Satellite Orbits

Keplerian motion
Perturbed motion
GPS satellites
GPS orbits

E. Calais
Purdue University - EAS Department
Civil 3273 – ecalais@purdue.edu
Orbits?
Satellite orbits: What for?

- Principle of GPS positioning:
  - Satellite 1 sends a signal at time $t_{e1}$
  - Ground receiver receives it signal at time $t_r$
  - The range measurement $\rho_1$ to satellite 1 is:
    - $\rho_1 = (t_r - t_{e1}) \times$ speed of light
    - We are therefore located on a sphere centered on satellite 1, with radius $\rho_1$
  - 3 satellites => intersection of 3 spheres

- The mathematical model is:
  $$\rho^s_r = \sqrt{(X_s - X_r)^2 + (Y_s - Y_r)^2 + (Z_s - Z_r)^2}$$
  - GPS receivers measure $\rho^s_r$
  - If the position of the satellites in an Earth-fixed frame $(X_s, Y_s, Z_s)$ is known,
  - Then one can solve for $(X_r, Y_r, Z_r)$ (if at least 3 simultaneous range measurements)
Dynamics of satellite orbits

• Basic dynamics in an inertial frame described by: \[ \sum \vec{F} = m\ddot{a} \]

• Case of two-body problem (two point masses), forces are:
  – Gravitational forces
  – Solar radiation pressure (drag is negligible for GPS)
  – Thruster firings (not directly modeled).

• Neglecting radiation pressure, one can write:

\[ \vec{F} = \frac{Gm_em_E}{r^2} \vec{r} \quad \text{and} \quad \vec{F} = m_s\ddot{a} \]

\[ \Rightarrow \ddot{a} - \frac{Gm_Em_E}{r^2} \vec{r} = 0 \leftrightarrow \frac{d^2 \vec{r}}{dt^2} = \frac{Gm_E}{r^2} \vec{r} \]

- \( r \) = geocentric position vector
- \( a = d^2r/dt^2 \), relative acceleration vector
- \( G \) = universal gravitational constant
- \( m_E \) = Earth’s mass
- \( m_S \) = satellite’s mass
\[ m \dddot{r}_s = -G \frac{m_s m_E}{r^2} \cdot \ddot{r} \]

\[ m \dddot{r}_E = G \frac{m_s m_E}{r^2} \cdot \ddot{r} \]

\[ r = \| \ddot{r}_s - \ddot{r}_E \| \]

\[ \ddot{r} = \frac{\ddot{r}_s - \ddot{r}_E}{r} \]

\[ \dddot{r}_s - \dddot{r}_E = -G \frac{m_E}{r^2} \ddot{r} - G \frac{m_s}{r^2} \ddot{r} \]

\[ \Rightarrow \dddot{r} = -G (m_s + m_E) \frac{\ddot{r}}{r^2} \]

\[ \dddot{r} = -\mu \frac{\ddot{r}}{r^2} \]

\[ \mu = G (m_s + m_E) \]
Keplerian motion

Analytical solution to this force model =
Keplerian (= osculating) orbit:

- Six integration constants => 6 orbital parameters
  or Keplerian (= osculating) elements
- In an **inertial** reference frame, orbits can be
described by an ellipse
- The orbit plane stays fixed in space
- One of the foci of the ellipse is the center of
  mass of the body
- These orbits are described by Keplerian
  elements:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Semi-major axis</td>
<td>Size and shape of orbit</td>
</tr>
<tr>
<td>e</td>
<td>Eccentricity</td>
<td></td>
</tr>
<tr>
<td>Ω</td>
<td>Right ascension of ascending node</td>
<td>Orientation of the orbital plane in the inertial system</td>
</tr>
<tr>
<td>ω</td>
<td>Argument of perigee</td>
<td></td>
</tr>
<tr>
<td>i</td>
<td>Inclination</td>
<td></td>
</tr>
<tr>
<td>T₀</td>
<td>Epoch of perigee</td>
<td>Position of the satellite in the orbital plane</td>
</tr>
</tbody>
</table>
Keplerian motion

• Third Kepler’s law (period^2/a^3=const.) relates mean angular velocity n and revolution period P:

\[ n = \frac{2\pi}{P} = \sqrt{\frac{GM_E}{a^3}} \]

