaneous elastic strain when stress applied
Strain creeps towards limit under constant stress
Stress relaxes towards limit under constant strain
Instantaneous elastic recovery when strain removed
Followed by gradual recovery to zero strain

http://hcgl.eng.ohio-state.edu/~ce552/3rdMat06_handout.pdf
www.mse.mtu.edu/~wangh/my4600/chapter4.ppt

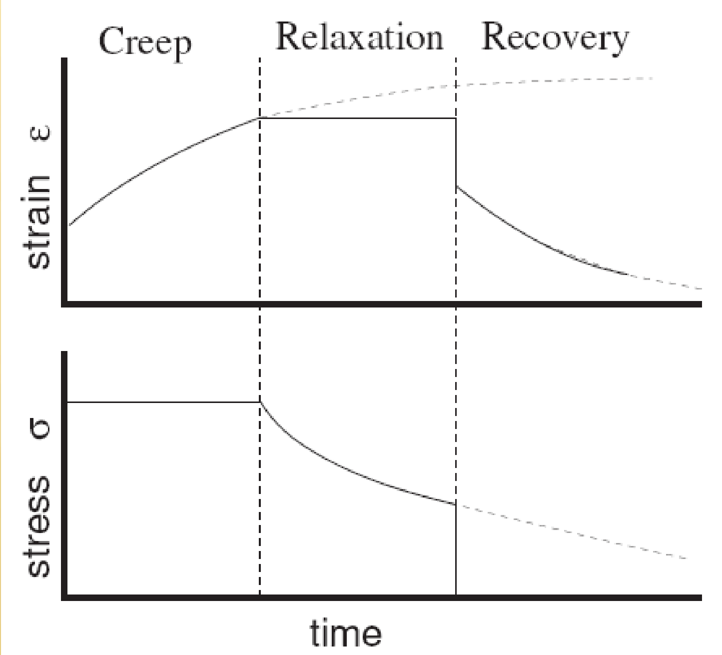
viscoelastic
Standard linear/Zener



Two time constants

- Creep/recovery under constant stress

- Relaxation under constant strain



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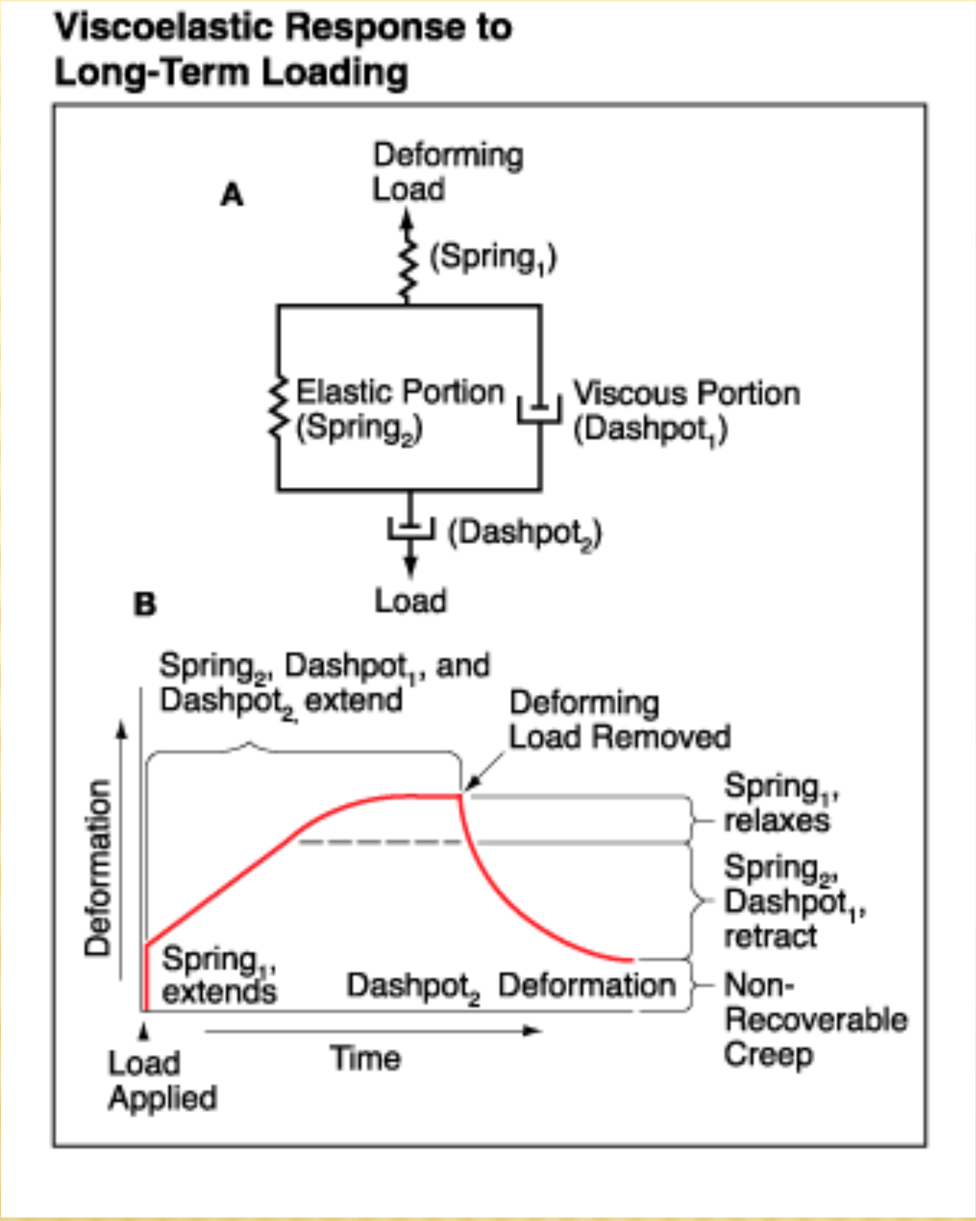


