

Circular or periodic convolution (what we usually DON'T want!
But be careful, in case we do want it!)

Remembering that convolution in the TD is multiplication in the FD
(and vice-versa) for both continuous and discrete infinite length
sequences, we would like to see what happens for periodic, finite-
duration sequences.

So let's form the product of the DFS in the FD and see what we
get after an IDFS back to the TD.

We have the FT pairs (the relationship is symmetric)

$$\tilde{x}_1(n) * \tilde{x}_2(n) \Leftrightarrow \tilde{X}_1(k) \tilde{X}_2(k)$$

$$\tilde{x}_1(n) \tilde{x}_2(n) \Leftrightarrow \tilde{X}_1(k) * \tilde{X}_2(k)$$

This is fine mathematically.

But now, unless we are careful, what we get on the computer is known as circular convolution.

It comes from the fact that the DFT is periodic, with the period equal to the length of the finite sequence.

Remember that for “regular” convolution - we padding the sequences with zeros, flipped one and slid them along one another, so for the sequences (0, 2, 1, 0) and (0, 3, 4, 0) we had

$$\begin{array}{cccc} \dots & 0 & 2 & 1 & 0 & \dots \\ & \bullet & \bullet & \bullet & & \\ \dots & 0 & 4 & 3 & 0 & \dots \\ \hline \end{array}$$

$$0 + 0 + 6 + 0 + 0 = 6 = z_0$$

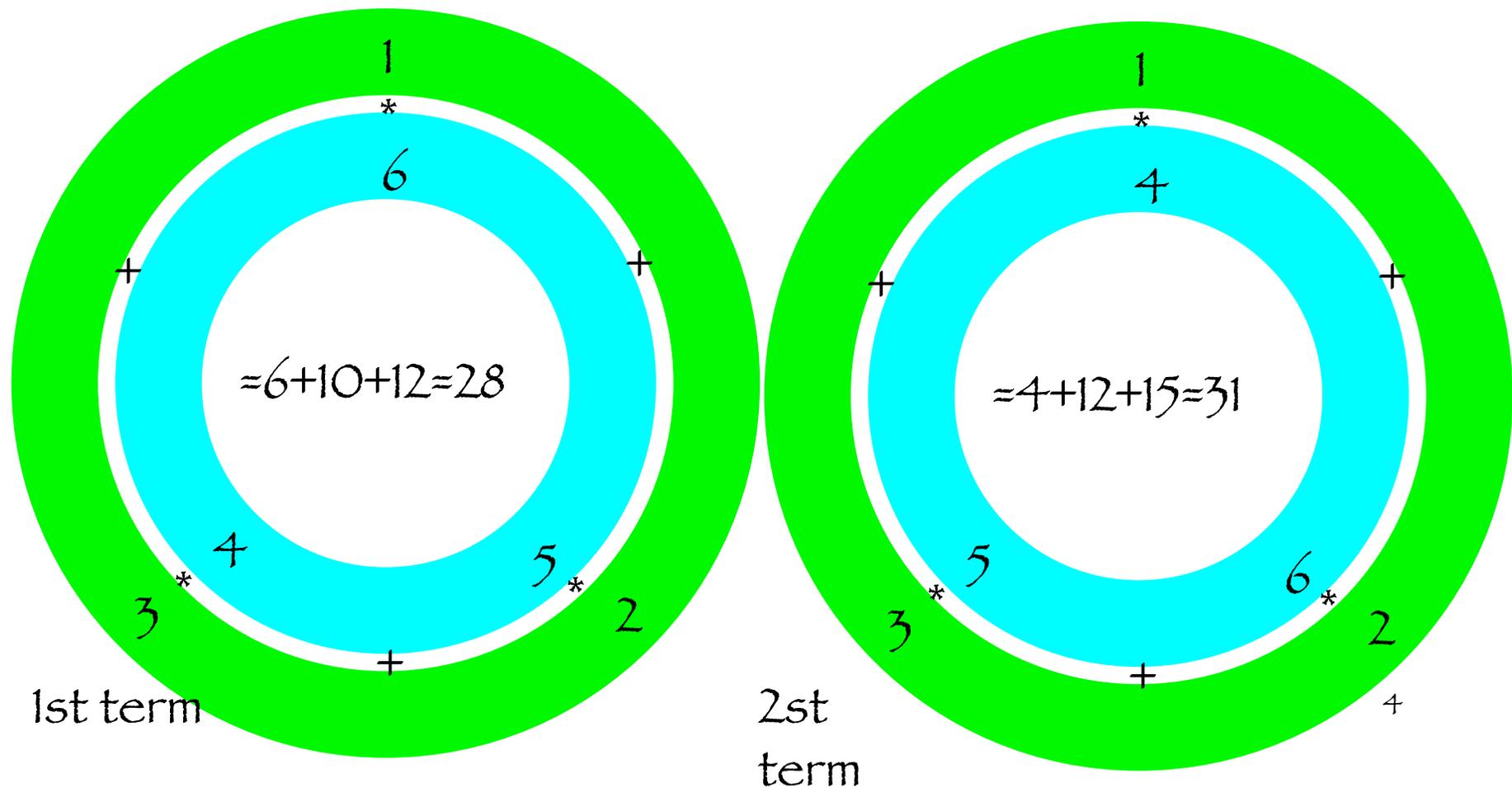
$$\begin{array}{cccc} \dots & 0 & 2 & 1 & 0 & \dots \\ & \bullet & \bullet & \bullet & \bullet & \\ \dots & 0 & 4 & 3 & 0 & \dots \\ \hline \end{array}$$