  \( M_E = \) Earth’s mass, \( GM_E = 3,986,005 \times 10^8 \text{ m}^3\text{s}^{-2} \)

• For GPS satellites:
  
  – Nominal semi-major axis \( a = 26,560,000 \text{ m} \)
  
  – Orbital period = 12 sidereal hours (= 11h28mn UT), \( v=3.87 \text{ km/s} \)
  
  – Therefore, positions of GPS satellites w.r.t. Earth’s surface repeat every sidereal day
Keplerian motion

- Instantaneous position of a satellite on its orbit is defined by angular quantities called "anomalies":
  - Mean anomaly: \( M(t) = n \times (t - To) \)
    with \( n = \) mean motion = number of orbits in 24 hours, \( To = \) time of perigee
  - Eccentric anomaly: \( E(t) = M(t) + e \sin E(t) \)
  - True anomaly: \( \nu(t) = 2 \tan^{-1} \left( \frac{(1+e)/(1-e)^{1/2}}{\tan(E(t)/2)} \right) \)

- In the coordinate system defined by the orbital plane:
  - \( e_1 = r \cos \nu = a \cos E - ae = a (\cos E - e) \)
  - \( e_2 = r \sin \nu = \frac{b}{a} a \sin E = b \sin E = a (1-e^2)^{1/2} \sin E \)
  - \( e_3 = 0 \)

\[
\vec{r} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} a(\cos E - e) \\ a \sqrt{1-e^2} \sin E \\ 0 \end{bmatrix}
\]

\[
\|\vec{r}\| = a(1-e \cos E)
\]
Keplerian motion

- GPS orbit in an inertial frame (+Earth rotation)
- To compute site position on the Earth, we need the satellite orbit to an Earth-fixed frame…

Fig. T. Herring (MIT)
Keplerian motion

In the **Earth centered inertial system** \((X_1^0, X_2^0=X_2, X_3^0=X_3)\), \(\rho\) relates to \(r\) through the combination of 3 rotations:

\[
\begin{align*}
\rho &= R \bar{r} \\
R &= R_3(-\Omega) R_1(-i) R_3(-\omega)
\end{align*}
\]

\[
R = \begin{bmatrix}
\cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i & -\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i & \sin \Omega \sin i \\
\sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i & -\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i & -\cos \Omega \sin i \\
\sin \omega \sin i & -\sin \omega \cos i & \cos i
\end{bmatrix}
\]

[think of what it takes for \((e_1, e_2, e_3)\) to align with \((X_1^0, X_2, X_3)\)]

\(\Omega\) = ascension of ascending node
\(\omega\) = argument of perigee
\(i\) = inclination
\(v\) = true anomaly
Keplerian motion

• With 3 rotations \((\Omega, i, \omega)\), we went from orbital plane coordinates \((e1, e3, e3)\) to Earth-centered inertial coordinates \((r)\).

• To convert \(r\) to an Earth fixed system \((X_1, X_2, X_3)\), we need an additional rotation of \(\Theta_o\) (related to the Greenwich Sidereal Time) about \(X_3\):

\[
\bar{\rho} = R' \bar{r}
\]

\[
R' = R_3\{\Theta_o\} R = R_3\{\Theta_o\} R_3\{-\Omega\} R_1\{-i\} R_3\{-\omega\}
\]

\[ \text{defining:} \quad l = \Omega - \Theta_o \]

\[
R' = \begin{bmatrix}
\cos l \cos \omega - \sin l \sin \omega \cos i & -\cos l \sin \omega - \sin l \cos \omega \cos i & \sin l \sin i \\
\sin l \cos \omega + \cos l \sin \omega \cos i & -\sin l \sin \omega + \cos l \cos \omega \cos i & -\cos l \sin i \\
\sin \omega \sin i & \cos \omega \cos i & \cos i
\end{bmatrix}
\]
GPS Orbits

1. Broadcast ephemerides (in GPS signal) contain orbital parameters $M, e, a, \Omega, i, \omega, n$ (=angular velocity)

2. Orbital parameters are used to compute orbit in inertial frame

3. Orbit in inertial frame is then rotated into terrestrial frame (Earth-fixed)

4. In order to do this accurately over long time periods, one need to account for variations in Earth orientation and rotation parameters => use precise UT1, nutations, and polar motion.

