Inverse Methods in Geophysics
CERI 7260/8260; CVL 7128/8128
Spring 2018
Problem Set #3
Due Wednesday 26 February 2018

1. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 1 & 3 \\ 4 & 6 & 7 & 11 \end{bmatrix}.$$

   Find bases for $N(\mathbf{A})$, $R(\mathbf{A})$, $N(\mathbf{A}^T)$, and $R(\mathbf{A}^T)$. What are the dimensions of the four subspaces?

2. Let **A** be an n by n matrix such that A^{-1} exists. What are $N(\mathbf{A})$, $R(\mathbf{A})$, $N(\mathbf{A}^T)$, and $R(\mathbf{A}^T)$?

3. Suppose that a nonsingular matrix A can be diagonalized as

$$\mathbf{A} = \mathbf{P}\Lambda\mathbf{P}^{-1}.$$

   Find a diagonalization of $\mathbf{A}^{-1}$. What are the eigenvalues of $\mathbf{A}^{-1}$?

4. Let $P_3[0,1]$ be the space of polynomials of degree less than or equal to 3 on the interval [0, 1]. The polynomials $p_1(x)=1$, $p_2(x)=x$, $p_3(x)=x^2$, and $p_4(x)=x^3$ form a basis for $P_3[0, 1]$, but they are not orthogonal with respect to the inner product

$$f \cdot g = \int_0^1 f(x) g(x)\, dx.$$

   Use the Gram-Schmidt orthogonalization process to construct an orthogonal basis for $P_3[0, 1]$. Once you have your basis, use it to find the third-degree polynomial that best approximates $f(x)=e^{-x}$ on the interval [0, 1].