Can make arbitrarily complicated to match many deformation/ strain/time

relationships

http://www.dow.com/styron/design/guide/modeling.htm

Three types faults and plate boundaries

- Faults -

Strike-slip
Thrust
Normal

- Plate Boundary -

Strike-slip
Convergent
Divergent

How to model

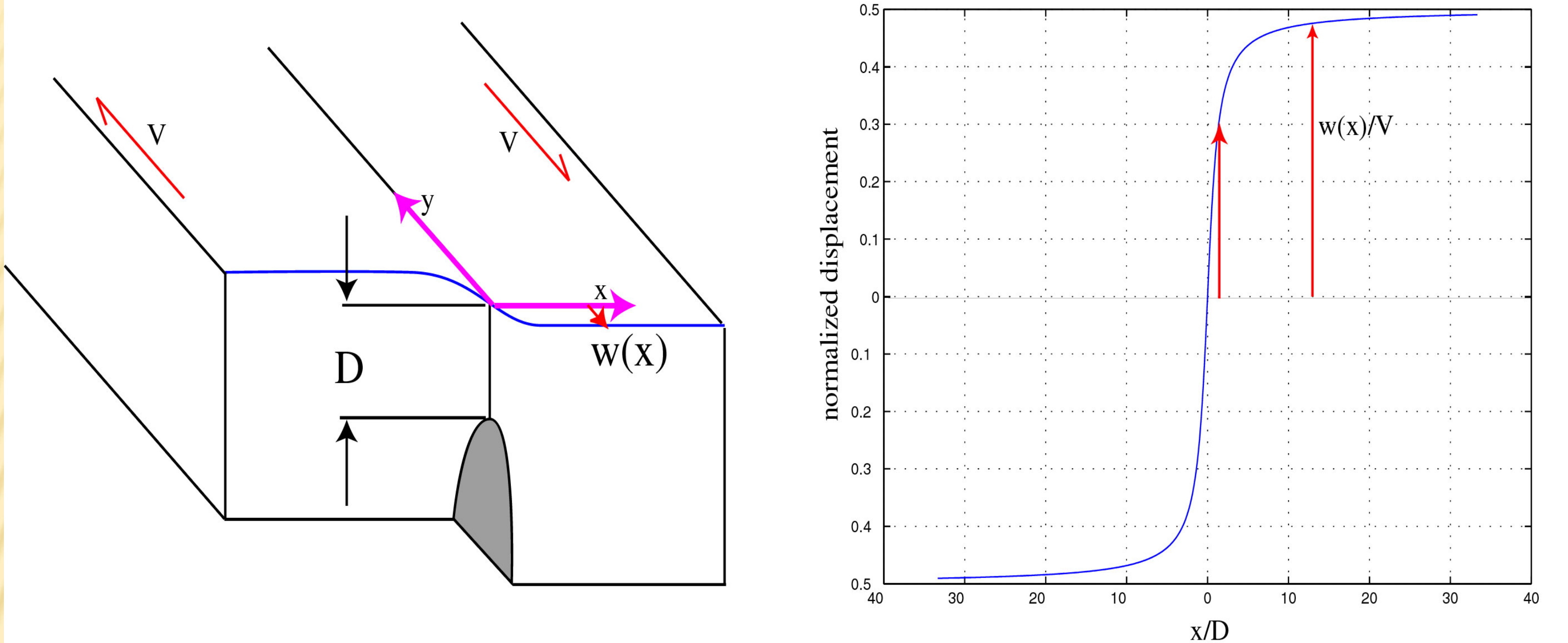
Elastic

Viscoelastic

Half space

Layers

Inhomogeneous

