

Earth Science Applications of Space Based Geodesy

DES-7355

Tu-Th

9:40-11:05

Seminar Room in 3892 Central Ave. (Long building)

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678-4929

Office Hours – Wed 14:00-16:00 or if I'm in my office.

http://www.ceri.memphis.edu/people/smalley/ESCI7355/ESCI_7355_Applications_of_Space_Based_Geodesy.html

Class 11

(incomplete)

look at

Applications of GPS

in Earth Sciences



carpincho
or
capybara



Use the Global Positioning System (GPS) to determine accurate positions (order mm) of “high stability” geodetic benchmarks over time to determine changes in relative positions (order mm/year).

Principal tenet/Central assumption of
plate tectonics:

plates (interiors) are rigid

-Observation -

Plates move with respect to one another

-Secondary tenet/assumption -

Interaction limited to (narrow) plate boundary zones
where deformation is allowed

Plate motions --- NUVEL vs GPS

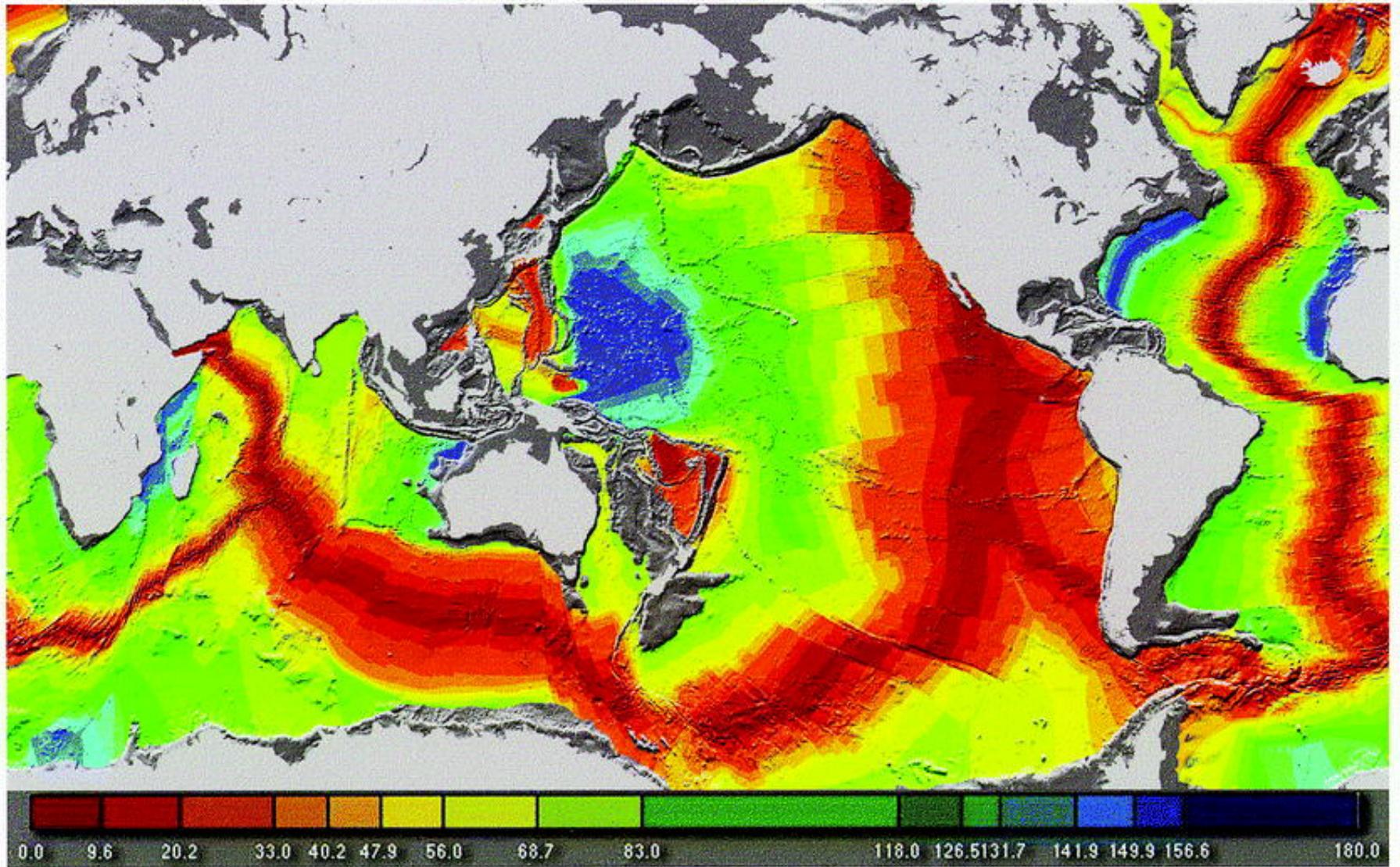
NUVEL – geologic

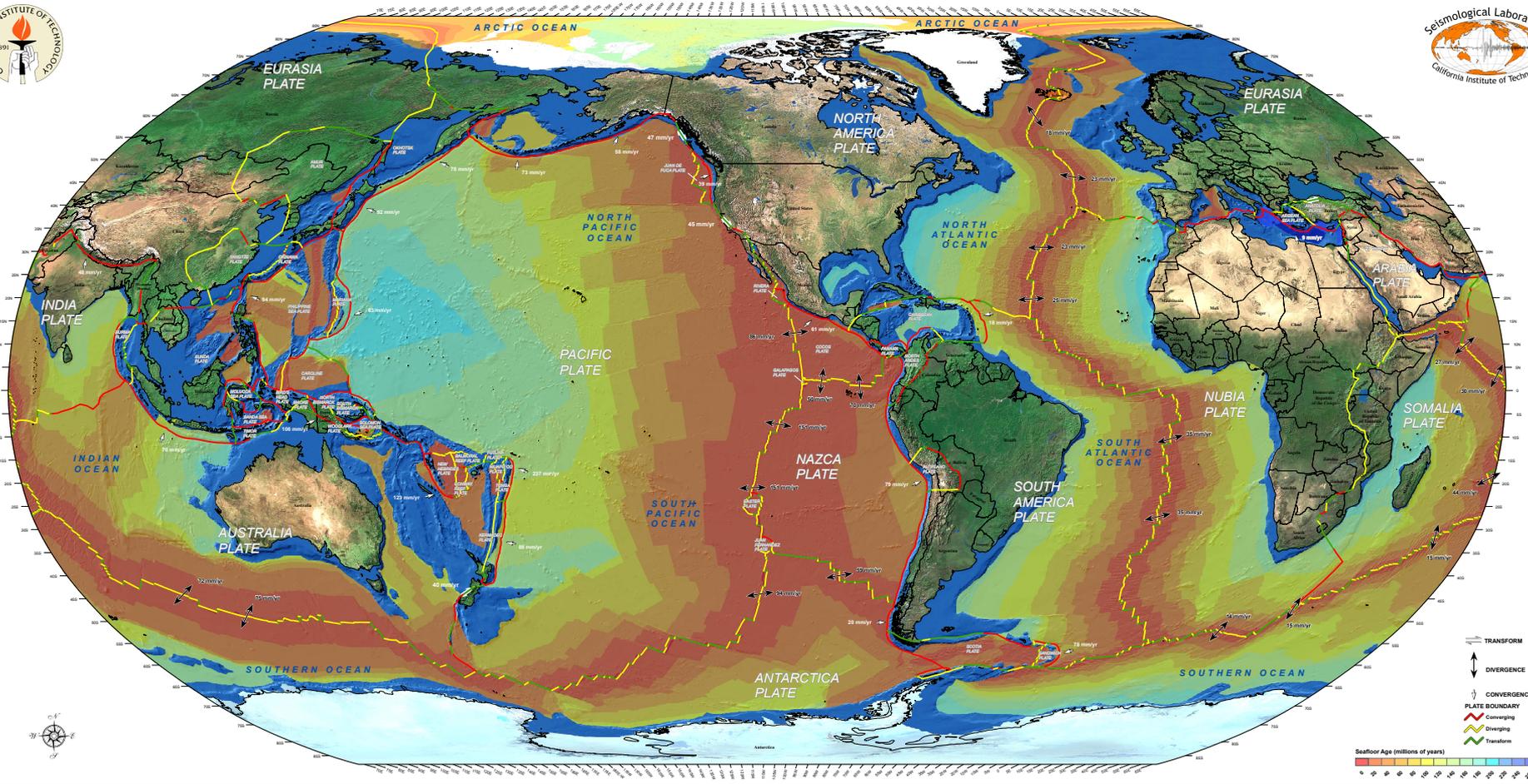
Spreading rate and orientation (Ma ave)
Transform fault orientation (no rate info, Ma ave)
Earthquake Focal mechanism (problem with slip
partitioning, 30 yr ave - actual)

GPS – non-geologic

Measures relative movement (20 yr ave – actual)
Can't test (yet) plate stability assumption

Strain rates in
stable plate interiors ~
bounded between
 10^{-12} - 10^{-11} and 10^{-10} year⁻¹.





This map builds on the Tectonic Plates map by adding seafloor age data. Note that the youngest seafloor ages are found at the mid-ocean spreading ridges, where new rock is constantly coming up from under the crust to heal the rifts formed as tectonic plates move away from one another. Most new crust forms at these mid-ocean spreading ridges.

The crust ages as it moves away from the spreading ridges, and eventually gets pushed back into the Earth in a subduction zone. Because oceanic crust subducts more easily than continental crust, all of the seafloor eventually is recycled by subduction while very little continental crust is subducted. The result is that the oldest oceanic crust is still much younger than the oldest continental crust.

The oldest seafloor in the world is found in the Mediterranean Sea. The next oldest seafloor ages are found in the northeastern Atlantic and the northeastern Pacific, far from any spreading ridges. The northeastern Pacific also has a long convergent boundary, where some of the oldest seafloor is now being subducted back into the interior of the Earth. In areas where spreading rates are slow, seafloor age changes quickly as you move away from the spreading ridge. Conversely, in areas where spreading rates are fast, seafloor age changes more slowly as you move away from the spreading ridge.

SEAFLOOR AGE

TRANSFORM
 ↑ ↓
DIVERGENCE
 ↓ ↑
CONVERGENCE
PLATE BOUNDARY
 → ← (Diverging)
 ← → (Converging)
 ↕ (Transform)

Seafloor Age (millions of years)

Plate Boundary:
 Referenced from Eric Platt (2002) An updated digital model of plate boundaries. (Open source project)

Plate Convergence Vectors:
 Referenced from United States Geological Survey. Convergence data are shown by arrow describing direction and speed, relative to the plate across the boundary.

Plate Divergence Vectors:
 Referenced from United States Geological Survey. Divergence data are shown by double arrow describing direction and speed.

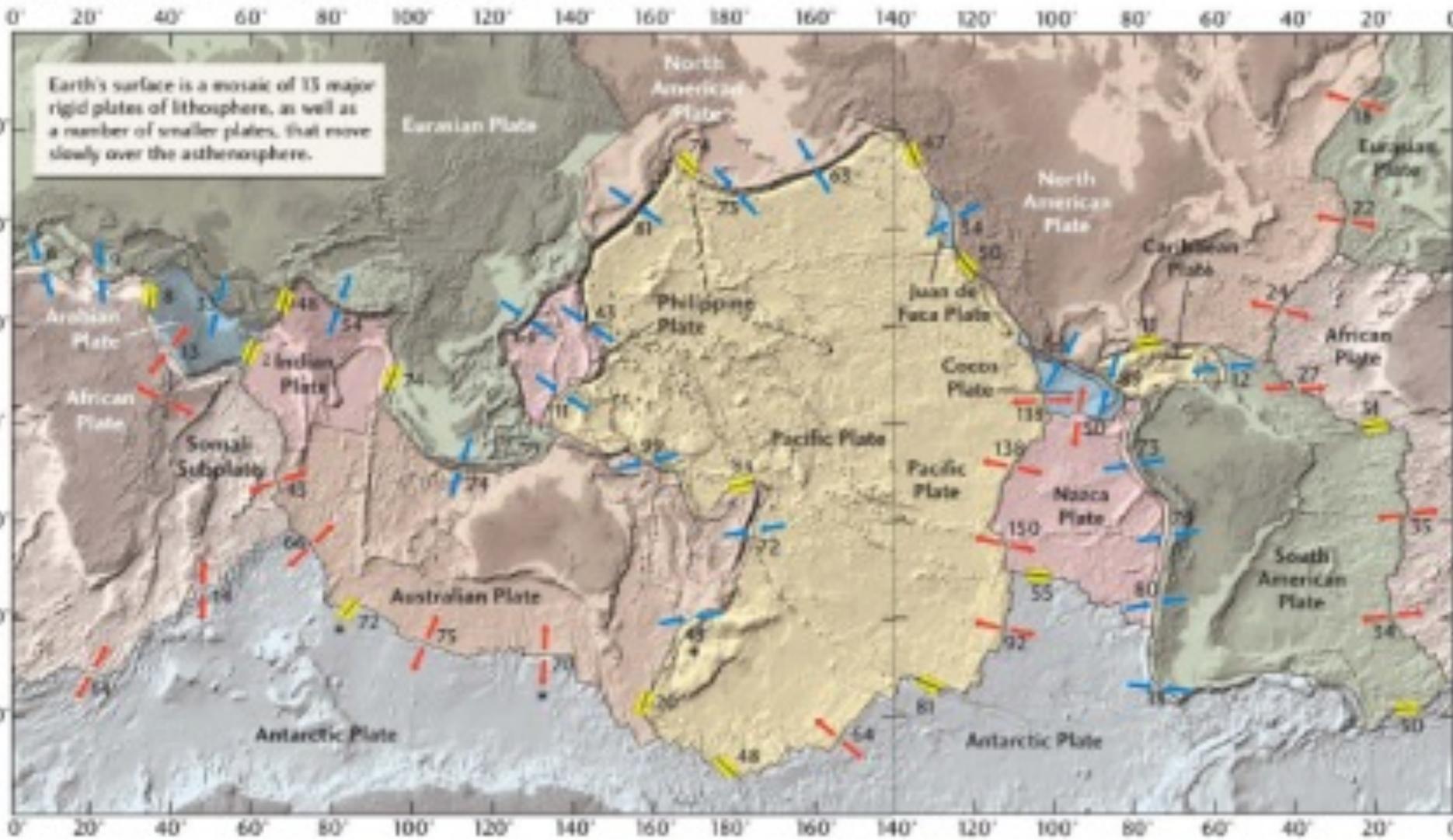
Seafloor Age:
 Referenced from Muller, R.D., M. Sdrolias, C. Gaina, and W.R. Ryan (2008). Age, subsidence and spreading geometry of the world ocean crust. *Geochim. Geophys. Res.* 13: Q08002. doi:10.1029/2007GC007143

Projection: Robinson (2004)

Produced by
 California Institute of Technology
 Seismological Laboratory
 April 2008

