

Earth Science Applications of Space Based Geodesy

DES-7355

Tu-Th

9:40-11:05

Seminar Room in 3892 Central Ave. (Long building)

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http://www.ceri.memphis.edu/people/smalley/ESCI7355/ESCI_7355_Applications_of_Space_Based_Geodesy.html

Class 10

Using double difference phase observations for relative positioning

First notice that if we make all double differences - even ignoring the obvious duplications

$$\nabla\Delta L_{AB}^{jk} = \nabla\Delta L_{AB}^{kj} = \nabla\Delta L_{BA}^{kj} = \nabla\Delta L_{BA}^{jk}$$

We get a lot more double differences than original data.

This can't be (can't create information).

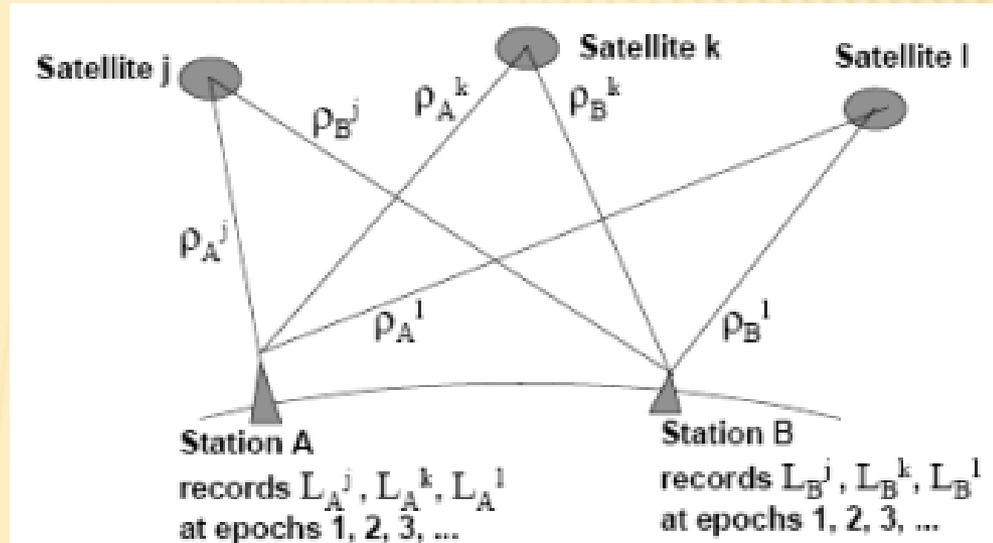
Consider the case of 3 satellites observed by 2 receivers.

Form the (non trivial)
double differences

$$L_{AB}^{jk} = (L_A^j - L_B^j) - (L_A^k - L_B^k)$$

$$L_{AB}^{jl} = (L_A^j - L_B^j) - (L_A^l - L_B^l)$$

$$L_{AB}^{lk} = (L_A^l - L_B^l) - (L_A^k - L_B^k)$$



Note that we can form any one
from a linear combination of the
other two

(linearly dependent)

We need a linearly independent set for Least Squares.

$$L_{AB}^{jk} = L_{AB}^{jl} - L_{AB}^{lk}$$

$$L_{AB}^{jl} = L_{AB}^{jk} - L_{AB}^{lk}$$

$$L_{AB}^{lk} = L_{AB}^{jk} - L_{AB}^{jl}$$

From the linearly dependent set

$$\left\{ L_{AB}^{jk}, L_{AB}^{jl}, L_{AB}^{lk} \right\}$$

We can form a number of linearly independent subsets

$$\left\{ L_{AB}^{jk}, L_{AB}^{jl} \right\} = \Lambda^j = \left\{ L_{AB}^{ab} \mid a = j; b \neq j \right\}$$

$$\left\{ L_{AB}^{kj}, L_{AB}^{kl} \right\} = \Lambda^k = \left\{ L_{AB}^{ab} \mid a = k; b \neq k \right\}$$

$$\left\{ L_{AB}^{lj}, L_{AB}^{lk} \right\} = \Lambda^l = \left\{ L_{AB}^{ab} \mid a = l; b \neq l \right\}$$

Which we can then use for our Least Squares estimation.

How to pick the basis?

All linearly independent sets are “equally” valid and should produce identical solutions.

Pick Λ' such that reference satellite / has data at every epoch

Better (but harder) approach is to select the reference satellite epoch by epoch

(if you have 24 hour data file, cannot pick one satellite and use all day – no satellite is visible all day)

For a single baseline (2 receivers) that observe s satellites,
the number of linearly independent double difference observations is

$$s-1$$

Next suppose we have more than 2 receivers.

We have the same situation

-all the double differences are not linearly independent.

As we just did for multiple satellites, we can pick a
reference station

that is common to all the double differences.