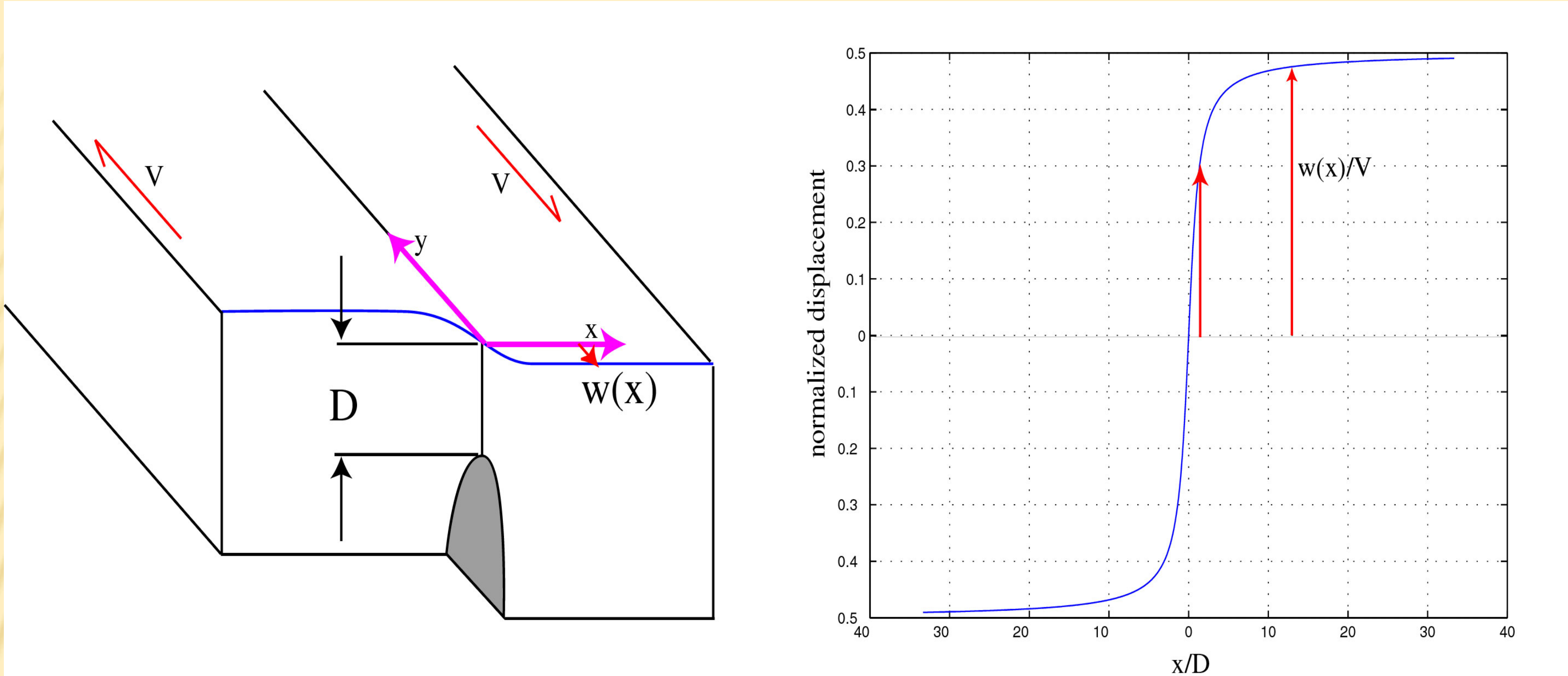


$w(x)=(V/\pi)\text{atan}(x/D)$

2-D model for strain across strike-slip fault in elastic half space.

Fault is locked from surface to depth D, then free to infinity.

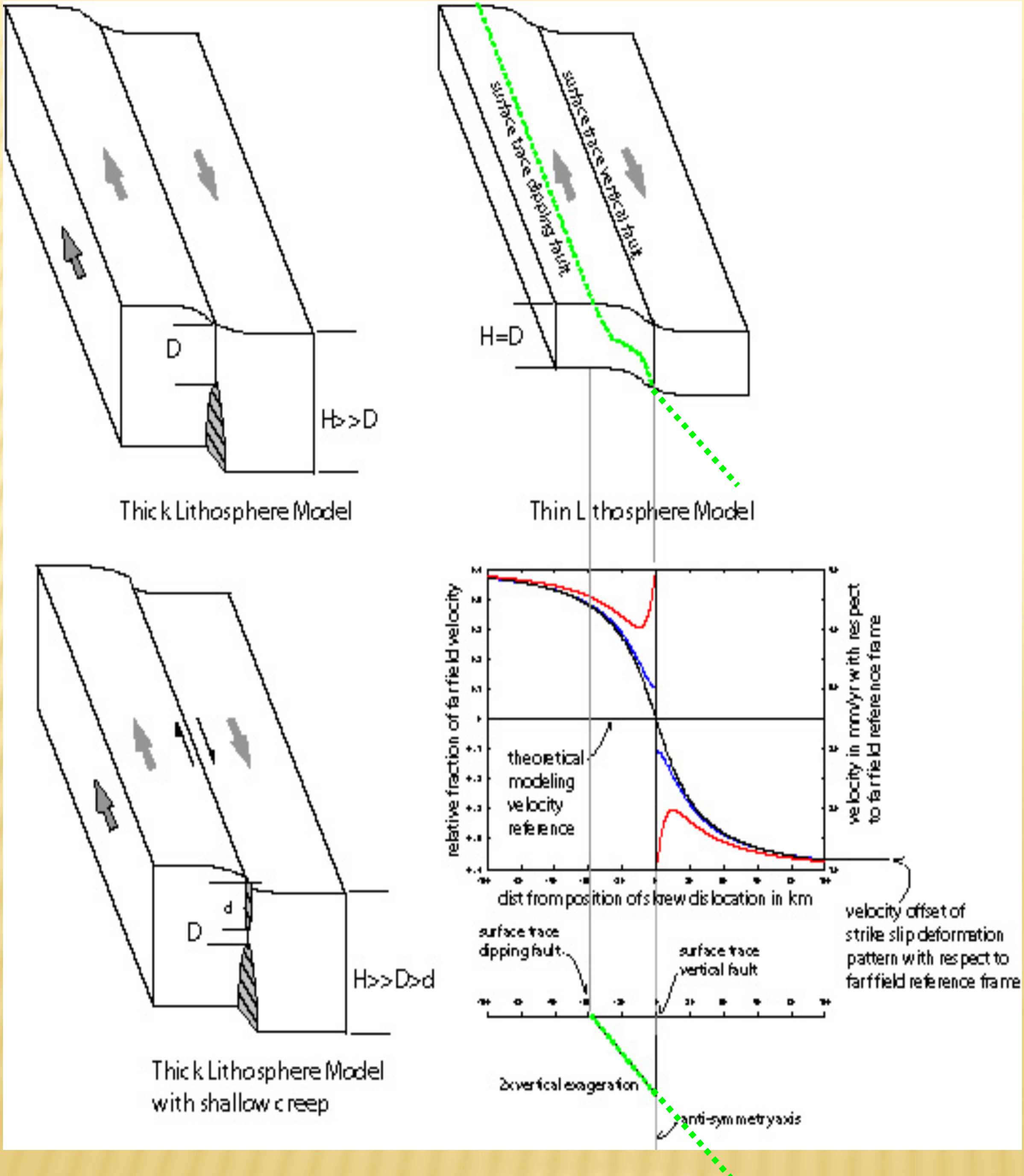
Far-field displacement, V, applied.



$w(x)=(V/\pi)\mathrm{atan}(x/D)$

w(x) is the equilibrium displacement parallel to y at position x.