NUVEL picture

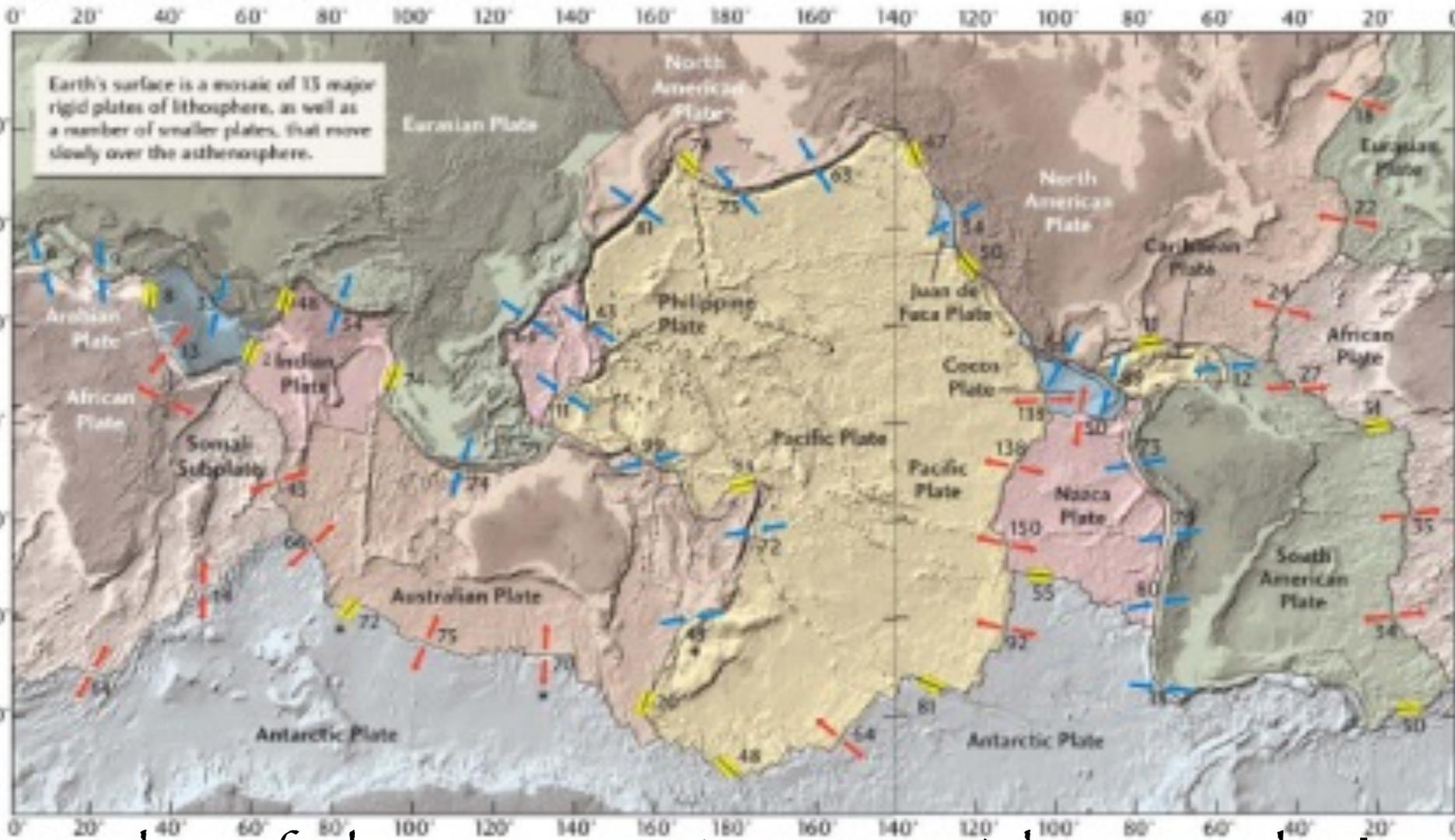
EARTH'S LITHOSPHERE IS MADE OF MOVING PLATES



Relative velocities across boundaries

NUVEL picture

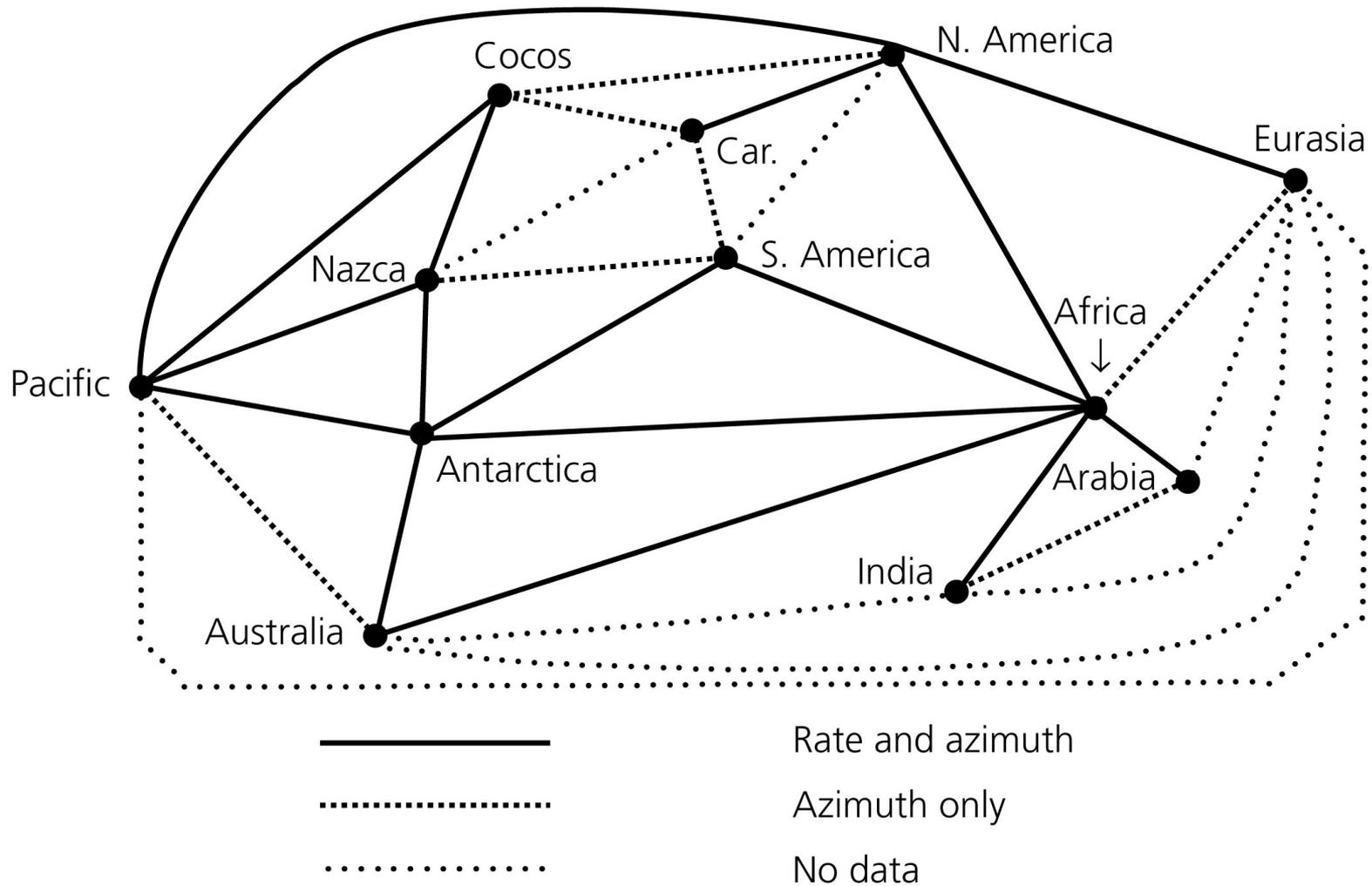
EARTH'S LITHOSPHERE IS MADE OF MOVING PLATES



Number of plates missing (e.g. Scotia) because don't have spreading boundaries (only place we can determine velocity). 11

NUVEL picture

Global plate circuit



Number of plates missing (e.g. Scotia) because don't have spreading boundaries (only place we can determine velocity) Stein and Wysession 12.

First big contribution of space based geodesy

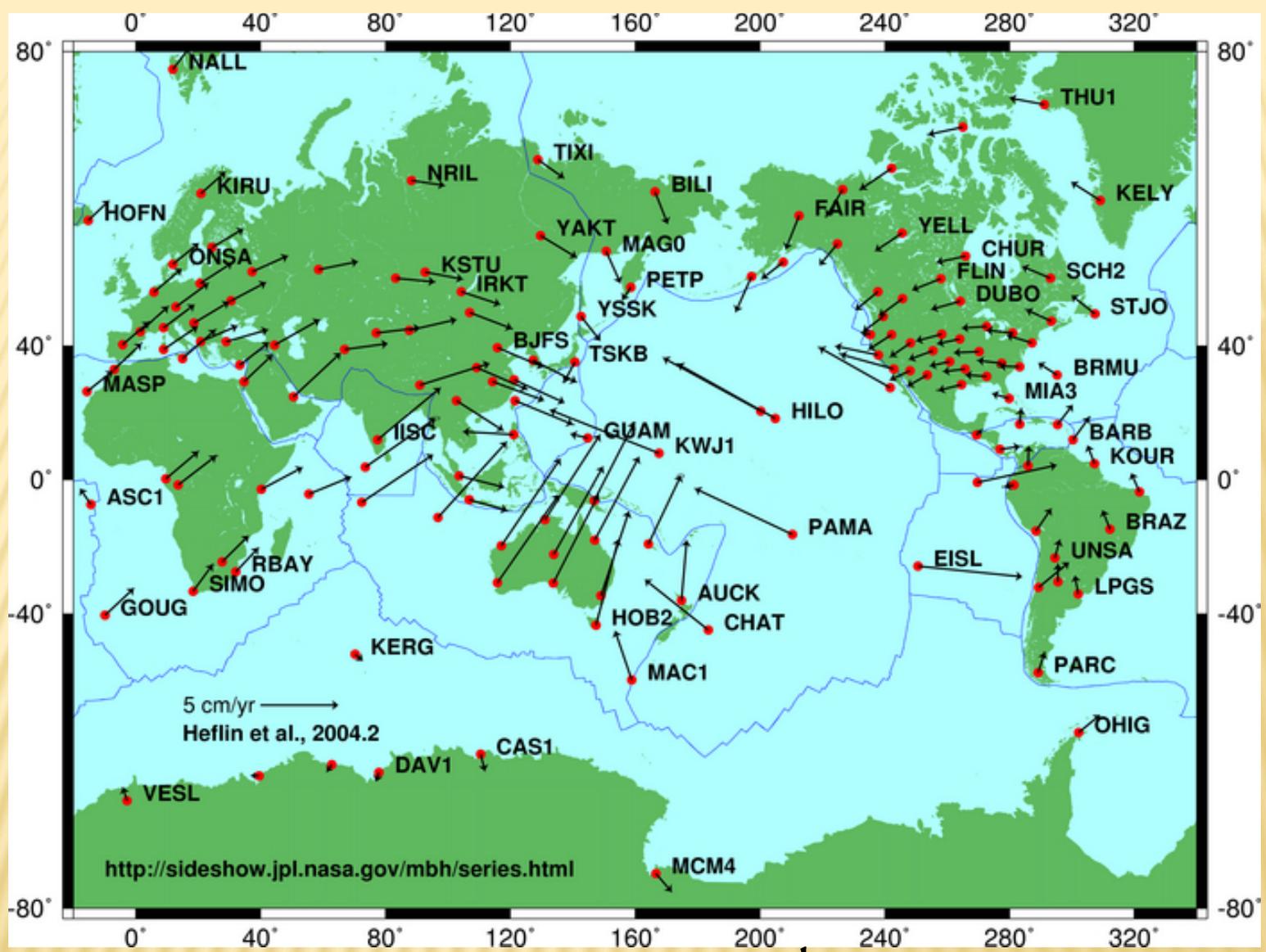
Motion of plates

(note –
plates

- have to be “pre-defined”

- are not part of how velocities of sites are computed,
-selected based on “rigidity” at level of GPS precision

Also VLBI, SLR, DORIS – space based, not limited to
GPS – results)



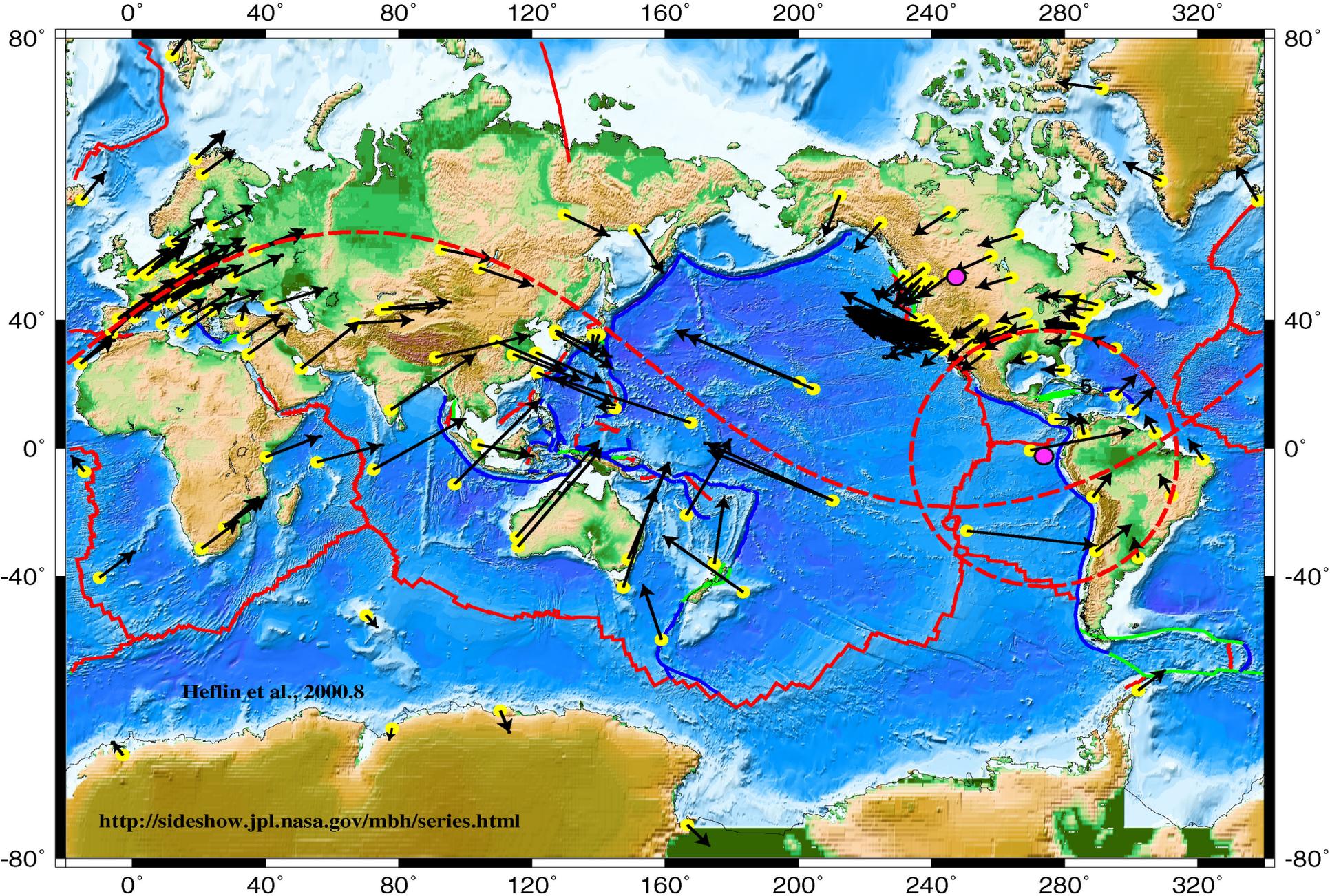
GPS picture – now motion with respect to some “absolute reference frame (ITRF), does not know about “plates”

two distinct reference systems:

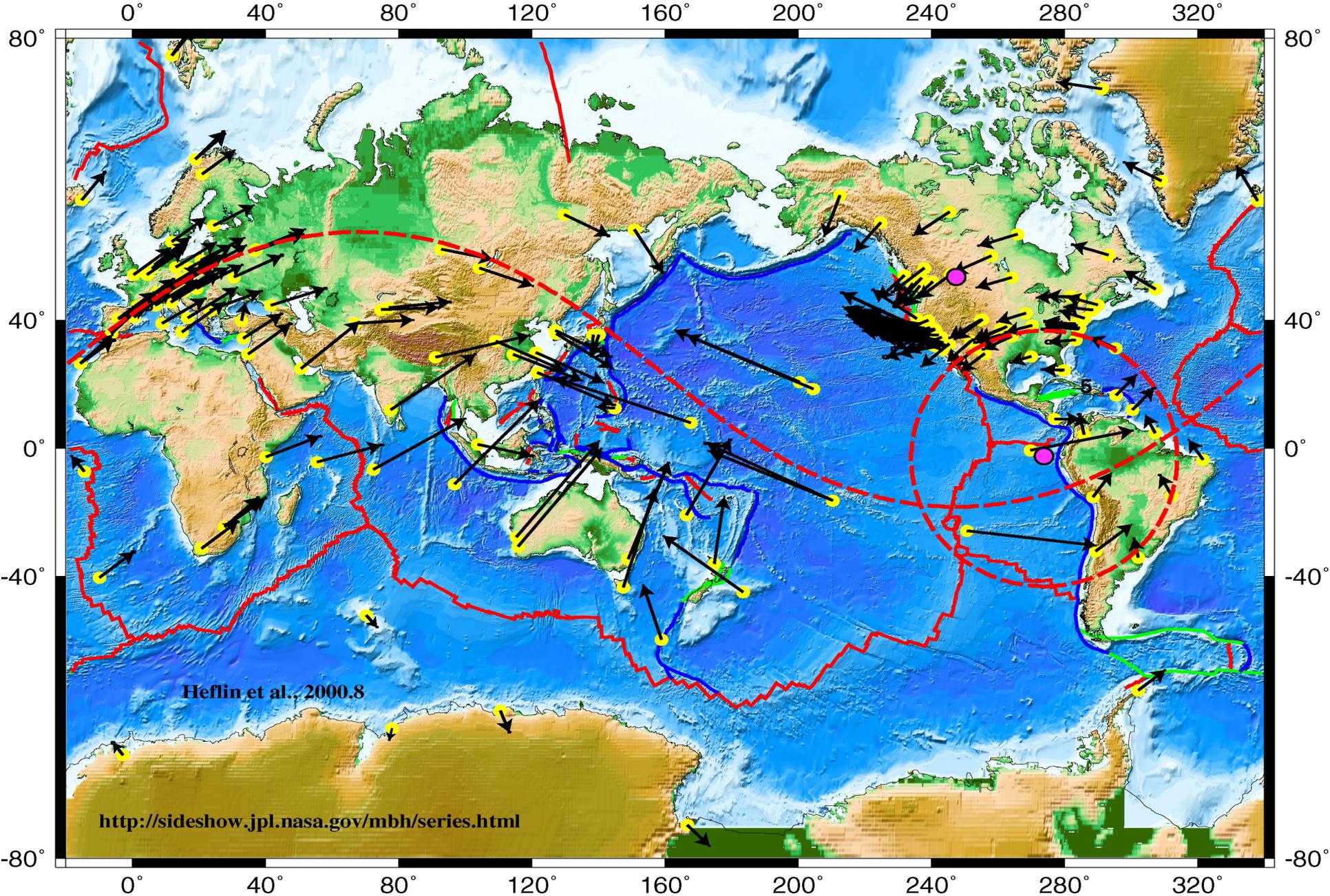
1. space-fixed (quasi) inertial system
(Conventional Inertial System CIS)
(Astronomy, VLBI in this system)
ITRF

2. Earth-fixed terrestrial system
(Conventional Terrestrial System CTS)

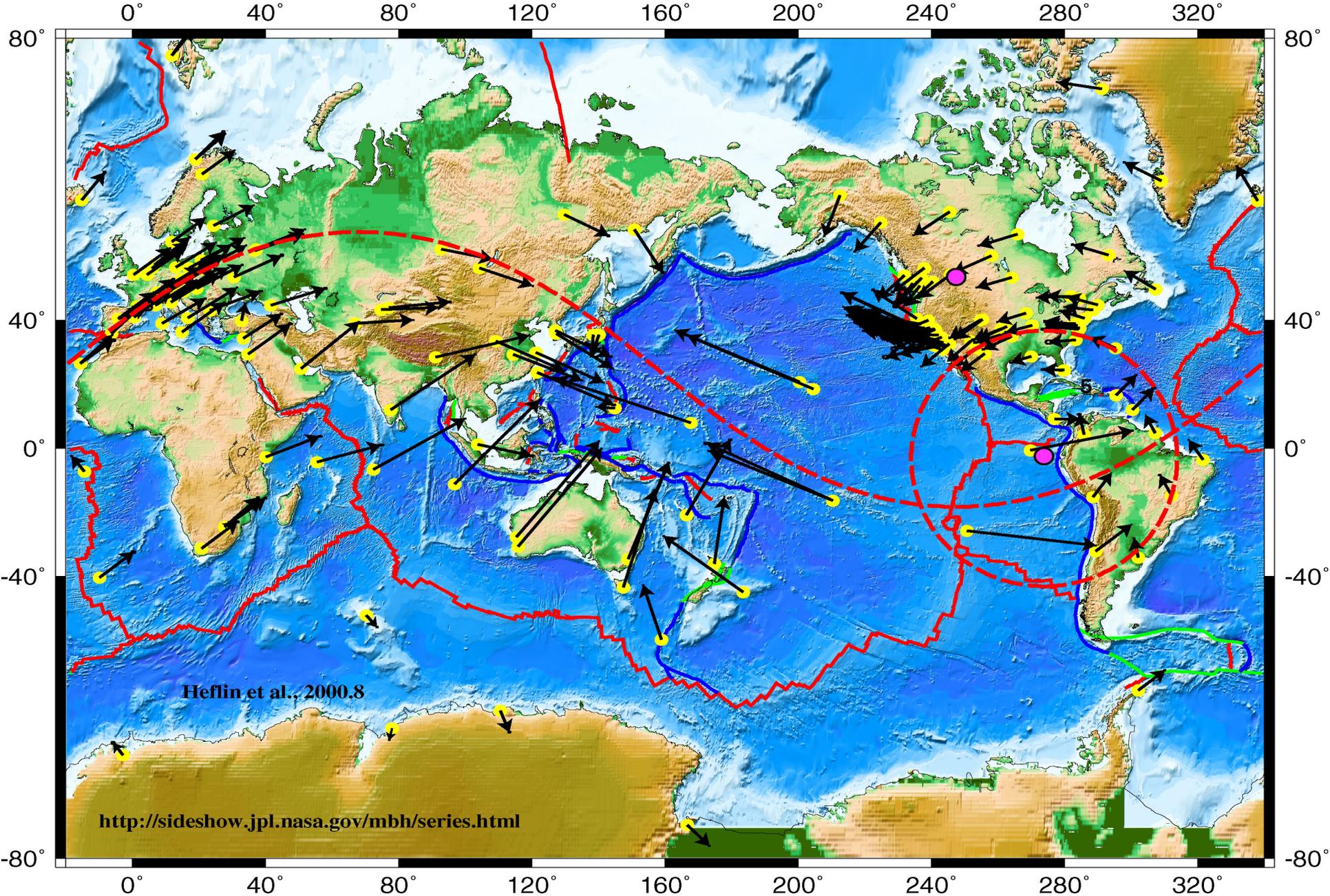
Both systems use center of earth and earth rotation in
definition and realization



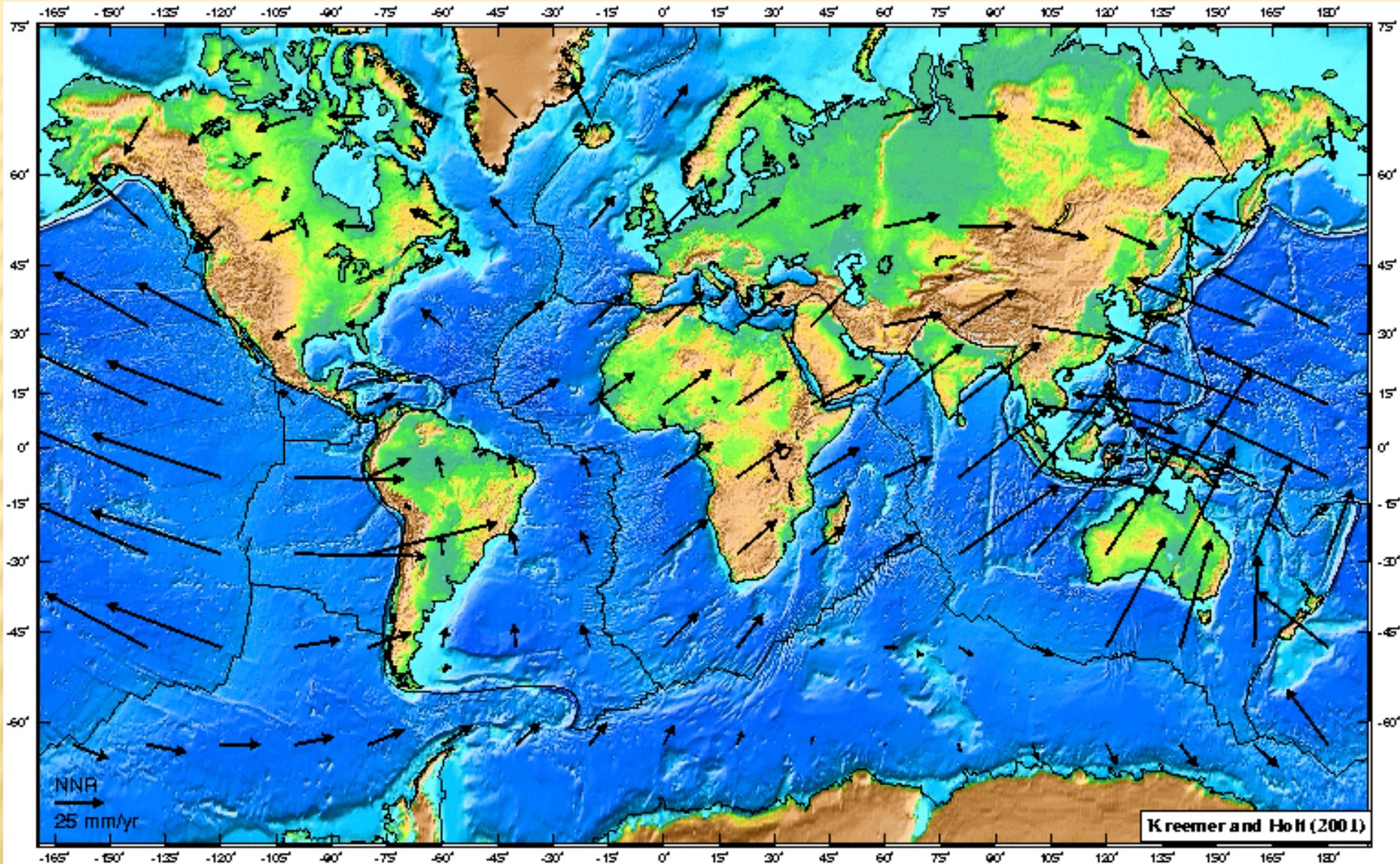
Velocities of IGS global tracking GPS sites in ITRF.



Small “circles” for European and N. American poles. 17



Velocities are tangent to small circles (look like windshield wiper streaks). 18



Gridded view of plate velocities in ITRF

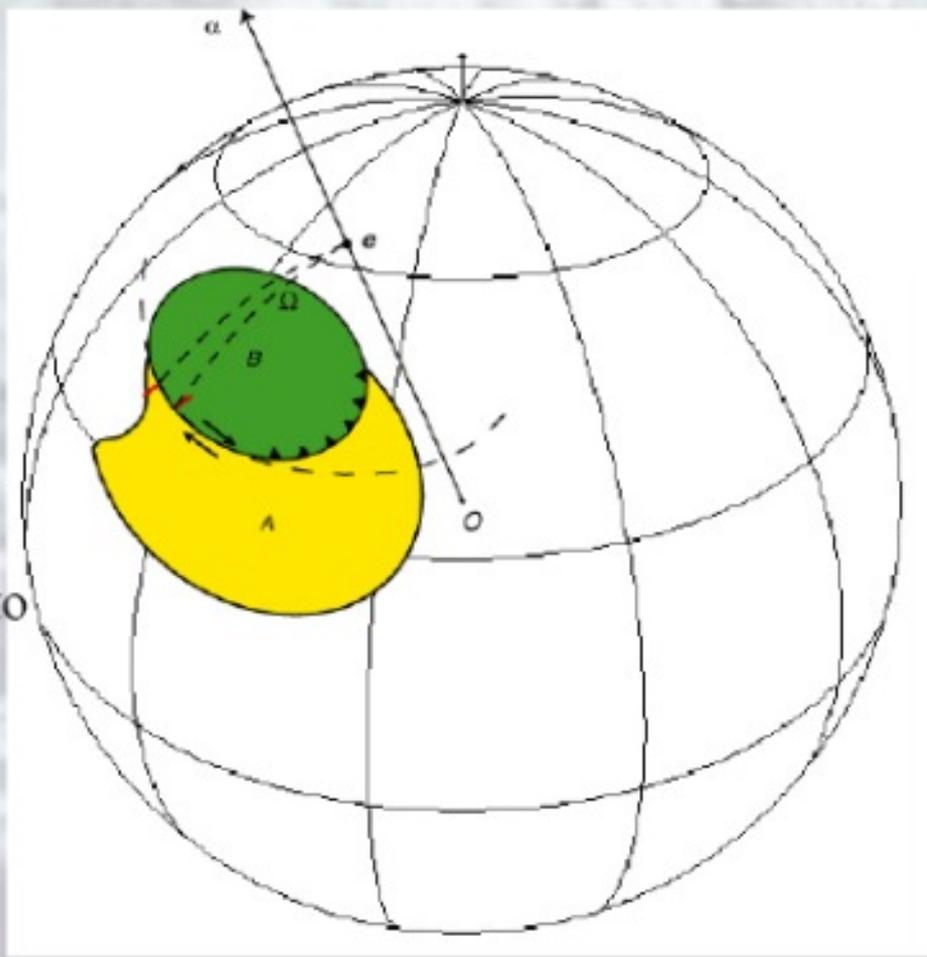
(approximates NUVEL, but does not “look like” NUVEL because NUVEL shows relative motions)



Rotation of
N. America
about Euler
pole.