For a network of r receivers,
the number of linearly independent double difference
observations is

$$r-1$$

So all together we have a total of

$$(s-1)(r-1)$$

Linearly independent double differences

So our linearly independent set of double differences is

$$\Lambda_C^j = \left\{ L_{AB}^{ab} \mid a = j; b \neq j; A = C, B \neq C \right\}$$

Reference station method has problems when all receivers can't see all satellites at the same time.

Choose receiver close to center of network.

Even this might not work when the stations are very far apart.

For large networks may have to pick short baselines that connect the entire network.

Idea is to not have any closed polygons (which give multiple paths and are therefore linearly dependent) in the network.

Can also pick the reference station epoch per epoch.

If all the receivers see the same satellites at each epoch,
and data weighting is done properly,
then it does not matter which receiver and satellite we
pick for the reference.

In practice, however,
the solution depends on our choices of reference
receiver and satellite.

(although the solutions should be similar)

(could process all undifferenced phase observations
and estimate clocks at each epoch – ideally gives
“better” estimates)

Double difference observation equations

Start with

$$\nabla \Delta L_{AB}^{jk} = \nabla \Delta \rho_{AB}^{jk} + \nabla \Delta Z_{AB}^{jk} - \nabla \Delta I_{AB}^{jk} - \nabla \Delta N_{AB}^{jk}$$

Simplify to

$$L_{AB}^{jk} = \rho_{AB}^{jk} - \lambda_0 N_{AB}^{jk}$$

By dropping the $\nabla \Delta$

And assuming $\nabla \Delta Z_{AB}^{jk}$ & $\nabla \Delta I_{AB}^{jk}$ are negligible

Processing double differences between two receivers
results in a

Baseline solution

The estimated parameters include the vector between
the two receivers (actually antenna phase centers).

May also include estimates of parameters to model
troposphere (statistical) and ionosphere (measured –
dispersion).