5. And take into account orbit perturbations (Earth’s gravity field, solar pressure, attraction from Sun and Moon): also provided in broadcast ephemerides

Fig. T. Herring (MIT)
GPS Orbits

Other representations of GPS orbits

GPS orbit in Earth-fixed frame = “ground track”

GPS orbit in topocentric frame = “sky plot”
Perturbed motion

Central gravitational force = main force acting on GPS satellites, but there are other significant perturbations:

- **Gravitational forces:**
  - Non sphericity of the Earth gravitational potential:
    - Major contribution from the Earth’s flattening ($J_2$)
    - GPS orbits high (20 000 km) and attraction force attenuates rapidly with altitude => only a few terms of the Earth’s gravitational potential are necessary for modeling GPS orbits
  - Third body effect: direct attraction of Moon and Sun => lunar and solar ephemerides necessary to model GPS orbits
  - [Tidal effects of Sun and Moon => deforms Earth => modifies Earth’s gravitational potential: negligible for GPS satellites]

- **Non-gravitational forces:**
  - Solar radiation pressure:
    - Impact on the satellite surfaces of photons emitted by the Sun and reflected by the Earth surface: can be modeled, knowing the 3D geometry and the attitude of the satellite.
    - Effects on GPS satellite position: 5-10 m
    - Eclipse periods = satellite in the Earth’s shadow (1-2/year, lasts about 1 hr): transition to Sun light difficult to model, usually this part of the orbit is simply edited out!
  - [Atmospheric drag = negligible for GPS satellites]
  - Satellite maneuvers
Solar radiation pressure

- Results from impact of Sun photons on the satellite’s surface.
- Depends on:
  - Effective area (surface normal to the incident radiation)
  - Surface reflectivity
  - Luminosity of the Sun
  - Distance to the Sun
- GPS satellites oriented so that the “Y-axis” is perpendicular to the direction of the Sun (solar sensors)
- For satellites in the Earth shadow region (eclipsing), the solar radiation pressure is zero.
- For precise orbit determination, the shadow region must be carefully determined using the relative positions of the Sun, Earth, and the satellites.

**Terminology**

Reflectivity ($\delta$) – the proportion of radiation incident on a surface that is reflected, the reflected radiation being separated into diffuse (scattered) and specular (beamed) components.

Specularity ($\mu$) – the proportion of reflected radiation that is reflected specularly. Specular reflection implies that the surface behaves like a perfect mirror.

Y-bias – a force acting along the spacecraft BFS Y-axis and believed to derive from NCF effects. A likely mechanism for the Y-bias is due to non-orthogonality of the solar panels with respect to the solar photon flux, as a result of attitude bias or variations. However, another possible contribution could come from heat dissipation effects of payload components.

![Diagram of solar radiation pressure](image)
## Perturbed motion

<table>
<thead>
<tr>
<th>Term</th>
<th>Acceleration (m/sec(^2))</th>
<th>Perturbation (after 2 days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central</td>
<td>0.6</td>
<td>-</td>
</tr>
<tr>
<td>(J_2)</td>
<td>5\times10^{-5}</td>
<td>14 km</td>
</tr>
<tr>
<td>Other gravity harmonics</td>
<td>3\times10^{-7}</td>
<td>0.1-1.5 km</td>
</tr>
<tr>
<td>Third body</td>
<td>5\times10^{-6}</td>
<td>1-3 km</td>
</tr>
<tr>
<td>Earth tides</td>
<td>10^{-9}</td>
<td>0.5-1 m</td>
</tr>
<tr>
<td>Ocean tides</td>
<td>10^{-10}</td>
<td>0-2 m</td>
</tr>
<tr>
<td>Drag</td>
<td>\sim0</td>
<td>-</td>
</tr>
<tr>
<td>Solar radiation</td>
<td>10^{-7}</td>
<td>0.1-0.8 km</td>
</tr>
<tr>
<td>Albedo radiation</td>
<td>10^{-9}</td>
<td>1-1.5 m</td>
</tr>
</tbody>
</table>
GPS orbits

- Orbit characteristics:
  - Semi-major axis = 26,400 km
  - Period = 12 sidereal hour
  - 6 orbital planes
  - Inclination = 55.5 degrees (except old block I sats = 63 deg., but all dead)
  - Eccentricity nearly 0 (largest 0.02) => quasi-circular orbits

- Full constellation = 24 satellites, completed March 9, 1994
GPS Orbital Constellation

- 27 satellites (24 operational + 3 spares = nominal constellation)
- 6 evenly spaced orbital planes (A to F), inclination 55°
- 4-6 satellites per plane, spacing for optimized visibility
- Period = 12 sidereal hours (= 11h58mn “terrestrial” hours) => in a terrestrial frame, the constellation repeats every 23h56mn.