Plate translation on a sphere

- Transcurrent and transform tectonic boundaries allow direct calculation of finite rotations by a combination of geological data and kinematic methods
- The strike-slip fault is modelled as a small circle arc about axis α
- The corresponding Euler pole e is calculated by fitting the modelled arc to plate boundary data
- The rotation angle Ω is determined geologically, through the identification of displaced markers (red lines)
- Finally, the timing of displacement is estimated stratigraphically or by other indirect methods



Sketch map illustrating the method of computation of finite rotations associated to strike-slip boundaries

Solving for Euler poles

Forward problem

Given rotation pole, \underline{R} , for movement of spherical shell
on surface of sphere

We can find the velocity of a point, \underline{X} , on that shell from

$$\vec{V} = \vec{R} \times \vec{X}$$

(review)

We can write this in matrix form

(in Cartesian coordinates)

as

$$\vec{V} = \vec{R} \times \vec{X}$$

$$\vec{V} = \Omega \vec{X}$$

Where Ω is the rotation matrix

$$\Omega = \begin{pmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{pmatrix}$$

(note – this is for infinitesimal, not finite rotations)

So – now we solve this

$$\vec{V} = \Omega \vec{X}$$


Hopefully with more data than is absolutely necessary
using Least Squares

(this is the remark you find in most papers –
Now we solve this by Least Squares)



But

known

known


$$\vec{V} = \Omega \vec{X}$$

And

we want to find



$$\vec{V} = \Omega \vec{X}$$

This is how we would set the problem up
if we know V and Ω and wanted to find X

So we have to recast the expression to put the knowns and unknowns into the correct functional relationship.

Start by multiplying it out

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$V_x = -r_z Y + r_y Z$$

$$V_y = r_z X - r_x Z$$

$$V_z = -r_y X + r_x Y$$

Now rearrange into the form

$$\vec{b} = A\vec{x}$$

Where \vec{b} and A are known

$$V_x = -r_z Y + r_y Z$$

$$V_y = r_z X - r_x Z$$

$$V_z = -r_y X + r_x Y$$

obtaining the following

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} 0 & Z & -Y \\ -Z & 0 & X \\ Y & -X & 0 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} 0 & Z & -Y \\ -Z & 0 & X \\ Y & -X & 0 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

$$\vec{V} = X\vec{R}$$

So now we have a form that expresses the relationship between the two vectors

V and R

With the “funny” matrix X.

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} 0 & Z & -Y \\ -Z & 0 & X \\ Y & -X & 0 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

$$\vec{V} = X\vec{R}$$

We have

3 equations and
3 unknowns

So we should be able to solve this
(unfortunately not!)

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} 0 & Z & -Y \\ -Z & 0 & X \\ Y & -X & 0 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

$$\vec{V} = X\vec{R}$$

You can see this two ways

1 - The matrix is singular (the determinant is zero)

2 - Geometrically, the velocity vector is tangent to a small circle about the rotation pole -

There are an infinite number of small circles (defined by a rotation pole) to which a single vector is tangent

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} 0 & Z & -Y \\ -Z & 0 & X \\ Y & -X & 0 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

$$\vec{V} = X\vec{R}$$

So there are an infinite number of solutions to this expression.

Can we fix this by adding a second data point?
(another \underline{X} , where \underline{V} is known)

Yes – or we would not have asked!

Following the lead from before in terms of the relationship between \underline{V} and \underline{R} we can write

$$\begin{pmatrix} V_{x_1} \\ V_{y_1} \\ V_{z_1} \\ V_{x_2} \\ V_{y_2} \\ V_{z_2} \end{pmatrix} = \begin{pmatrix} 0 & Z_1 & -Y_1 \\ -Z_1 & 0 & X_1 \\ Y_1 & -X_1 & 0 \\ 0 & Z_2 & -Y_2 \\ -Z_2 & 0 & X_2 \\ Y_2 & -X_2 & 0 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

$$\vec{V} = X\vec{R}$$

Where V is now the “funny” thing on the left.

Geometrically

Given two points we now have

Two tangents to the same small circle

And

(assuming they are not incompatible – i.e. contradictory
resulting in no solution.)

we can find a single (actually there is a 180° ambiguity)

Euler pole

For n data points we obtain

$$\begin{pmatrix} V_{x_1} \\ V_{y_1} \\ V_{z_1} \\ V_{x_2} \\ V_{y_2} \\ V_{z_2} \\ \vdots \\ V_{x_n} \\ V_{y_n} \\ V_{z_n} \end{pmatrix} = \begin{pmatrix} 0 & Z_1 & -Y_1 \\ -Z_1 & 0 & X_1 \\ Y_1 & -X_1 & 0 \\ 0 & Z_2 & -Y_2 \\ -Z_2 & 0 & X_2 \\ Y_2 & -X_2 & 0 \\ \vdots & \vdots & \vdots \\ 0 & Z_n & -Y_n \\ -Z_n & 0 & X_n \\ Y_n & -X_n & 0 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

$$\vec{V} = X\vec{R}$$

Which we can solve by Least Squares

We actually saw this earlier when we developed the Least Squares method and wrote $y = mx + b$ as

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix}$$

$$\vec{y} = G\vec{m}$$

Where

\vec{y} is the data vector (known)

\vec{m} is the model vector (unknown parameters, what we want)

G is known

Pretend leftmost thing is “regular” vector and solve same way as linear least squares

$$\begin{pmatrix} V_{x_1} \\ V_{y_1} \\ V_{z_1} \\ V_{x_2} \\ V_{y_2} \\ V_{z_2} \\ \vdots \\ V_{x_n} \\ V_{y_n} \\ V_{z_n} \end{pmatrix} = \begin{pmatrix} 0 & Z_1 & -Y_1 \\ -Z_1 & 0 & X_1 \\ Y_1 & -X_1 & 0 \\ 0 & Z_2 & -Y_2 \\ -Z_2 & 0 & X_2 \\ Y_2 & -X_2 & 0 \\ \vdots & \vdots & \vdots \\ 0 & Z_n & -Y_n \\ -Z_n & 0 & X_n \\ Y_n & -X_n & 0 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix}$$

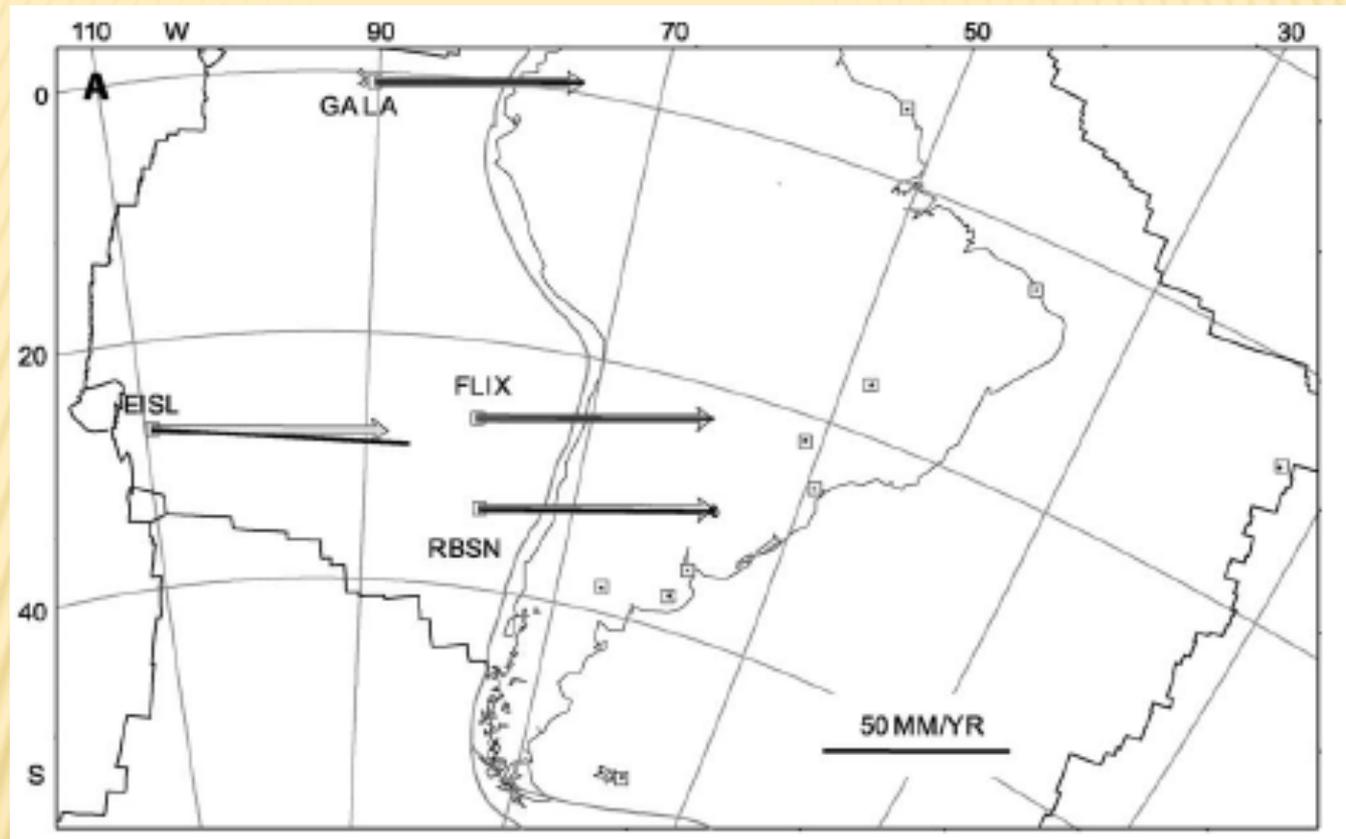
$$\vec{y} = G\vec{m}$$

$$\vec{V} = X\vec{R}$$

$$\vec{R} = (X^T X)^{-1} X^T \vec{V}$$

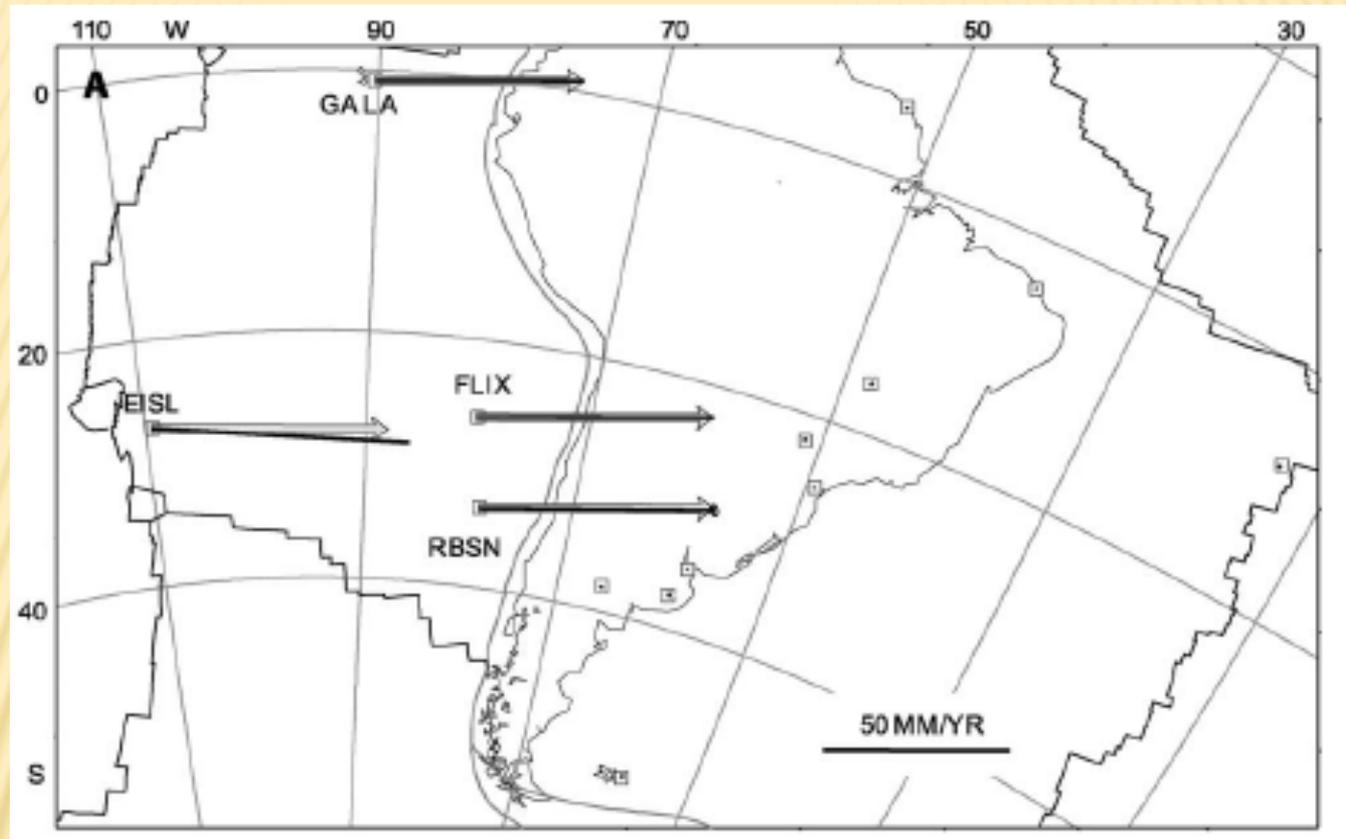
$$\vec{m} = (G^T G)^{-1} G^T \vec{d}$$

Example: Nazca-South America Euler pole



Data plotted in South America reference frame
(points on South America plate have zero – or near zero
– velocities.)