Also have to estimate the

Integer Ambiguities

For each set of satellite-receiver double differences

We are faced with the same task we had before when we
used

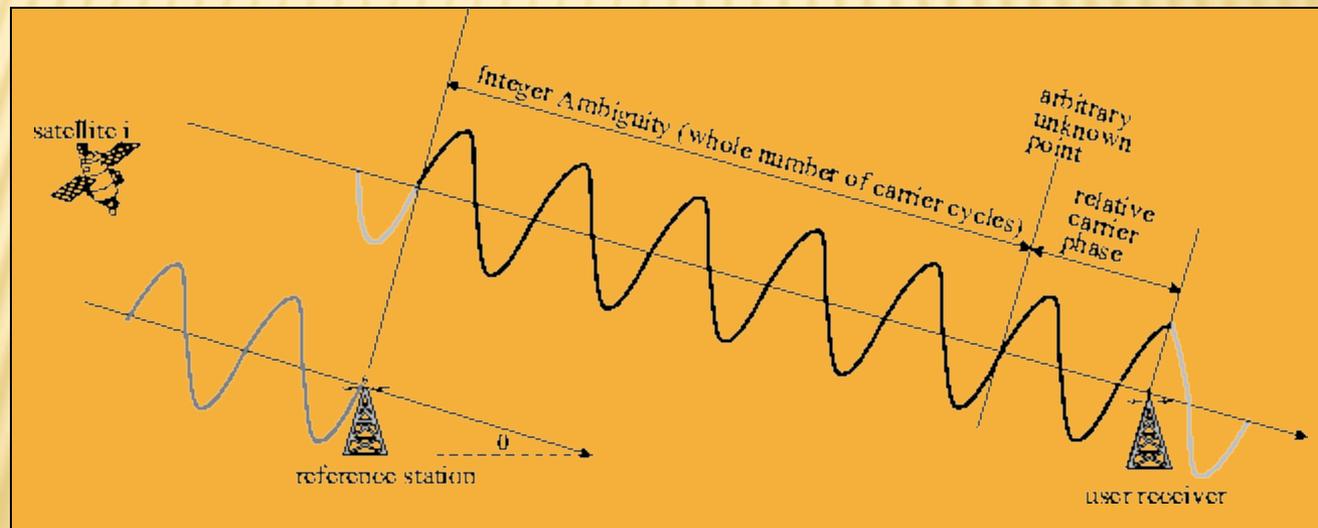
pseudo range

We have to

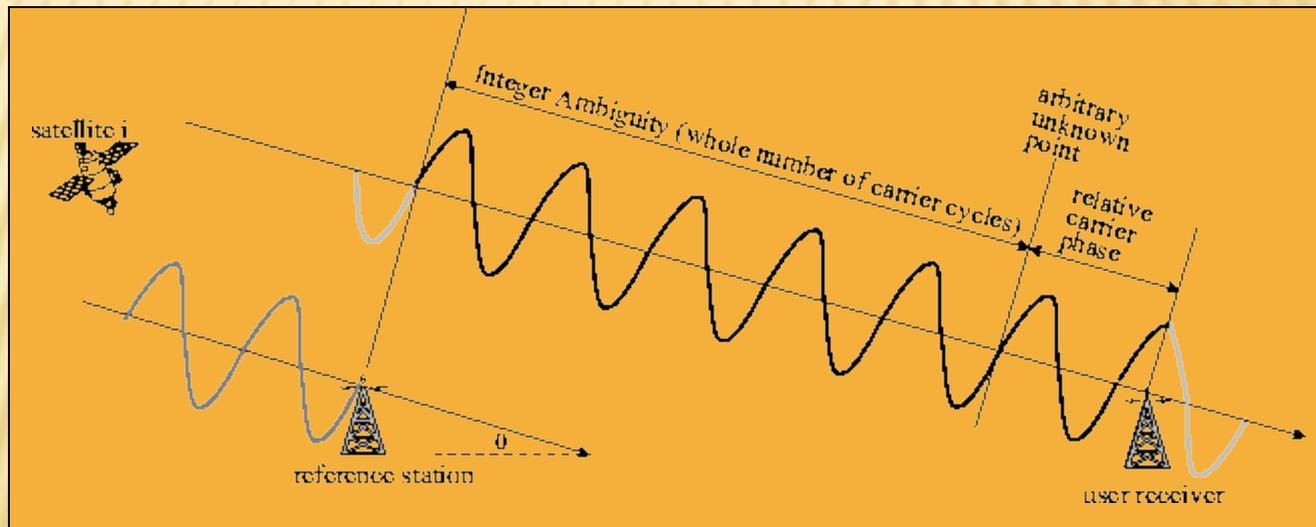
linearize

the problem in terms of the parameters we want to
estimate

A significant difference between using the pseudo range, which is a stand alone method, and using the Phase, is that the phase is a differential method (similar to VLBI).

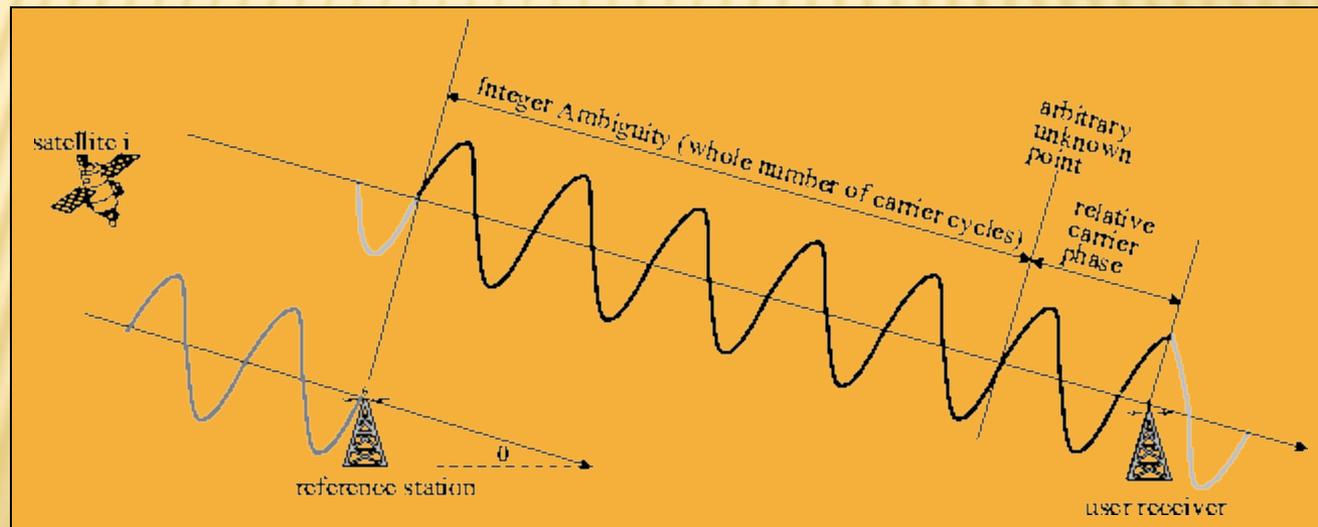


So far we have cast the problem in terms of the distances
to
the satellites,
but we could recast it in terms of the relative distances
between stations.



So now we will need multiple receivers.
We will also have to use (at least one) as a reference station.

In addition to knowing where the satellites are,
We need to know the position of the reference station(s)
to the same level of precision as we wish to estimate the
position of the other stations.



fiducial positioning

Fiducial

Regarded or employed as a standard of reference, as in surveying.

