Geodetic Coordinate Conversions

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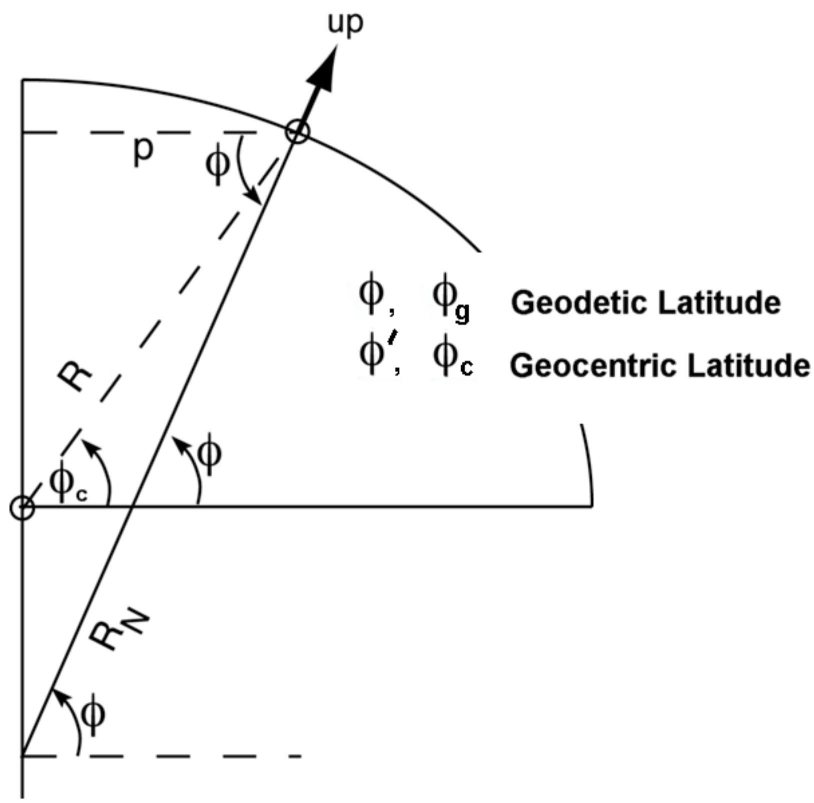
I. Geodetic to/from Geocentric Latitude



Geodetic and Geocentric Latitude

A. Geodetic Latitude (ϕ, or ϕ_g) to Geocentric Latitude (ϕ', or ϕ_c)

There are many equations that can be used. One of the most common involves the tangent of the latitude. At a geodetic or ellipsoidal height h,

$$\tan\phi_c = \left[1 - e^2 \frac{R_N}{R_N + h}\right] \tan\phi$$

where the radius of curvature in the prime vertical, R_N, is given by

$$R_N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$

B. Geocentric Latitude (ϕ', or ϕ_c) to Geodetic Latitude (ϕ, or ϕ_g)

This case uses the same equations. The value of R_N is found using the geocentric latitude. The error in this approximation is second order in smallness and is usually ignored. The ratio factor is, of course divided into the right hand side of the tangent equation in this case

$$\tan\phi = \left[1 - e^2 \frac{R_N}{R_N + h}\right]^{-1} \tan\phi_c$$

Note on common errors:

The heights used in these equations is ellipsoidal or geodetic height. It is not the height seen on maps and may differ from that height by 100 m. See the note on heights for more details. However as the height only enters in a ratio after being added to a quantity approximately the radius of the earth, the result is fairly insensitive to small errors in h.

A similar magnitude error happens if you use geocentric latitude in computing Rn. (A 70 m error at 45 deg latitude). In both cases the maximum North-South position error occurs at an altitude of 1 earth radius at 45 deg latitude. This error is 1.1 meter. One iteration on latitude types makes this much less than a cm. The error remains if you use the orthometric height even if you iterate. Both these errors result in zero position error on the ellipsoid.

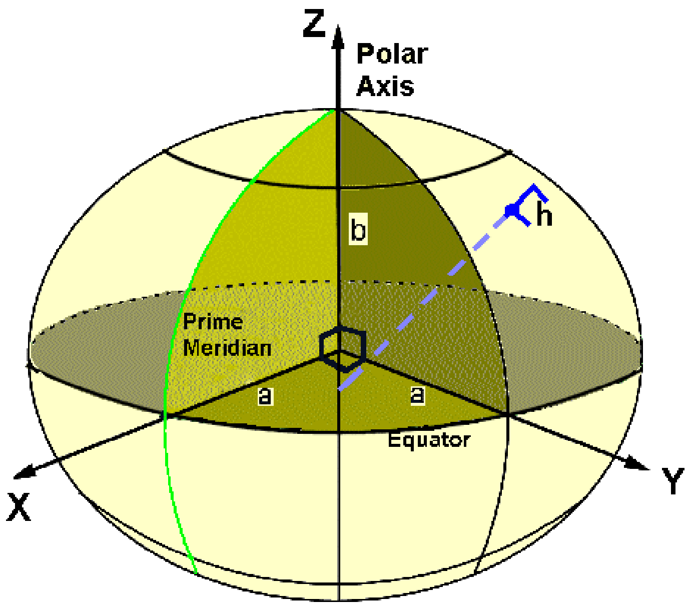
There is a common approximation for small h (near earth),

$$\tan\phi_c = [1 - e^2 + 1.1x10^{-9} h(m)] \tan\phi$$

where h must be in meters.