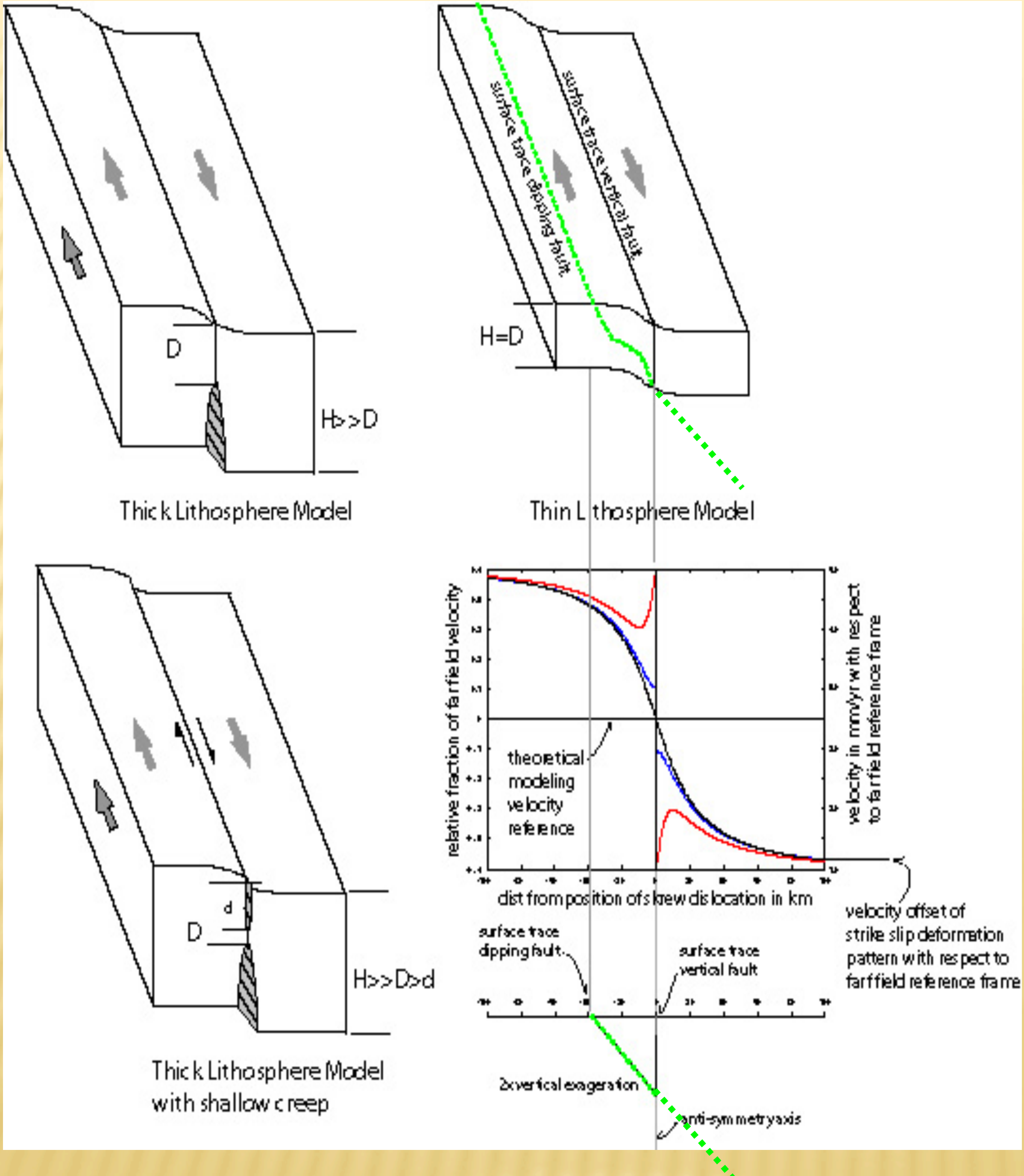
|w| is 50% max at x/D=.93; 63% at x/D=1.47 & 90% at x/D=6.3



Effect of fault dip.

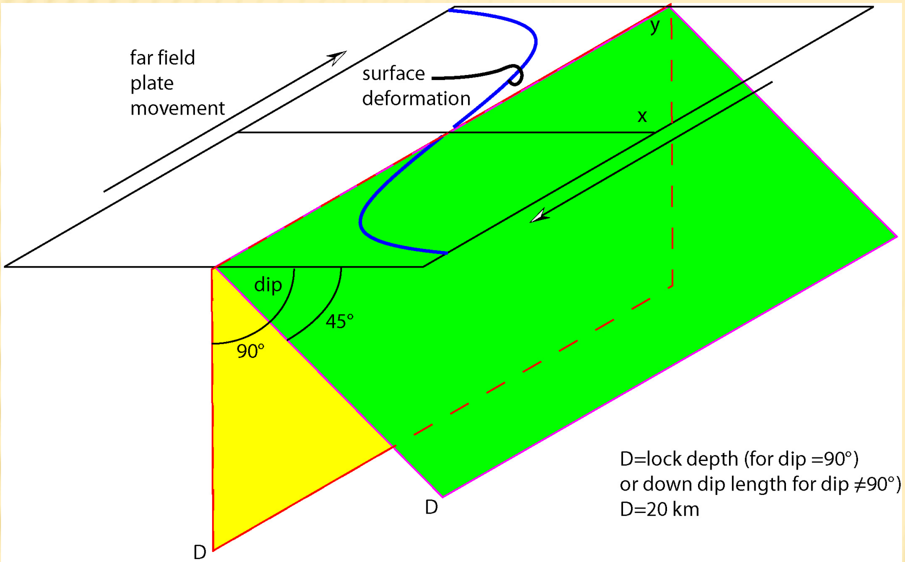
The fault is locked from the surface to a depth D (not a down dip length of D).