So now we have to assign the location of our fiducial station(s)

Can do this with

RINEX header position

VLBI position

Other GPS processing

etc.

So we have to

Write down the equations

Linearize

Solve

Double difference observation equations

Start with

$$\nabla \Delta L_{AB}^{jk} = \nabla \Delta \rho_{AB}^{jk} + \nabla \Delta Z_{AB}^{jk} - \nabla \Delta I_{AB}^{jk} - \nabla \Delta N_{AB}^{jk}$$

Simplify to

$$L_{AB}^{jk} = \rho_{AB}^{jk} - \lambda_0 N_{AB}^{jk}$$

By dropping the $\nabla \Delta$

And assuming $\nabla \Delta Z_{AB}^{jk}$ & $\nabla \Delta I_{AB}^{jk}$ are negligible

So we have to

Write down the equations

Linearize

Solve

Let the “reference” (also KNOWN) station be A

We want to estimate (x_B, y_B, z_B)

Using observations of satellites 1, 2, 3, and 4
(common observations at all epochs)

We also need to pick a “reference” satellite
(position of all satellites known)
Pick satellite 2.

(we have to pick the reference station and satellite to
properly form a linearly independent set of double
differences)

For each epoch i

We have the following 3 linearly independent sets of double difference observations

$$\Lambda_A^2(i) = \{L_{AB}^{ab}(i) | a = 2; b \neq 2\}$$

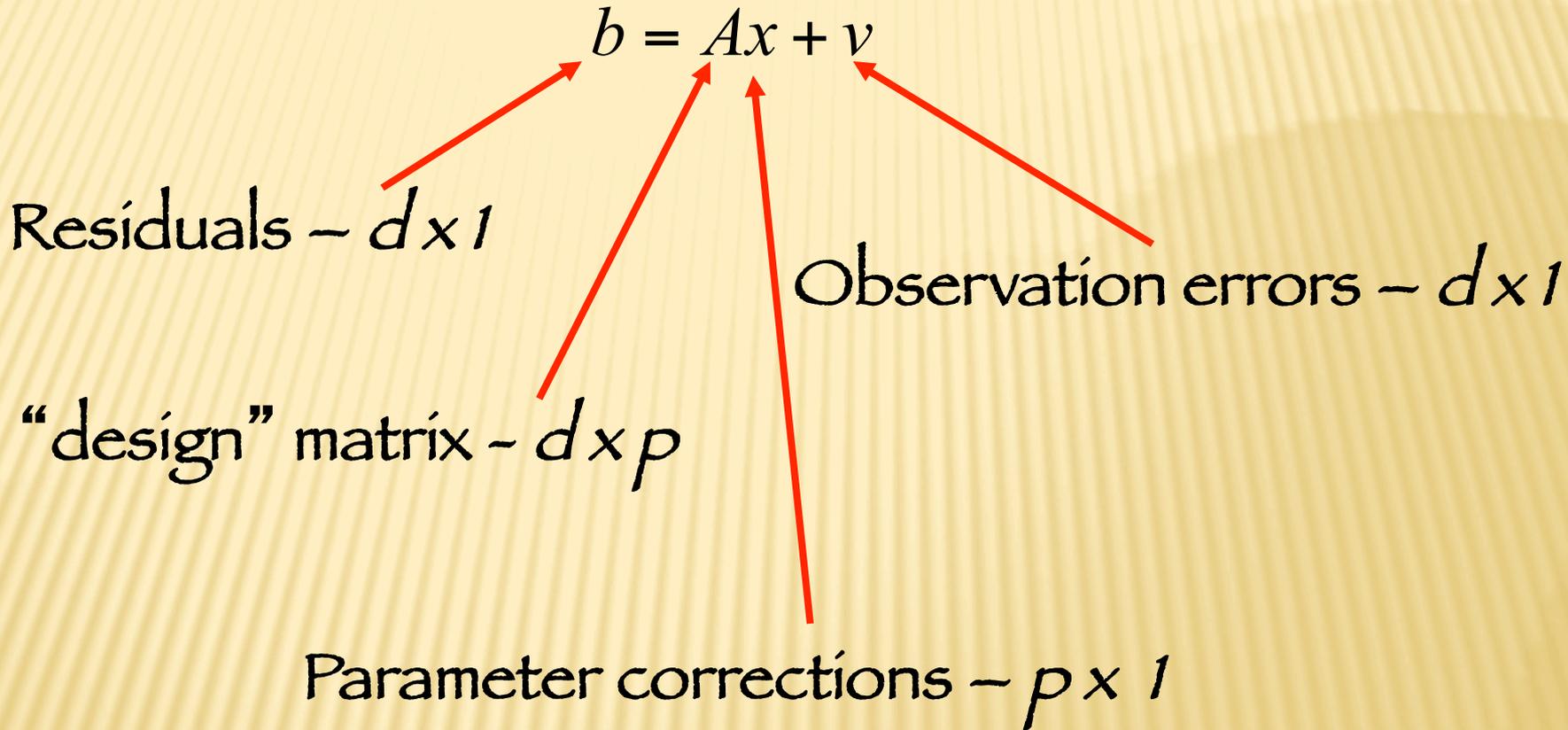
$$\Lambda_A^2(i) = \{L_{AB}^{21}(i), L_{AB}^{23}(i), L_{AB}^{24}(i)\}$$