The constant multiplying h is e^2/Rn and has the units of inverse length. Clearly this approximation will not work well at altitudes that are a significant fraction of a earth radius, such as GPS or geosynchronous satellites.

II. Latitude, Longitude and Height to/from ECEF (x,y,z)



Latitude, Longitude, Height
To/From (X,Y,Z)

A. Latitude, Longitude, Height to ECEF xyz

There is a closed form solution for this transformation. Given geodetic latitude, ϕ, (what you find on maps), longitude, λ, and ellipsoidal height h, then

$$\begin{aligned} x &= (R_N + h)\cos\phi\cos\lambda \\ y &= (R_N + h)\cos\phi\sin\lambda \\ z &= ([1-e^2]R_N + h)\sin\phi \end{aligned}$$

Note that the longitude will be East Longitude. This is the convention for geodesy.

B. ECEF xyz to Latitude, Longitude, Height

There is no closed form solution for this transformation if the altitude is not zero. The problem is that the radius R_N is needed to find geodetic height h and geodetic latitude is needed to find R_N. The usual procedure is to iterate beginning with the assumption that there is no difference between geodetic and geocentric latitude.

First compute the longitude, which can be precisely done.

$$\begin{aligned}\lambda &= a\tan(y/x)\\ &= a\tan 2(y,x)\end{aligned}$$

The second form is the usual computer call for a 4-quadrant arctangent. Note that computer code usually returns angles in radians. These must be converted to degrees. Note also that this procedure produces East Longitude.

Next the physical radius of the point and the radius in the x-y plane are computed and used in an initial estimate of the altitude.

$$\begin{aligned}r &= \sqrt{x^2+y^2+z^2}\\ p &= \sqrt{x^2+y^2}\end{aligned}$$

The geocentric latitude is computed exactly, and used as the initial value for the geodetic latitude in the iteration loop.

$$\begin{aligned}\phi_c &= a\tan(p/z)\\ &= a\tan 2(p,z)\\ \phi_{now} &= \phi_c\end{aligned}$$

The loop is:

$$\begin{aligned}h &= \frac{p}{\cos\phi_{now}} - R_N(\phi_{now})\\ \phi_{next} &= a\tan\left[\frac{z}{p}\left(1-e^2\frac{R_N}{R_N+h}\right)^{-1}\right]\end{aligned}.$$

This converges in a few iterations (4 at most) to a few centimeters. This is for positions even at earth satellite altitudes. After the geodetic latitude, ϕ, is found the ellipsoidal height, h is obtained from