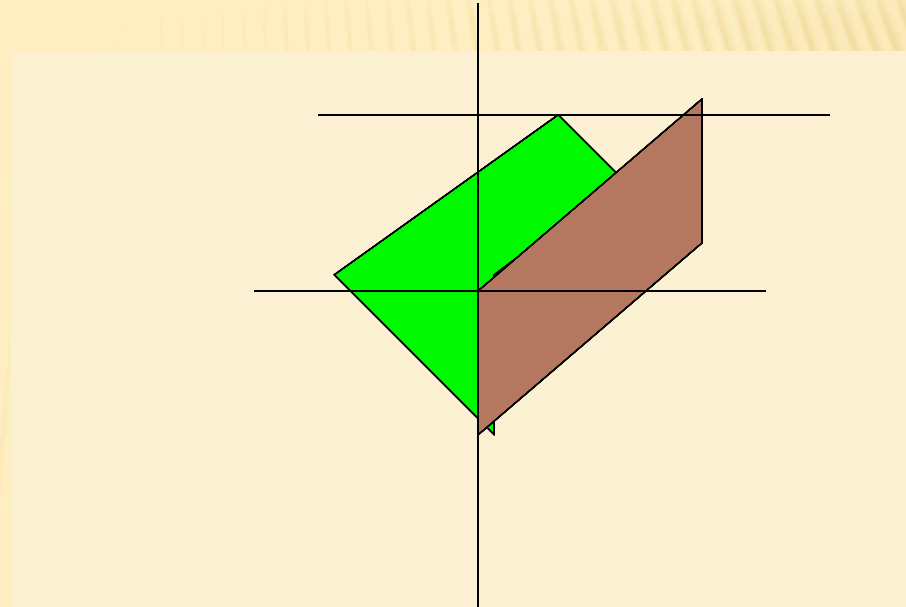
The fault is free from this depth to infinity.

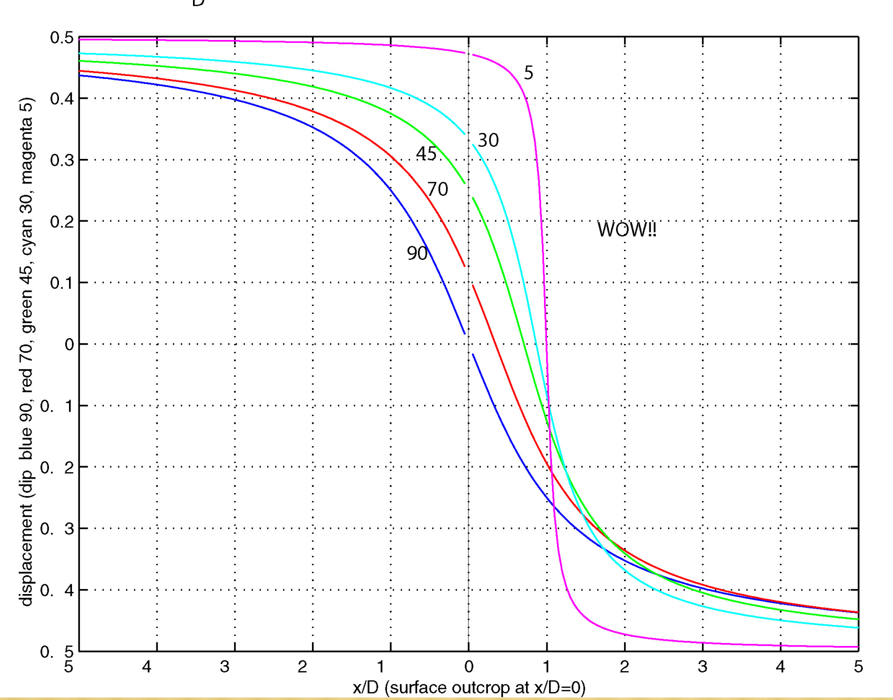


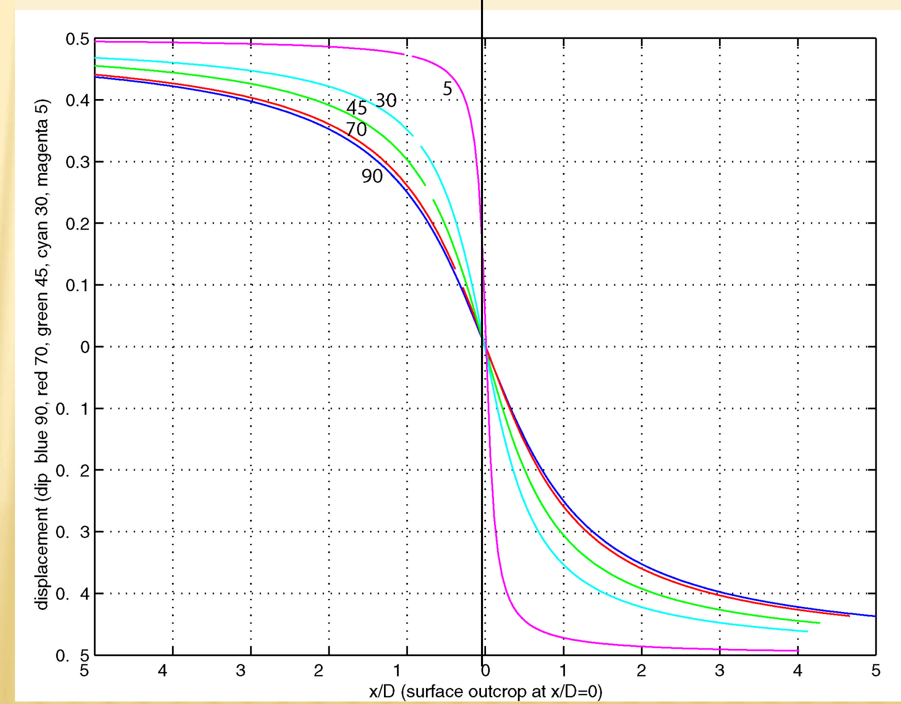
Surface deformation pattern is <u>SAME</u> as for vertical fault, but centered over <u>down dip end</u> of dipping fault.