To estimate the parameter set

$$\{x_B, y_B, z_B, N_{AB}^{21}, N_{AB}^{23}, N_{AB}^{24}\}$$

(if there were no cycle slips, else we would have to estimate additional $N_{AB}^{jj}(k)$ term for each cycle slip, k .)

As before, the linearized observation equations can be written in terms of the “usual suspects”



d – number linearly independent observables

p – number of parameters to estimate

In comparison to the pseudo range data,
where we assumed the errors in the observables were
independent,

the errors in double differenced data
are not – the errors are correlated.

This means that we should use
Weighted Least Squares

The WLS solution to the normal equations is

$$\hat{x} = (A^T W A)^{-1} A^T W \vec{b}$$

Where W is (an appropriately formed)
data weight matrix.

The covariance matrix is now given by
(does this look familiar?)

$$C_x = (A^T W A)^{-1}$$

The covariance matrix now has information about both the geometry (as before)

And new (information or effects due to) correlations between the observables.

(if we assume, as for pseudo range, that the error in measurement of the phase is the same for all measurements – we can factor out a σ ,

But the differencing introduces a correlation between the “independent” measurements that makes the errors “leak” from one observable to another)

Again, one can get important information from the
Covariance matrix

If it is not invertable mathematically
(linearly dependent)

If it is not invertable practically/numerically
(almost linearly dependent, large condition number)

Practically, can tell if all the integer ambiguities can be
fixed.

If so, get statistically better estimations.

Coefficients of the design matrix

Look at one row.

$$\Lambda_{AB}^{24}(i) = \left\{ \frac{\partial L_{AB}^{24}(i)}{\partial x_B}, \frac{\partial L_{AB}^{24}(i)}{\partial y_B}, \frac{\partial L_{AB}^{24}(i)}{\partial z_B}, \frac{\partial L_{AB}^{24}(i)}{\partial N_{AB}^{21}}, \frac{\partial L_{AB}^{24}(i)}{\partial N_{AB}^{23}}, \frac{\partial L_{AB}^{24}(i)}{\partial N_{AB}^{24}} \right\}$$

$$L_{AB}^{jk} = \rho_{AB}^{jk} - \lambda_0 N_{AB}^{jk}$$

$$\Lambda_{AB}^{24}(i) = \left\{ \frac{\partial \rho_{AB}^{24}(i)}{\partial x_B}, \frac{\partial \rho_{AB}^{24}(i)}{\partial y_B}, \frac{\partial \rho_{AB}^{24}(i)}{\partial z_B}, 0, 0, -\lambda_0 \right\}$$

Coefficients of the design matrix

Look at one derivative.

$$\frac{\partial \rho_{AB}^{24}(i)}{\partial x_B} = \frac{\partial}{\partial x_B} \left(\rho_A^2(i) - \rho_B^2(i) - \rho_A^4(i) + \rho_B^4(i) \right)$$

$$\frac{\partial \rho_{AB}^{24}(i)}{\partial x_B} = \frac{\partial \rho_A^2(i)}{\partial x_B} - \frac{\partial \rho_B^2(i)}{\partial x_B} - \frac{\partial \rho_A^4(i)}{\partial x_B} + \frac{\partial \rho_B^4(i)}{\partial x_B}$$

Independent of x_B

$$\frac{\partial \rho_{AB}^{24}(i)}{\partial x_B} = \frac{\partial \rho_B^4(i)}{\partial x_B} - \frac{\partial \rho_B^2(i)}{\partial x_B}$$

$$\frac{\partial \rho_{AB}^{24}(i)}{\partial x_B} = \frac{x_{B0} - x^4(i)}{\rho_B^4(i)} - \frac{x_{B0} - x^2(i)}{\rho_B^2(i)}$$

Coefficients of the design matrix

Finally one can use the relationship between

Range and Time

and

Time and Phase (what we measured).

$$\rho_A^j(i) = c(T_A(i) - T^j(i))$$

$$\phi(T) = f_0 T + \phi_0$$

To write everything in terms of the observables.