$$h = \frac{p}{\cos\phi} - R_N .$$

This equation for h diverges at the poles. There are two alternatives. One is

$$h = \frac{L}{\sin\phi} - R_N ,$$

and the other is

$$h = \sqrt{L^2 + z^2} - R_N ,$$

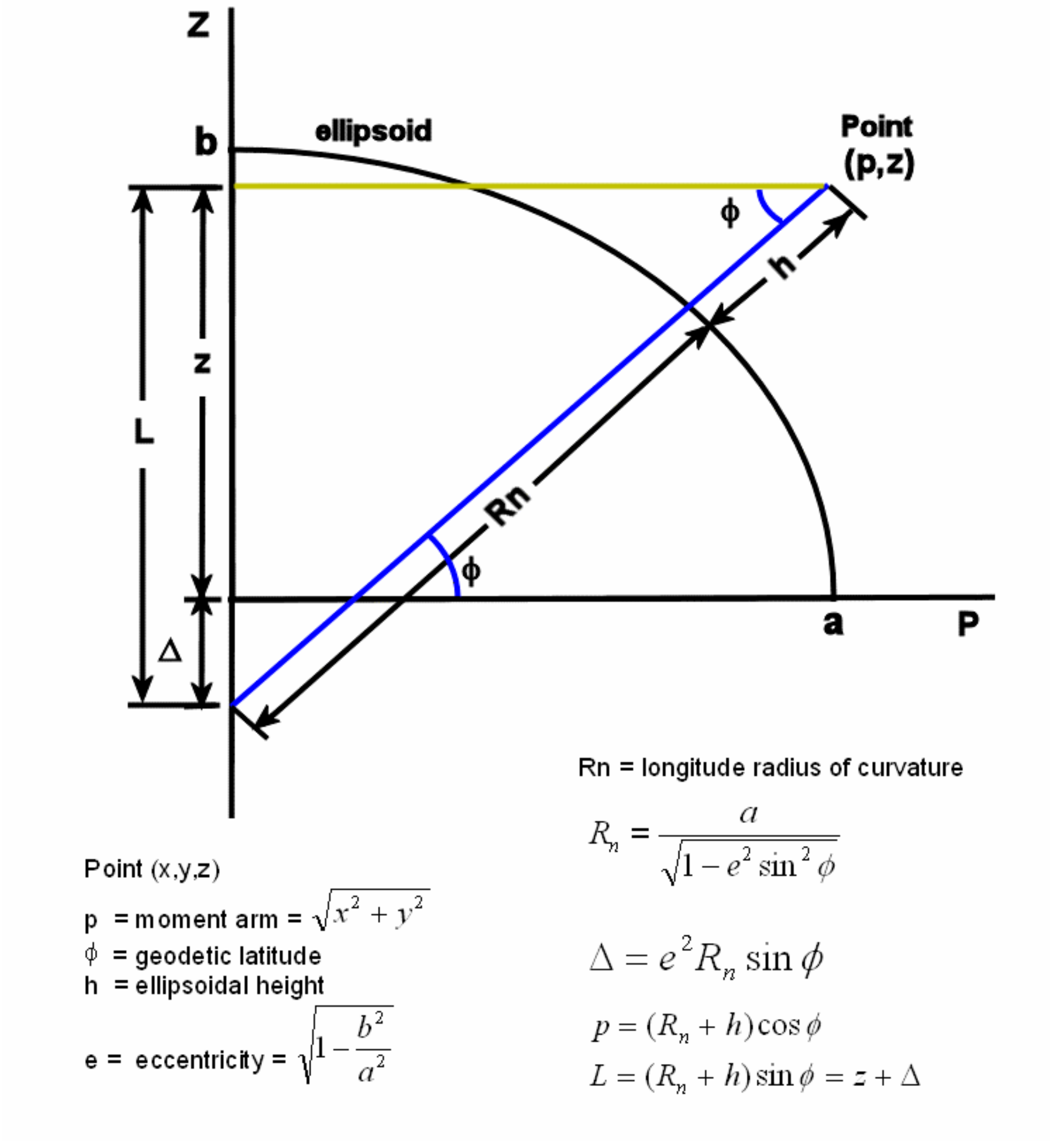
where

$$L = z + e^2 R_N \sin\phi .$$

Where

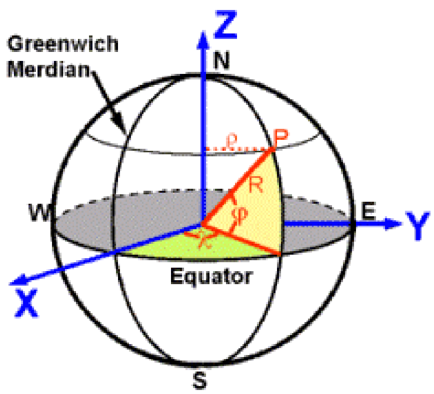
$L = z + e^2 R_N \sin\phi$

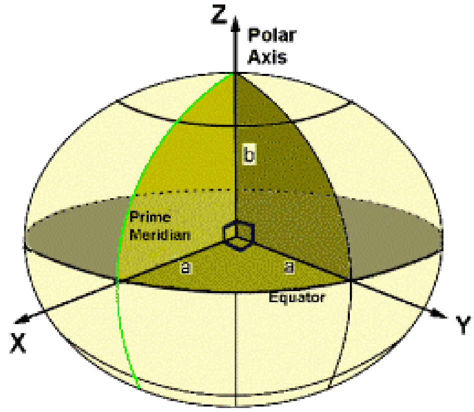
These equations are based on the geometry in the figure below.



Geometry For
XYZ to Latitude Longitude Height

Cartesian to Angular





$$
\begin{aligned}
x &= (r+h)\cos\phi\cos\lambda \\
y &= (r+h)\cos\phi\sin\lambda \\
z &= (r+h)\sin\phi
\end{aligned}
$$

Spherical

$$
\begin{aligned}
x &= (R_N+h)\cos\phi\cos\lambda \\
y &= (R_N+h)\cos\phi\sin\lambda \\
z &= [(1-e^2)R_N+h]\sin\phi
\end{aligned}
$$

Ellipsoidal

NOTE: The heights used in these equations is ellipsoidal or geodetic height. It is not the height seen on maps and may differ from that height by 100 m. See the note on heights for more details.