Dip estimation from center of deformation pattern to surface trace and locking depth.

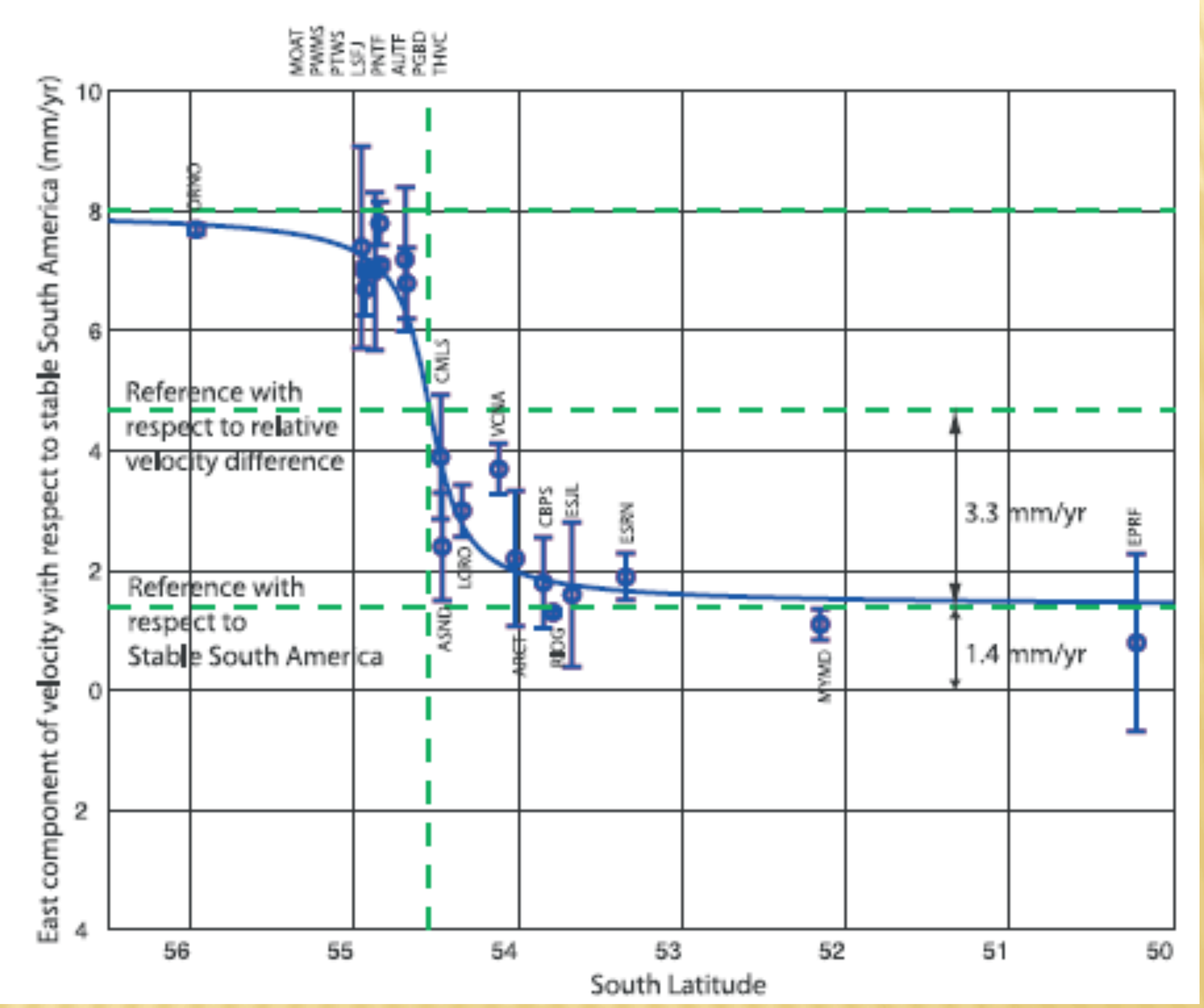

far field
plate
movement
surface
deformation
y
x
dip
45°
90°
D
D
D=lock depth (for dip =90°)
or down dip length for dip ≠90°)
D=20 km