Final detail

Minimum data requirements

Necessary (but not sufficient condition) that

Number of data

Exceed

Number of parameters to estimate.

So we have

$$d \geq p$$

(allowing perfect solution $d=p$)

If all receivers track the same satellites there are

$$d = q(r-1)(s-1)$$

Linearly independent double differences

Where

q is the number of epochs

r the number of receivers

s the number of satellites

Assuming no cycle slips

$$p = 3 + (r-1)(s-1)$$

So

$$d = q(r-1)(s-1) \geq 3 + (r-1)(s-1)$$

$$(q-1)(r-1)(s-1) \geq 3$$

So for $r=2, s=2$

$$q \geq 4$$

(gives one double difference per epoch)

Common-mode Cancellations

Observation	Effects eliminated	Effects reduced	Option
Single differences.	Satellite <u>or</u> station clock (first order).	Orbit errors. GDOP. ionosphere	Constrain ambiguity.
Double differences.	Satellite <u>and</u> station clock (first order).	Orbit errors. GDOP. Ionosphere.	Constrain ambiguity.
Triple differences.	Satellite <u>and</u> station clock (first order).		Ambiguity eliminated. Find-fix cycle slips

RINEX files

Receiver Independent Exchange files

(standard GPS, now GNSS, observables – data – file)

ASCII files

(text – you can read them)

New competitor – may replace RINEX –

BINEX

Binary Exchange files

(binary – can't read files without program, much more general == complicated)

RINEX Files have two basic parts

Header

Data
(observables)

RINEX Header

TABLE A1
GPS OBSERVATION DATA FILE - HEADER SECTION DESCRIPTION

HEADER LABEL (Columns 61-80)	DESCRIPTION	FORMAT
RINEX VERSION / TYPE	- Format version (2.10) - File type ('O' for Observation Data) - Satellite System: blank or 'G': GPS 'R': GLONASS 'S': Geostationary signal payload 'T': NNSS Transit 'M': Mixed	F9.2,11X, A1,19X, A1,19X
PGM / RUN BY / DATE	- Name of program creating current file - Name of agency creating current file - Date of file creation	A20, A20, A20
* COMMENT	Comment line(s)	A60
MARKER NAME	Name of antenna marker	A60
* MARKER NUMBER	Number of antenna marker	A20
OBSERVER / AGENCY	Name of observer / agency	A20,A40

RINEX Header

REC # / TYPE / VERS	Receiver number, type, and version (Version: e.g. Internal Software Version)	3A20
ANT # / TYPE	Antenna number and type	2A20
APPROX POSITION XYZ	Approximate marker position (WGS84)	3F14.4
ANTENNA: DELTA H/E/N	- Antenna height: Height of bottom surface of antenna above marker - Eccentricities of antenna center relative to marker to the east and north (all units in meters)	3F14.4
WAVELENGTH FACT L1/2	- Default wavelength factors for L1 and L2 1: Full cycle ambiguities 2: Half cycle ambiguities (squaring) 0 (in L2): Single frequency instrument - zero or blank The default wavelength factor line is required and must precede satellite-specific lines.	2I6, I6

RINEX Header

* WAVELENGTH FACT L1/2	- Wavelength factors for L1 and L2 1: Full cycle ambiguities 2: Half cycle ambiguities (squaring) 0 (in L2): Single frequency instrument	2I6,	*
	- Number of satellites to follow in list for which these factors are valid.	I6,	
	- List of PRNs (satellite numbers with system identifier)	7(3X,A1,I2)	
	These optional satellite specific lines may follow, if they identify a state different from the default values.		
	Repeat record if necessary.		

RINEX Header

# / TYPES OF OBSERV	- Number of different observation types stored in the file	I6,
	- Observation types	9(4X,A2)
	If more than 9 observation types: Use continuation line(s)	6X,9(4X,A2)
	The following observation types are defined in RINEX Version 2.10:	
	L1, L2: Phase measurements on L1 and L2	
	C1 : Pseudorange using C/A-Code on L1	
	P1, P2: Pseudorange using P-Code on L1,L2	
	D1, D2: Doppler frequency on L1 and L2	
	T1, T2: Transit Integrated Doppler on 150 (T1) and 400 MHz (T2)	
	S1, S2: Raw signal strengths or SNR values as given by the receiver for the L1,L2 phase observations	
	Observations collected under Antispoofing are converted to "L2" or "P2" and flagged with bit 2 of loss of lock indicator (see Table A2).	