Example: Nazca-South America Euler pole (relative)

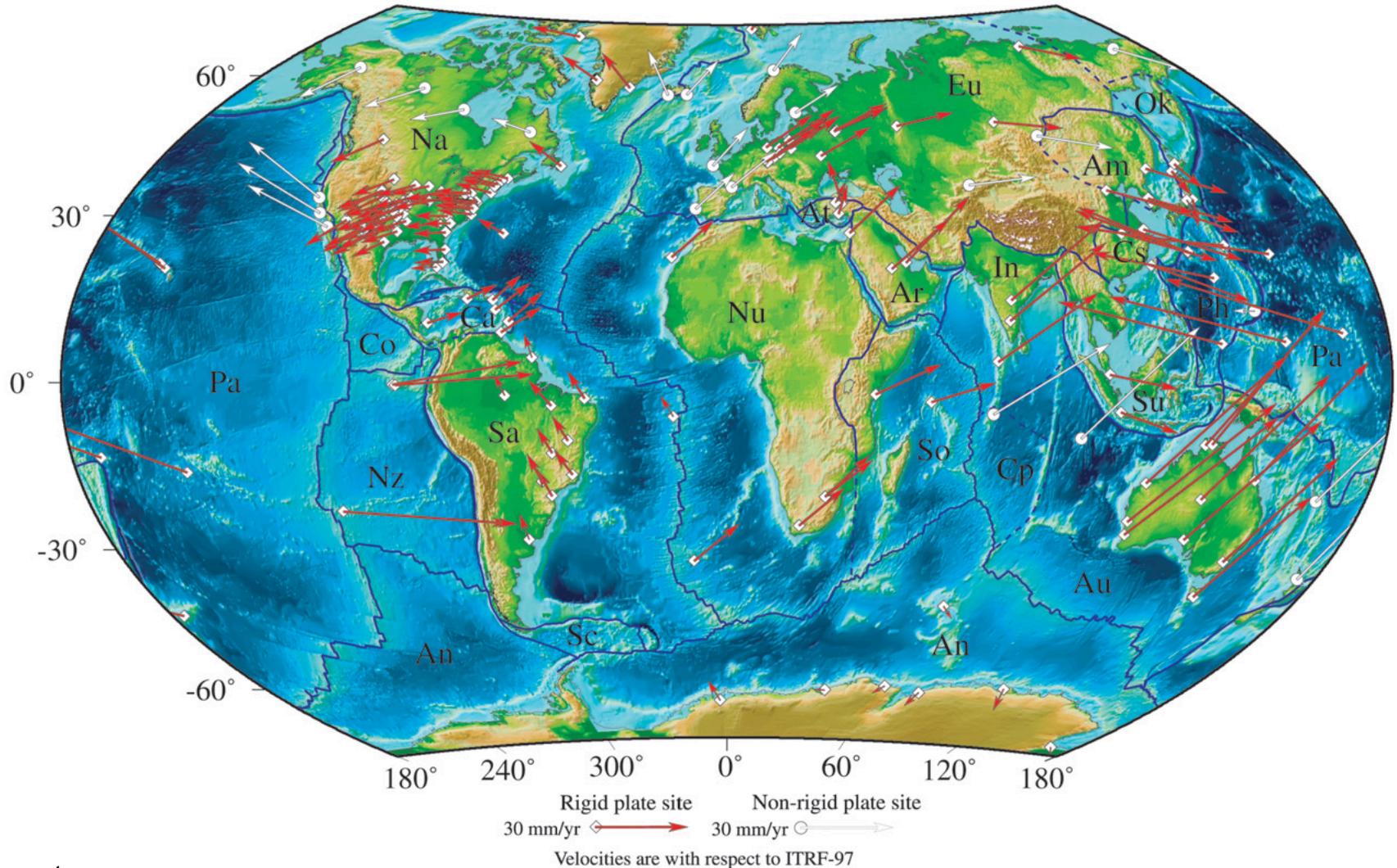


Also plotted in
Oblique Mercator projection
about Nazca-South America Euler pole

REcent VELOCities – from GPS

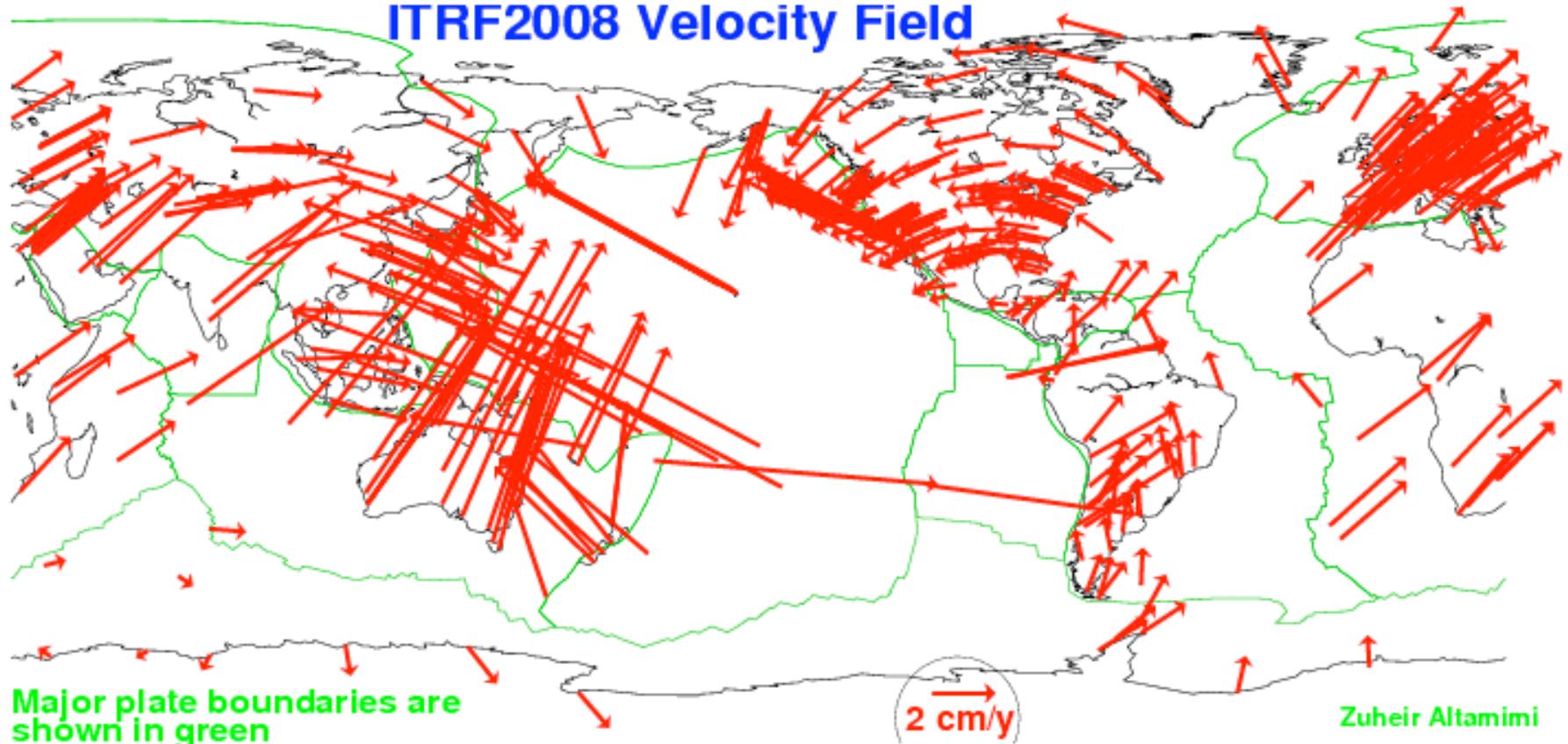
(note holes – Scotia Plate for example)

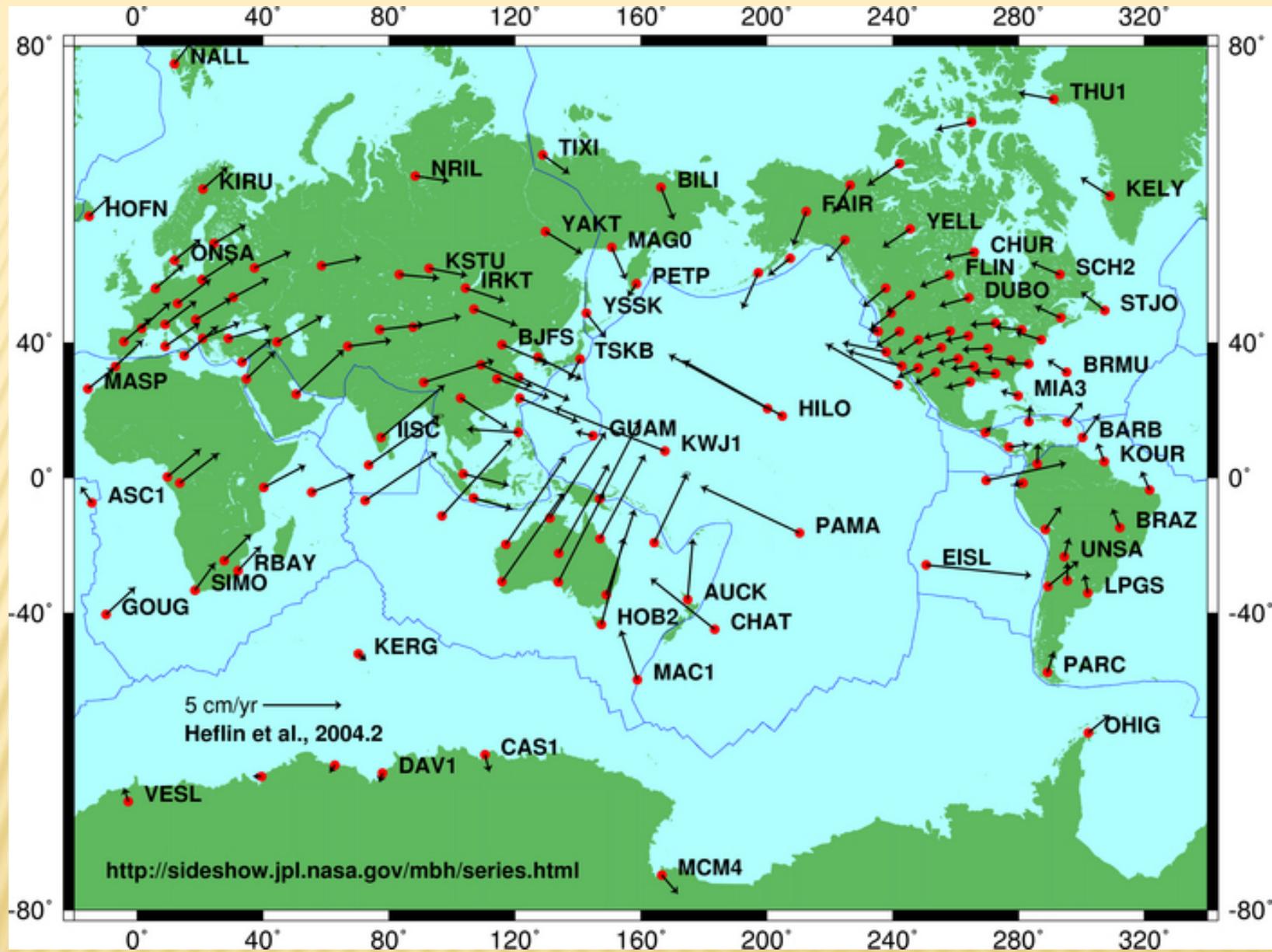
REVEL-2000



ITRF-2008

ITRF2008 Velocity Field





GPS picture – Scotia Plate still missing (also missing from NUVEL-1, “included, but not constrained in NUVEL-1A)

Combine GPS and Geology to define motion
Scotia plate.

Scotia plate “missing” from NUVEL-1

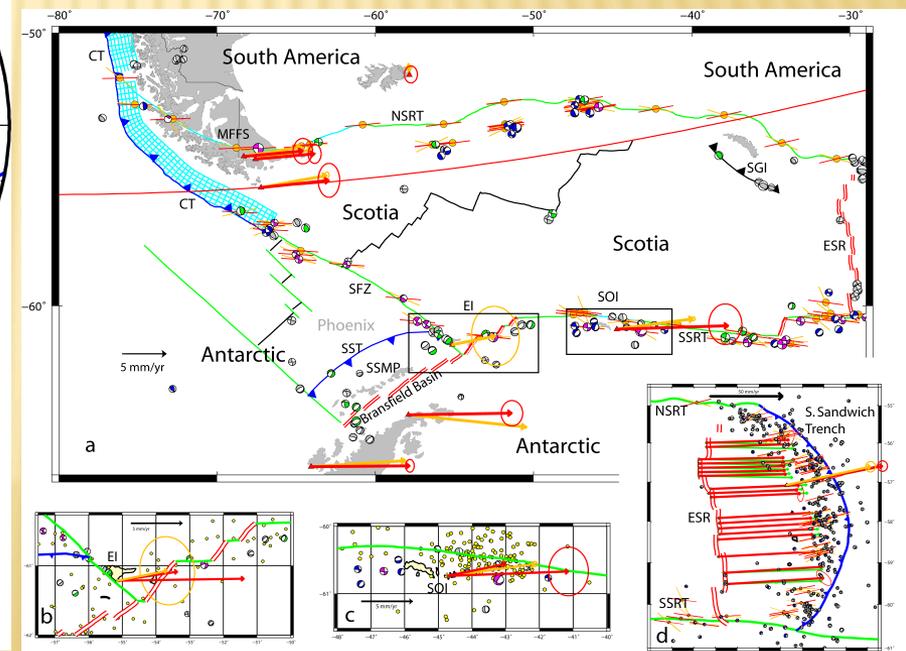
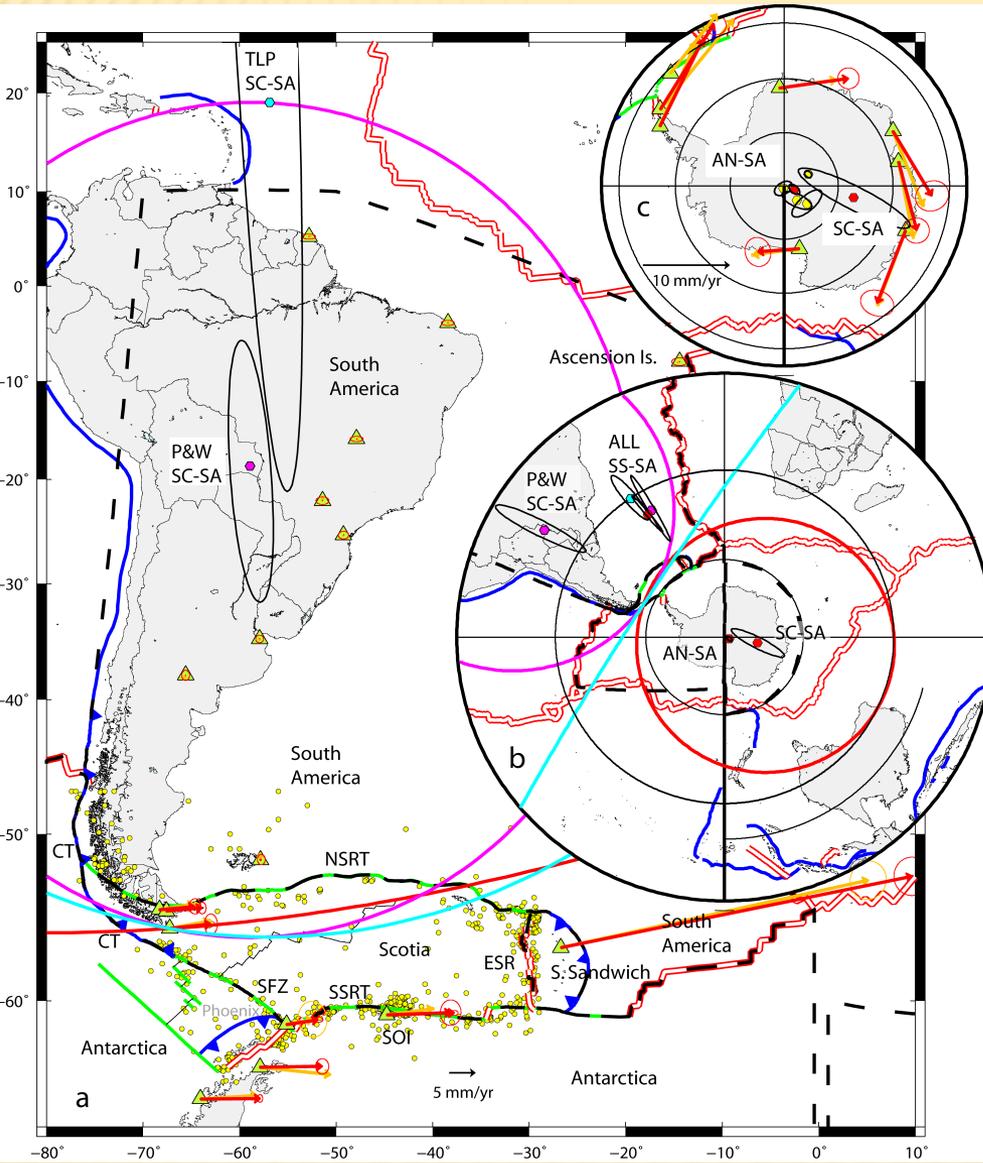
(is in NUVEL-1A but estimated from closure)