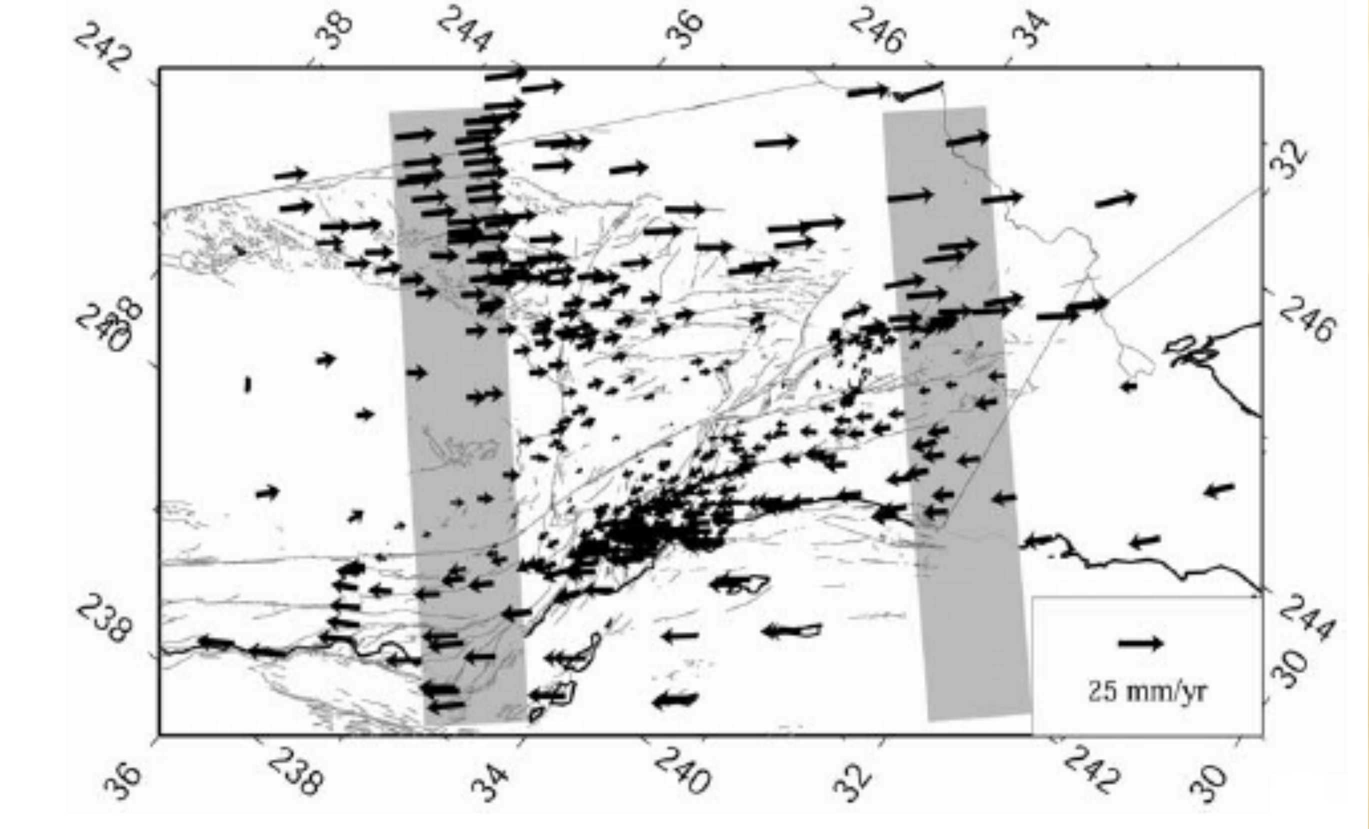



displacement (dip blue 90, red 70, green 45, cyan 30, magenta 5)
x/D (surface outcrop at x/D=0)
5
30
45
70
90
WOW!!


displacement (dip blue 90, red 70, green 45, cyan 30, magenta 5)
x/D (surface outcrop at x/D=0)
5
30
45
70
90

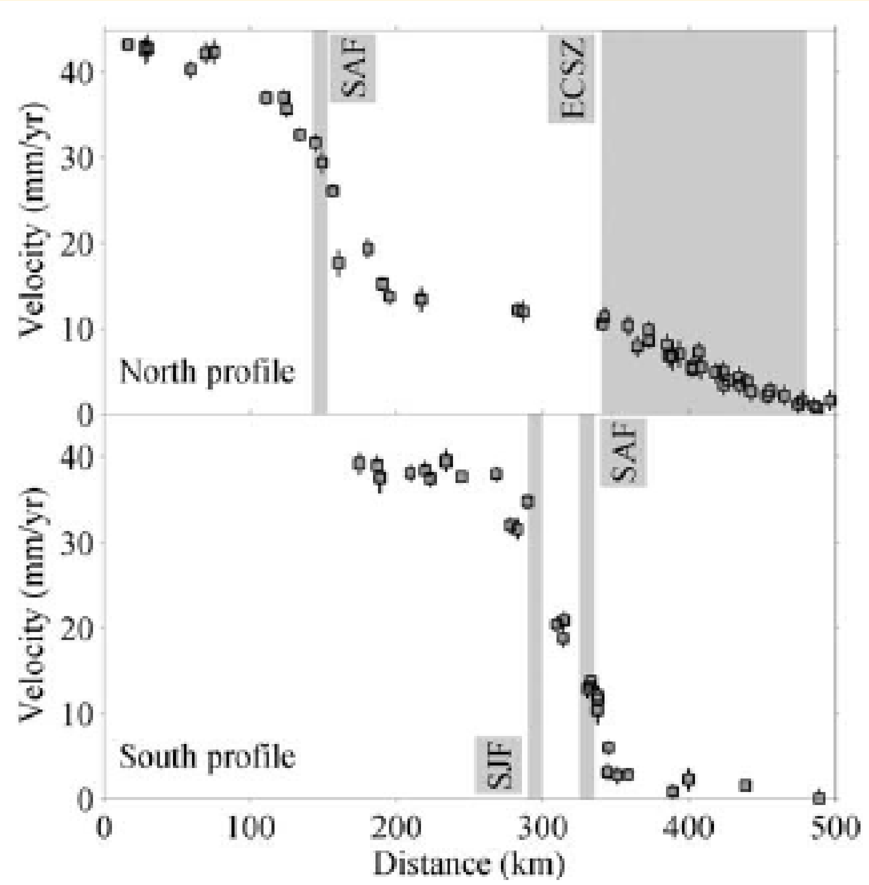

69W
68W
67W
66W
65W
10 mm/yr
ESRN
ESJL
RIOG
CBPS
ARCT
VCNA
LCRO
ASND
CMLS
PGBD
THVC
PNTF
AUTF
LSFJ
PWMS
PTWS
MOAT
ORNO
54S
55S
56S


MOAT
PWMS
PTWS
LSFJ
PNTF
AUTF
PGBD
THVC
East component of velocity with respect to stable South America (mm/yr)
South Latitude
Reference with respect to relative velocity difference
Reference with respect to Stable South America
3.3 mm/yr
1.4 mm/yr
ORNO
CMLS
VCMA
ASND
LCRO
ARCT
CBPS
RIOG
ESJL
ESRN
MYMD
EPRF