RINEX Header

```
Units : Phase      : full cycles  
       Pseudorange : meters  
       Doppler      : Hz  
       Transit      : cycles  
       SNR etc      : receiver-dependent
```

```
The sequence of the types in this record  
has to correspond to the sequence of the  
observations in the observation records
```

RINEX Header

* INTERVAL	Observation interval in seconds	F10.3	*
TIME OF FIRST OBS	<ul style="list-style-type: none"> - Time of first observation record (4-digit-year, month, day, hour, min, sec) - Time system: GPS (=GPS time system) GLO (=UTC time system) Compulsory in mixed GPS/GLONASS files Defaults: GPS for pure GPS files GLO for pure GLONASS files	5I6,F13.7, 5X,A3	
* TIME OF LAST OBS	<ul style="list-style-type: none"> - Time of last observation record (4-digit-year, month, day, hour, min, sec) - Time system: Same value as in TIME OF FIRST OBS record 	5I6,F13.7, 5X,A3	*
* RCV CLOCK OFFS APPL	Epoch, code, and phase are corrected by applying the realtime-derived receiver clock offset: 1=yes, 0=no; default: 0=no Record required if clock offsets are reported in the EPOCH/SAT records	I6	*
* LEAP SECONDS	Number of leap seconds since 6-Jan-1980 Recommended for mixed GPS/GLONASS files	I6	*
* # OF SATELLITES	Number of satellites, for which observations are stored in the file	I6	*

RINEX Header

* PRN / # OF OBS	PRN (sat.number), number of observations for each observation type indicated in the "# / TYPES OF OBSERV" - record. If more than 9 observation types: Use continuation line(s) This record is (these records are) repeated for each satellite present in the data file	3X,A1,I2,9I6 6X,9I6	*
END OF HEADER	Last record in the header section.	60X	

Records marked with * are optional

Header example

```
2.10      OBSERVATION DATA      M (MIXED)
BLANK OR G = GPS,  R = GLONASS,  T = TRANSIT,  M = MIXED
XXRINEXO V9.9      AIUB           24-MAR-01 14:43
EXAMPLE OF A MIXED RINEX FILE

A 9080
9080.1.34
BILL SMITH          ABC INSTITUTE
X1234A123          XX           ZZZ
234                YY

  4375274.         587466.         4589095.
      .9030             .0000             .0000
1      1
1      2      6      G14      G15      G16      G17      G18      G19
0
4      P1      L1      L2      P2
18.000
2001      3      24      13      10      36.0000000
```

```
RINEX VERSION / TYPE
COMMENT
PGM / RUN BY / DATE
COMMENT
MARKER NAME
MARKER NUMBER
OBSERVER / AGENCY
REC # / TYPE / VERS
ANT # / TYPE
APPROX POSITION XYZ
ANTENNA: DELTA H/E/N
WAVELENGTH FACT L1/2
WAVELENGTH FACT L1/2
RCV CLOCK OFFS APPL
# / TYPES OF OBSERV
INTERVAL
TIME OF FIRST OBS
END OF HEADER
```

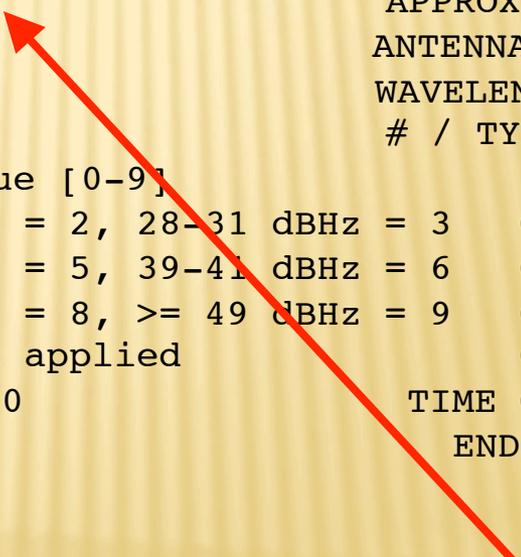
(I've not seen many headers with the "time of last observation" line)

Another header example

```

2.10          OBSERVATION DATA      G (GPS)          RINEX VERSION / TYPE
teqc 2005Feb10  You don't know?      20050411 15:07:57UTC PGM / RUN BY / DATE
Linux 2.0.36|Pentium II|gcc|Linux|486/DX+ COMMENT
BIT 2 OF LLIGFLAGS DATA COLLECTED UNDER A/S CONDITION COMMENT
CJTR          MARKER NAME
-Unknown-    -Unknown-              OBSERVER / A ENCY
664          ASHTECH Z-12            REC # / TYPE / VERS
943          -Unknown-              ANT # / TYPE
          0.0000          0.0000          0.0000          APPROX POSITION XYZ
          0.0000          0.0000          0.0000          ANTENNA: DELTA H/E/N
1           1           WAVELENGTH FACT L1/2
5          L1          L2          C1          P1          P2          # / TYPES OF OBSERV
SNR is mapped to RINEX snr flag value [0-9] COMMENT
L1 & L2:  2-19 dBHz = 1, 20-27 dBHz = 2, 28-31 dBHz = 3 COMMENT
          32-35 dBHz = 4, 36-38 dBHz = 5, 39-41 dBHz = 6 COMMENT
          42-44 dBHz = 7, 45-48 dBHz = 8, >= 49 dBHz = 9 COMMENT
pseudorange smoothing corrections not applied COMMENT
2004        12        26         0         0        30.0000000 TIME OF FIRST OBS
END OF HEADER