Get small circles from transform plate boundaries
(so theoretically can get location of pole) but no
tie into spreading system for velocity.

Use GPS to get velocity.

Results for GPS-Geologic combination for Scotia Arc.

Use Combination of GPS (velocity and azimuth, focal mechanisms (azimuth), Scotia-South Sandwich spreading.



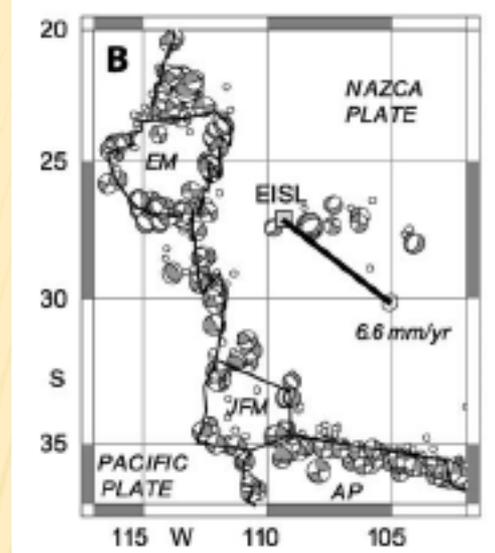
NUVEL-1A & GPS differences

Rotation rates of

- India, Arabian and Nubian plates wrt Eurasia are
30, 13 and 50% slower
 - Nazca-South America 17% slower
 - Caribbean-North America 76% faster
- than NUVEL-1A

Question – is Easter Island on “stable” Nazca Plate

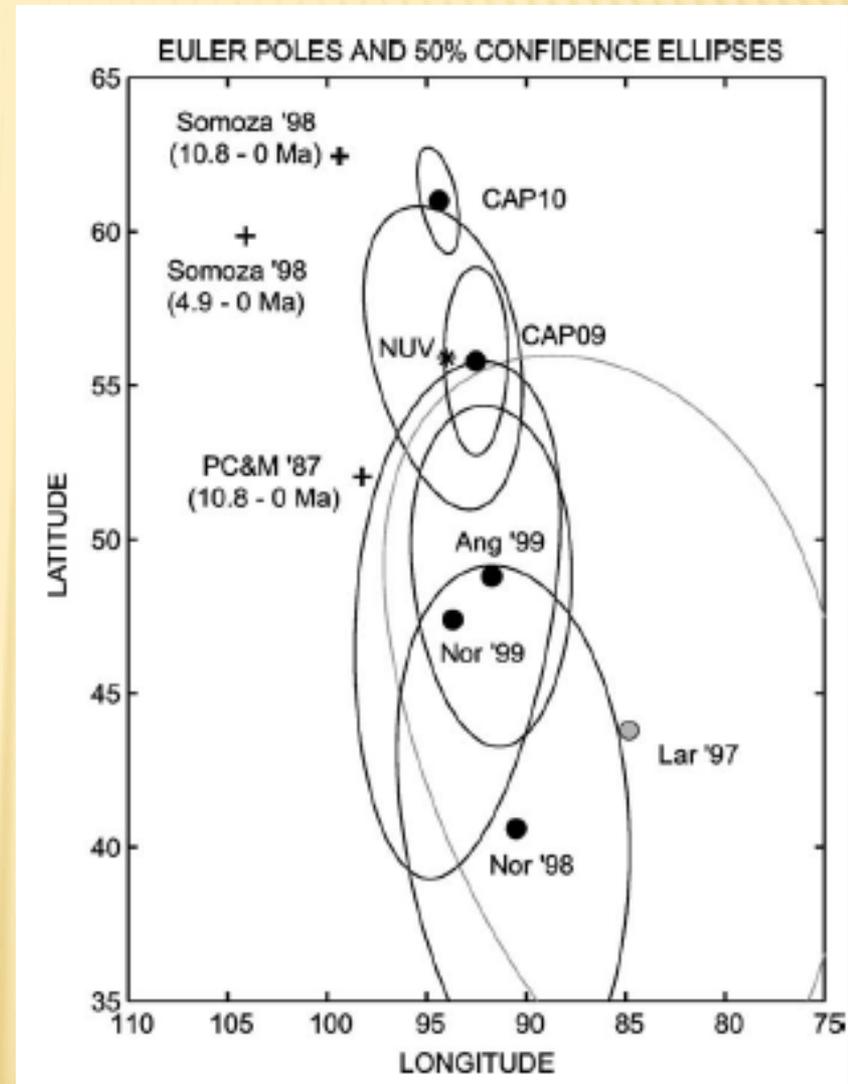
We think not.



Only 4 points total on Nazca Plate (no other islands!)

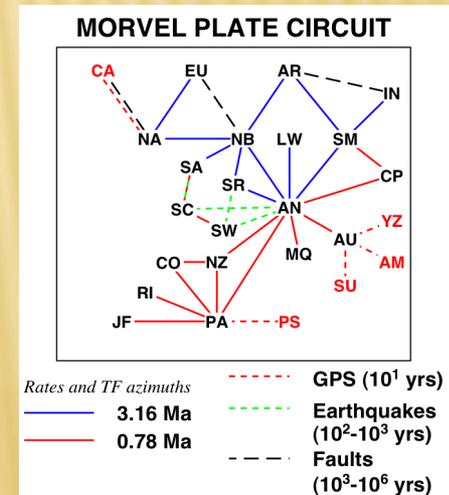
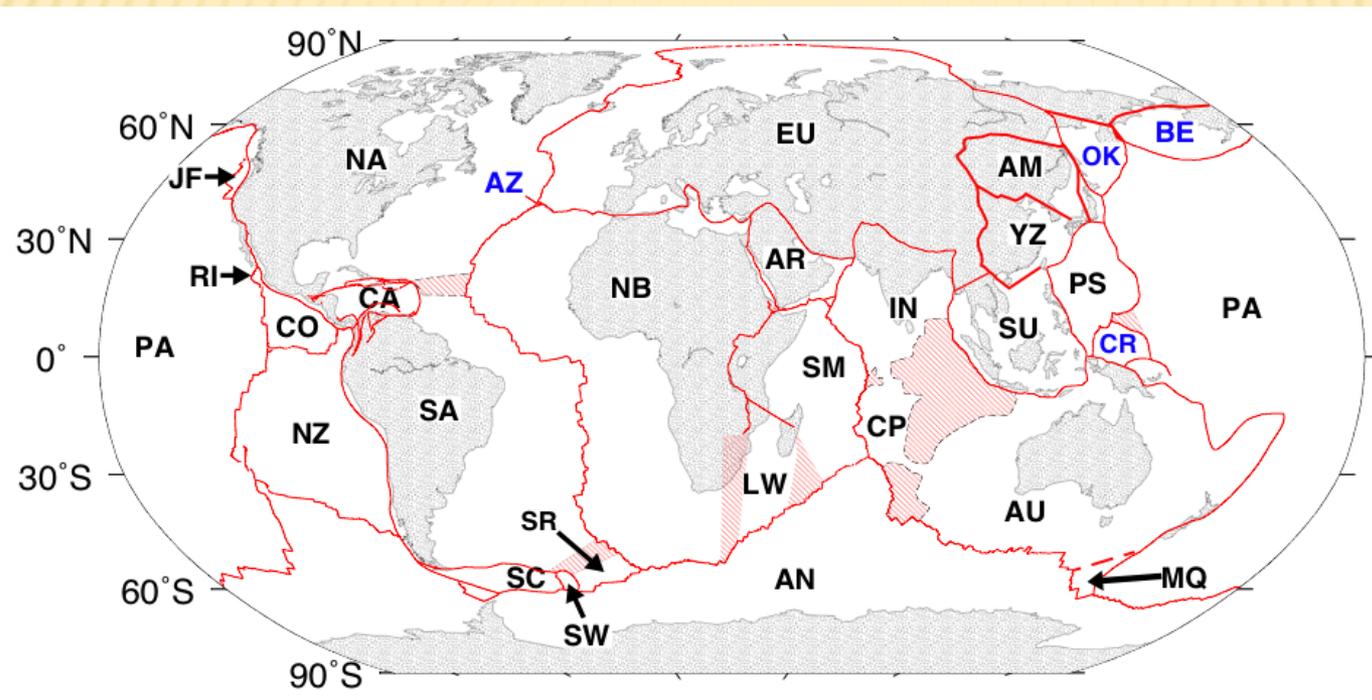
Galapagos and Easter Island part of IGS (continuous)

FLIX and RBSN campaign



Latest results - Combine Geology (3 Myr average) and GPS for places geology does not work

MORVEL



Complications to simple model in plate interiors

Horizontal deformations associated with post glacial rebound

$$\vec{V} = \Omega \vec{X} + \gamma \vec{V}_{pgr}$$

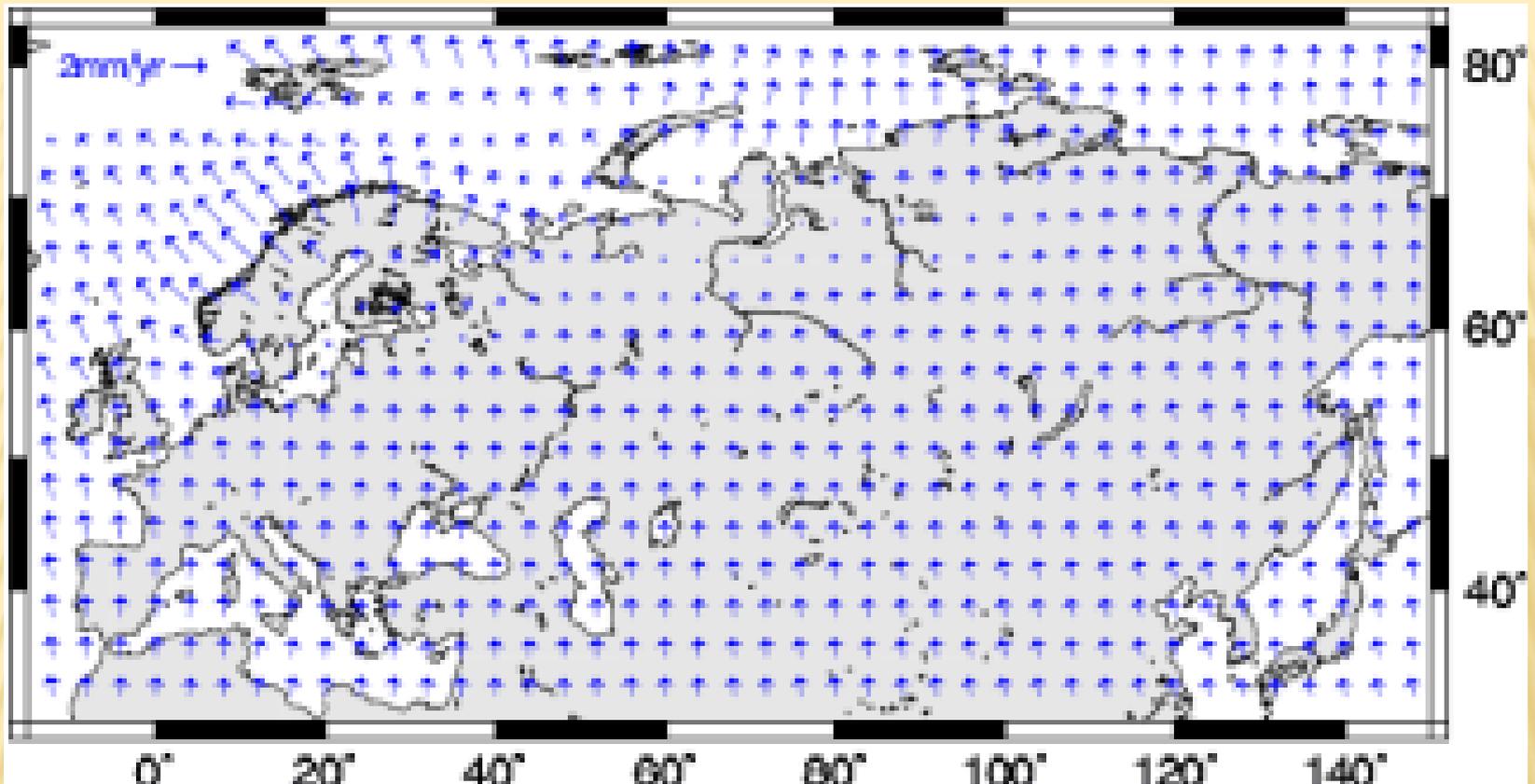
(problem for N. America and Eurasia)

Other effects

Other causes horizontal movement/deformation
(tectonics, changes in EOP?)