$$0 + 8 + 3 + 0 + 0 = 11 = z_1$$

$$\begin{array}{cccc} \dots & 0 & 2 & 1 & 0 & \dots \\ & & \bullet & \bullet & \bullet & \\ \dots & & 0 & 4 & 3 & 0 & \dots \\ \hline \end{array}$$

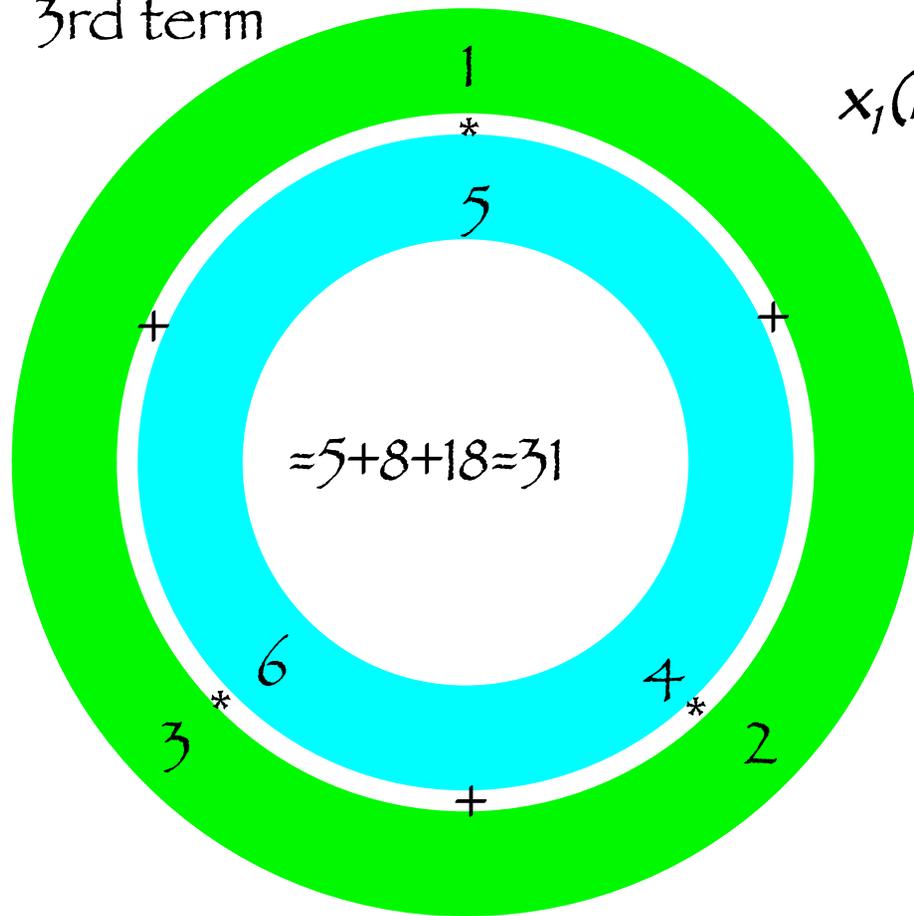
$$0 + 0 + 4 + 0 + 0 = 4 = z_2$$

In circular or periodic convolution we can look at the N point sequences as being distributed on a circle due to the periodicity. Now we do the same thing (line up, multiply and add, then shift), but with concentric circles. Let's convolve $x_1(n)=(1,2,3)$ and $x_2(n)=(4,5,6)$. One sequence is distributed clockwise and the other counterclockwise and the shift of the inner circle is clockwise.



So $y(n)$ is obtained in a manner reminiscent of convolution with the modifications that $x_1(m)$ and $x_2(m-n)$ are periodic in m with period N (this makes the “circular” part) and consequently so is their “product” (periodic in m with period N and circular). Also remember that the summation is carried out over only ONE period.

3rd term



$$x_1(n) * x_2(n) = (1, 2, 3) * (4, 5, 6) = (18, 31, 31).$$

We have the same symmetry as before

$$\tilde{y}(n) = \sum_{k=0}^{N-1} \tilde{x}_1(k) \tilde{x}_2(n-k)$$

$$\tilde{y}(n) = \sum_{k=0}^{N-1} \tilde{x}_2(k) \tilde{x}_1(n-k)$$