```



Not having an XO estimate makes processing more difficult

RINEX Observations (data)

TABLE A2
GPS OBSERVATION DATA FILE - DATA RECORD DESCRIPTION

OBS. RECORD	DESCRIPTION	FORMAT
EPOCH/SAT or EVENT FLAG	<ul style="list-style-type: none"> - Epoch : - year (2 digits, padded with 0 if necessary) - month, day, hour, min, - sec - Epoch flag 0: OK <li style="padding-left: 2em;">1: power failure between previous and current epoch <li style="padding-left: 2em;">>1: Event flag - Number of satellites in current epoch - List of PRNs (sat.numbers with system identifier, see 5.1) in current epoch - receiver clock offset (seconds, optional) <li style="padding-left: 2em;">If more than 12 satellites: Use continuation line(s) <li style="padding-left: 2em;">If epoch flag 2-5: 	<p>1X,I2.2, 4(1X,I2), F11.7, 2X,I1, I3, 12(A1,I2), F12.9 32X, 12(A1,I2)</p>

RINEX Observations (data)

- | | |
|---|----------|
| - Event flag: | [2X,I1,] |
| 2: start moving antenna | |
| 3: new site occupation (end of kinem. data)
(at least MARKER NAME record follows) | |
| 4: header information follows | |
| 5: external event (epoch is significant,
same time frame as observation time tags) | |
|
 | |
| - "Number of satellites" contains number of
special records to follow.
Maximum number of records: 999 | [I3] |
|
 | |
| - For events without significant epoch the
epoch fields can be left blank | |
|
 | |
| If epoch flag = 6: | |
| 6: cycle slip records follow to optionally
report detected and repaired cycle slips
(same format as OBSERVATIONS records;
slip instead of observation; LLI and
signal strength blank or zero) | |

OBSERVATIONS

- Observation | rep. within record for
- LLI | each obs.type (same seq
- Signal strength | as given in header)

m(F14.3,
I1,
I1)

If more than 5 observation types (=80 char):
continue observations in next record.

This record is (these records are) repeated for
each satellite given in EPOCH/SAT - record.

Observations:

Phase : Units in whole cycles of carrier

Code : Units in meters

Missing observations are written as 0.0
or blanks.

Phase values overflowing the fixed format F14.3
have to be clipped into the valid interval (e.g.
add or subtract 10^{*9}), set LLI indicator.

Loss of lock indicator (LLI). Range: 0-7

0 or blank: OK or not known

Bit 0 set : Lost lock between previous and
current observation: cycle slip
possible

Bit 1 set : Opposite wavelength factor to the
one defined for the satellite by a
previous WAVELENGTH FACT L1/2 line.
Valid for the current epoch only.

RINEX Observations (data)

Bit 2 set : Observation under Antispoofing
(may suffer from increased noise)

Bits 0 and 1 for phase only.

Signal strength projected into interval 1-9:

1: minimum possible signal strength

5: threshold for good S/N ratio

9: maximum possible signal strength

0 or blank: not known, don't care

Phase in cycles, Range in meters

```

04 12 26 0 0 30.0000000 0 9G 4G24G 5G17G 6G10G30G 2G29
-7408143.20348 -5712212.12343 23722895.4574 23722895.8514 23722901.0124
-11151164.34848 -8348759.79145 23027140.6794 23027140.3024 23027147.6974
-17702667.27649 -13496720.20047 21946318.4604 21946318.0704 21946325.1504
-20607717.25049 -16031193.33649 20980332.7214 20980332.1484 20980339.2334
-10697009.82948 -8319281.13543 23671597.2204 23671597.2244 23671604.0324
-25994074.45749 -20224979.69249 20080903.8494 20080902.8804 20080910.1054
-17497598.39549 -13604851.76347 21641129.8624 21641129.7384 21641136.2574
-24900942.06749 -19353992.61648 20874424.4194 20874423.9874 20874428.6824
-2640345.03446 -1780147.16442 24402022.2324 24402021.1924 24402029.2134

```

L1

L2

C1

P1

P2

Input format is “fortranny” (Hollerith)
 (fixed number of digits per data entry field, in fixed
 “card columns”, can leave field blank for zero or no
 data)

Plus more for

Navigation

“met” (METEOROLOGICAL)

Tilt

Other?