Most vertical movements – tidal, atmospheric, etc. , as in
case of PGR - have some “cross talk” to horizontal

$$\vec{V} = \Omega \vec{X} + \sum_i \vec{V}_i^{\text{geologic effects}}$$

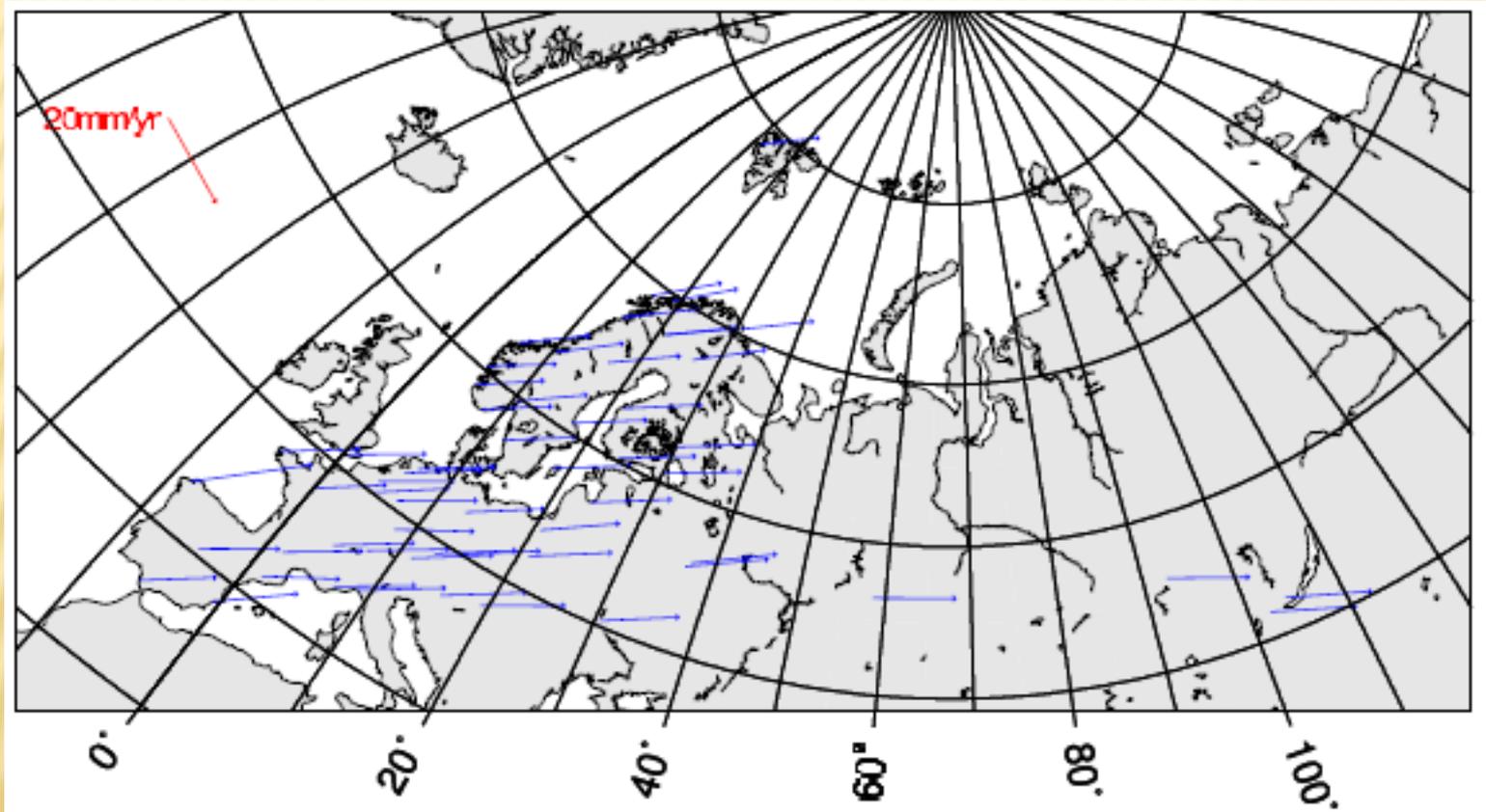


Predicted horizontal velocities in northern Eurasia from PGR

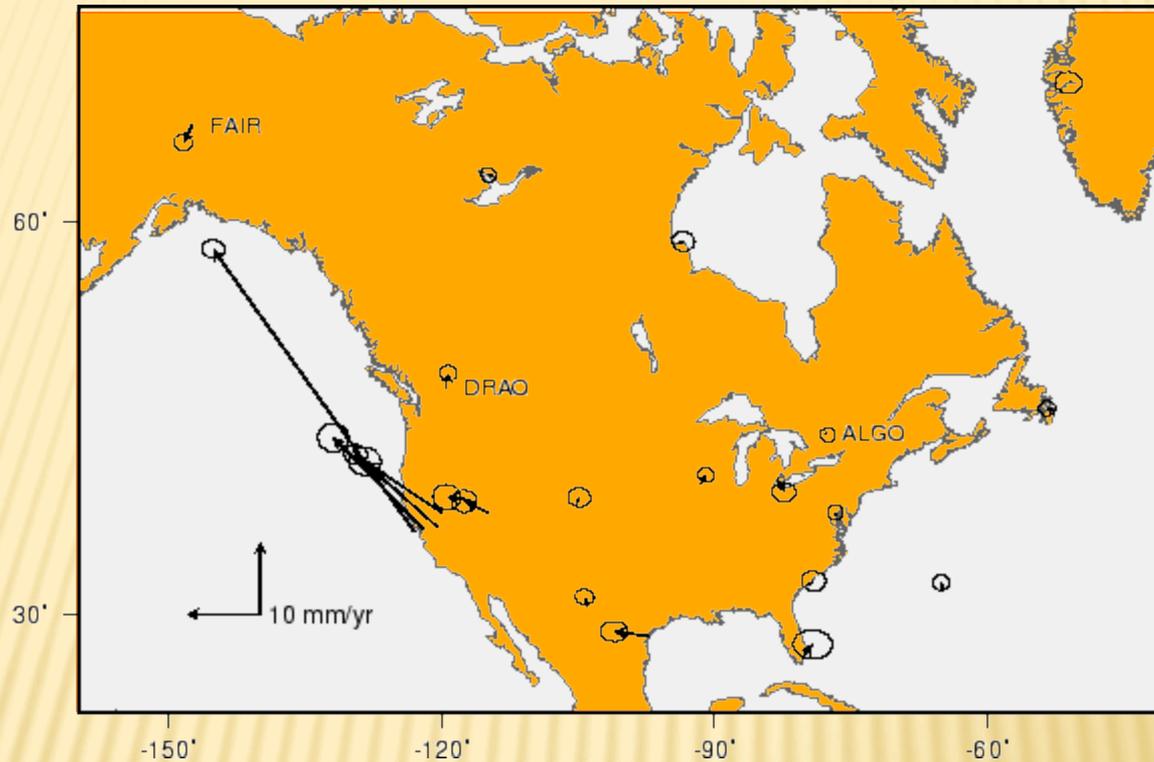
(No velocity scale! Largest are order 3 mm/yr away from center of ice load, figure does not seem to agree with discussion in paper)

Results for Eurasia

Site velocities plotted in oblique Mercator projection
(should be horizontal)



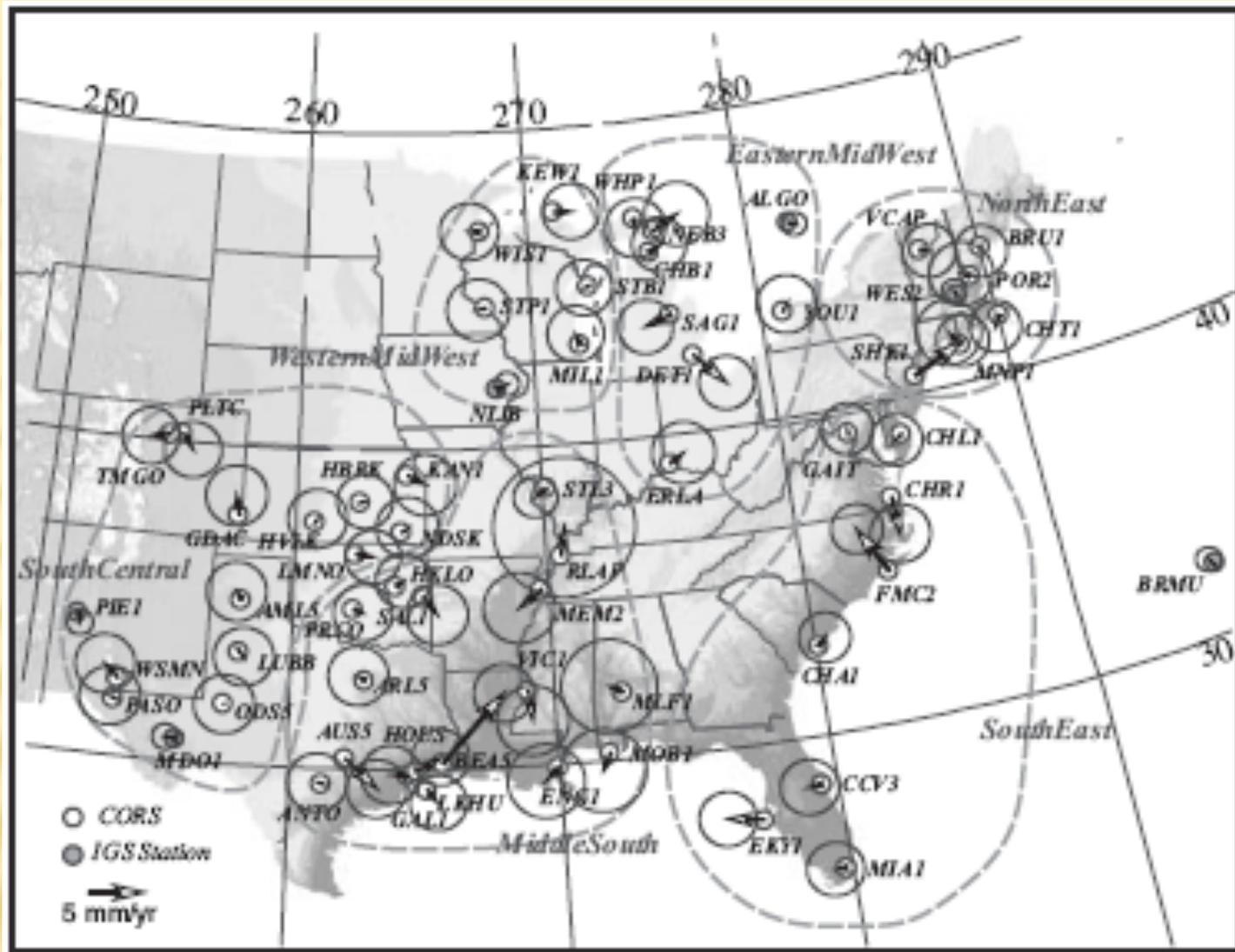
For North America



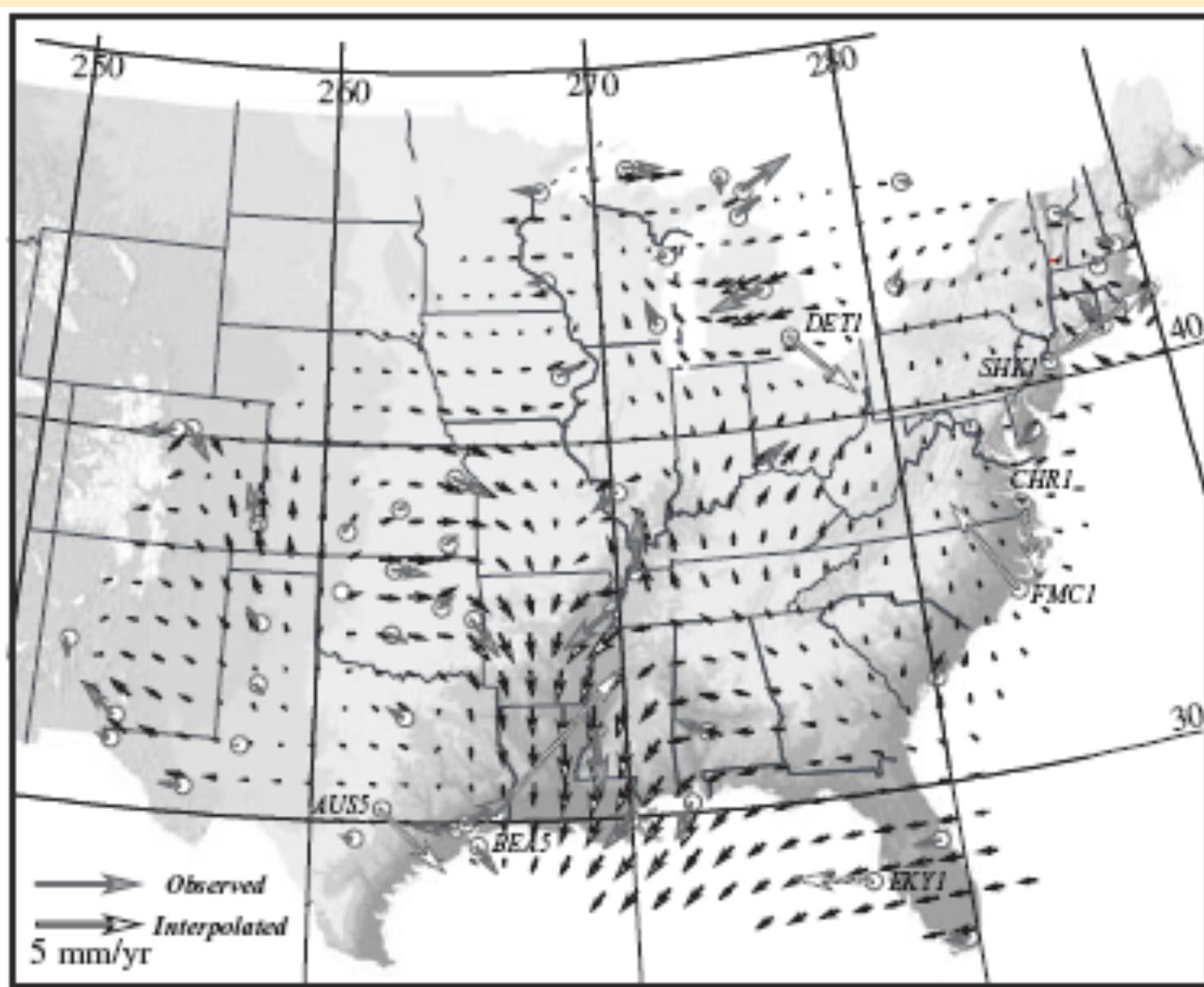
Stable North America Reference Frame (SNARF)