Interseismic velocities in southern California from GPS

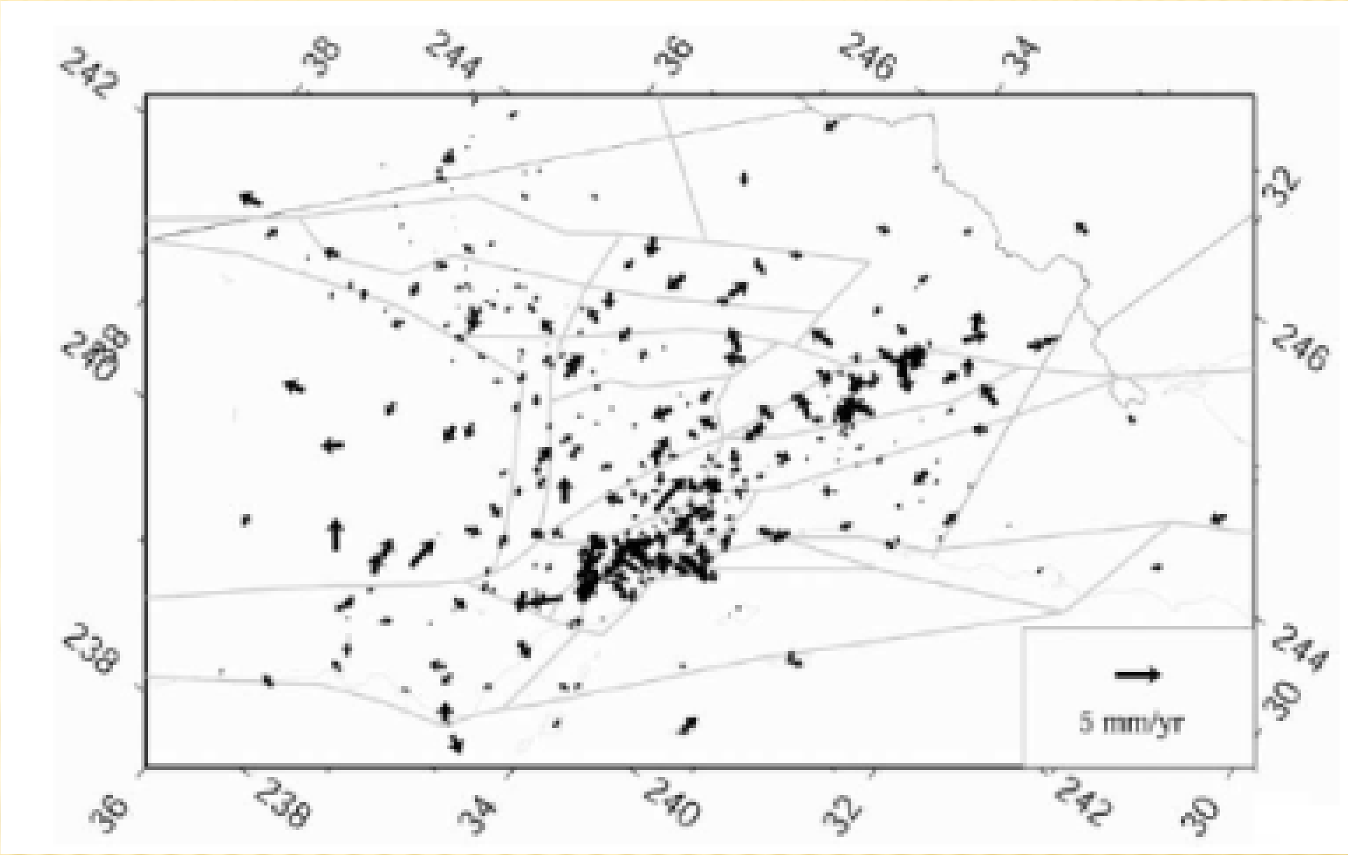
Meade and Hager, 2005



Fault parallel velocities for northern and southern "swaths".

Total change in velocity ~42mm/yr on both.

Meade and Hager, 2005



Residual (observed-model) velocities for block fault model (faults in grey)

Meade and Hager, 2005

Modeling velocities in California

$$\vec{V}(\vec{r}) = \Omega(\vec{r}) \times \vec{r} + \sum_{f=1}^{F} G \bullet s_f$$

Ω is the angular velocity vector

effect of interseismic strain accumulation is given by an elastic Green's function G

response to backslip distribution, s, on each of, f, faults.

Modeling Broadscale Deformation From Plate Motions and Elastic Strain Accumulation, Murray and Segall, USGS NEHRP report.

$$\vec{V}(\vec{r}) = \Omega(\vec{r}) \times \vec{r} + \sum_{f=1}^{F} G \bullet s_f$$

In general, the model can accommodate zones of distributed horizontal deformation if **Ω** varies within the zones

latter terms can account both for the Earth's sphericity and viscoelastic response of the lower crust and upper mantle.

Modeling Broadscale Deformation From Plate Motions and Elastic Strain Accumulation, Murray and Segall, USGS NEHRP report.

$$\sum_{f=1}^{F} G \bullet s_f \rightarrow -\frac{a}{\pi} \sum_{f=1}^{F} \Delta\omega_f \sin\phi_f \tan^{-1}\left(\frac{d_f}{a\left(\phi - \phi_f\right)} \right)$$

Where

a is the Earth radius

distance from each fault located at ϕ_f is $a(\phi - \phi_f)$.

Each fault has deep-slip rate

$a\Delta\omega_f \sin\phi_f$,

where $\Delta\omega_f$ is the difference in angular velocity rates on either side of the fault.

Modeling Broadscale Deformation From Plate Motions and Elastic Strain Accumulation, Murray and Segall, USGS NEHRP report.