Over 300 continuous GPS sites available in Central and Eastern US (and N. America)

(unfortunately most are garbage)

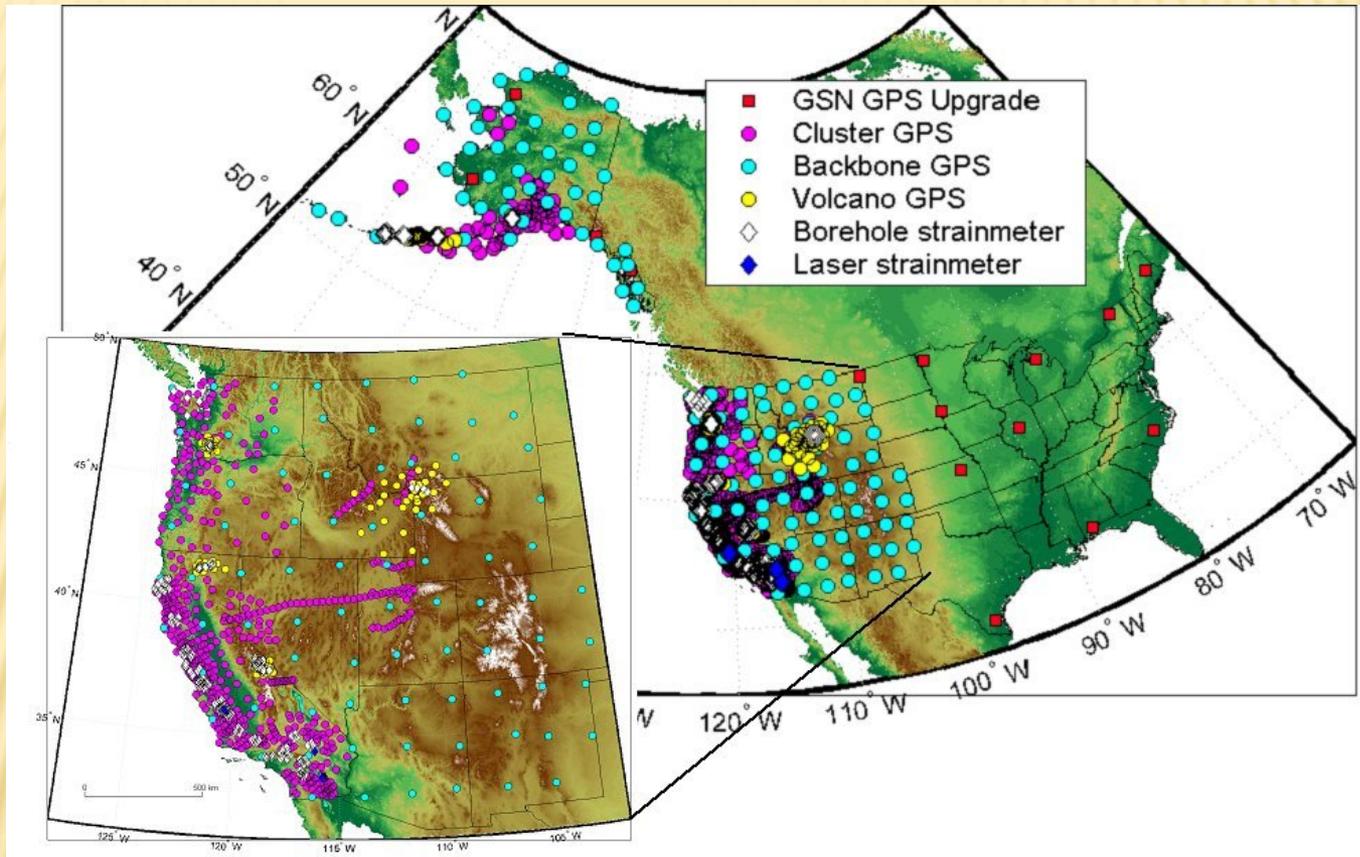


Analysis of CORS plus other continuous GPS data for intraplate deformation



Contoured (interpolated) velocity field
(ready for tectonic interpretation!)

PBO Needs



- What are PBO reference frame needs?
- How can we meet those needs?

More things to do with GPS

Deformation in plate boundary zones

(other main assumption of plate tectonics)

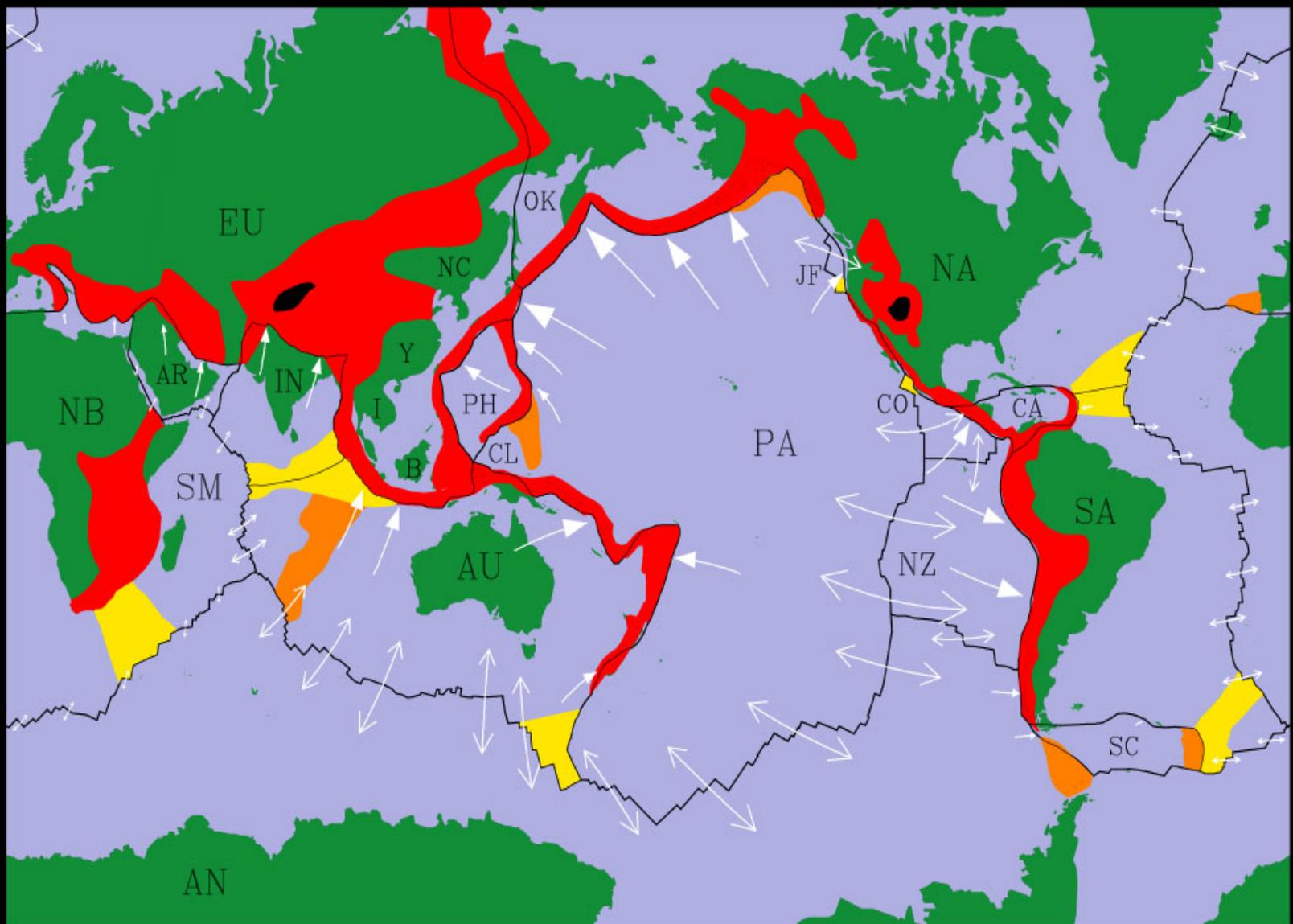
Narrowness of plate boundaries

contradicted by many observations,
in both continents and oceans.

Some diffuse plate boundaries exceed dimensions of
1000 km on a side.

Diffuse plate boundaries cover 15% of Earth's surface.

T. Shoberg and P. Stoddard, after R. Gordon and S. Stein,
1992



- Submarine Lithosphere Deformation  Inferred from plate motion data and seismicity
- Submarine Lithosphere Deformation  Inferred from seismicity
- Subaerial Lithosphere Deformation  Inferred from seismicity, topography, and faulting

Diffuse plate boundaries

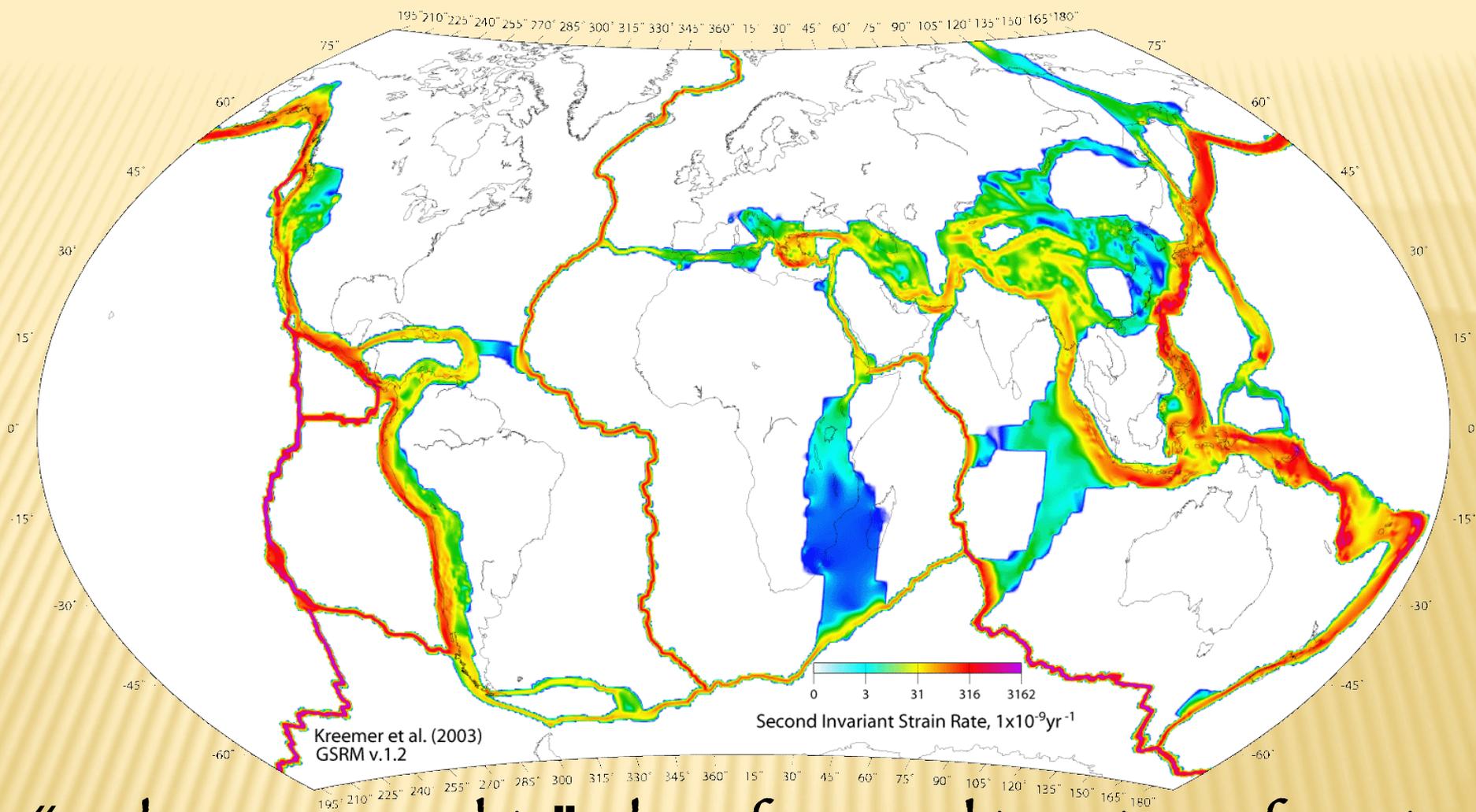
Maximum speed (relative) across diffuse plate boundaries

2 to 15 mm/year

Strain rates in diffuse plate boundaries
as high as 10^{-8} year

25 times higher than upper bound on strain rates of
stable plate interiors

600 times lower than lowest strain rates across typical
narrow plate boundaries.



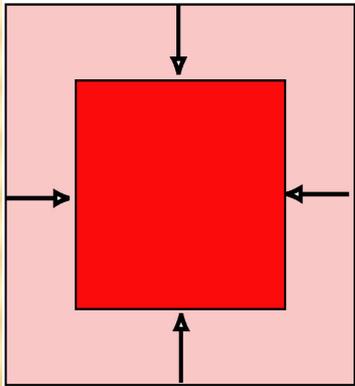
“Color topographic” plot of second invariant of strain rate tensor. Quantified version of previous figure.

Shows how fast the deforming regions are straining.
(Red fastest, blue slowest)

Mechanical work: $Work = Force \cdot distance$

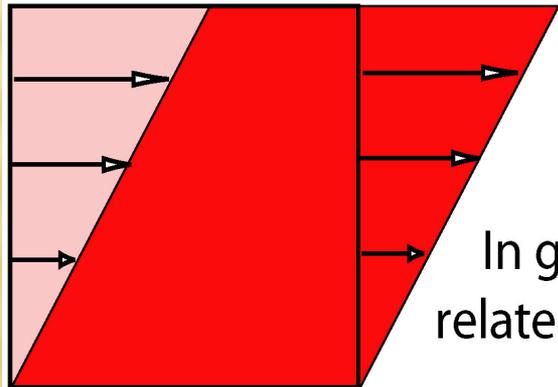
In an elastic medium it takes work to deform (change the shape of) a body: the force to create a deformation (change in distances) is a function of the deformation .

Work is therefore a function of the deformation (strain) squared.



Work related to volume change - first invariant of strain tensor - trace. Work is a function of the first invariant squared.

In general this deformation and work is not related to failure.



Work related to change in shape - second invariant - sum of cofactors. (individual terms are strain squared)

In general it is this deformation and work that is directly related to failure. (Von-Mises yield criterion).