- As Earth orbits around the Sun => eclipse periods (solar radiation pressure = 0, transition to shadow difficult to model, often simply edited out)
GPS satellite visibility

- Satellite visible up to 6 hours in a row (from rising to setting)
- In practice, 6-12 satellites are visible simultaneously, depending on:
  - Constellation geometry
  - User location
  - Elevation cut-off angle (chosen by the user)
- Problematic environments:
  - Forest
  - “urban canyons”
  - Mission planning may become necessary: determine best time of day to make measurements
- Site selection:
  - Must account for masks (minimal)
  - A! masks can grow (trees, houses)
GPS constellation


GPS satellites

- Solar powered
- S-Band (SGLS) communications for control and telemetry + UHF cross-link between spacecraft.
- Two L-Band navigation signals at 1575.42 MHz (L1) and 1227.60 MHz (L2).
- Each spacecraft carries 2 rubidium or 2 cesium clocks.
- Several generations of GPS satellites have been built and launched (next slide):
  - Different masses and phase center
  - Different capabilities
GPS satellites

  – 11 satellites launched between 1978 and 1985 Life expectancy = 4.5 years, actual mean life = 7.1 years
  – Signal entirely accessible to civilian users
  – Last block I satellite died on Feb. 28, 1994
• Block II (II,IIA,IIR, IIR-M)
  – First launch 1989, latest launch December 20, 2007
  – Possibility to degrade the signal for civilian users (= selective availability)
• Next generation: Block III, 2010
  – Selective availability eliminated
  – Additional navigation signals
• Details on: http://www.spaceandtech.com/spacedata/constellations/navstar-gps_consum.shtml

<table>
<thead>
<tr>
<th></th>
<th>Block I</th>
<th>Block II/IIA</th>
<th>Block IIR</th>
<th>Block IIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>11</td>
<td>28</td>
<td>21</td>
<td>12</td>
</tr>
<tr>
<td>First launch</td>
<td>1978</td>
<td>1989</td>
<td>1997</td>
<td>2005</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>760</td>
<td>1660</td>
<td>2000</td>
<td>?</td>
</tr>
<tr>
<td>Power (W)</td>
<td>410</td>
<td>710</td>
<td>1100</td>
<td>2400</td>
</tr>
<tr>
<td>Design life (yr)</td>
<td>5</td>
<td>7.5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Cost</td>
<td>?</td>
<td>43 M$</td>
<td>30 M$</td>
<td>28 M$</td>
</tr>
<tr>
<td>Contractor</td>
<td>Rockwell</td>
<td>Rockwell</td>
<td>Lockheed Martin</td>
<td>Rockwell</td>
</tr>
</tbody>
</table>
Satellite transmissions

- GPS satellites broadcast continuously on 2 frequencies in the L-band
  - 1575.42 MHz (L1)
  - 1227.60 MHz (L2)
- GPS antennas point their transmission antenna to the center of the Earth (controlled by solar sensors).
- Main beam = 21.4/23.4 (L1/L2) half width.
- Phase center of antenna does not coincide with center of gravity of satellite.
Satellite clocks

• Frequencies broadcast by GPS satellites are derived from a fundamental frequency of 10.23 Mhz

• Fundamental frequency provided by 2 or 4 atomic clocks (Ce/Rb)
  – Clocks run on GPS time = UTC not adjusted for leap seconds
  – Clock stability over 1 day = $10^{-13}$ (Rb) to $10^{-14}$ (Ce), ~ 1 ns/day
  – Clocks synchronized between all satellites

• Relativistic effects:
  – Clocks in orbit appear to run faster (38.3 microsec/day = 11.5 km/day!) =>
    tuned at 10.22999999543 MHz before launching (g.)
  – Clocks speed is a function of orbit eccentricity (45 nsec = 14 m) =>
    corrected at the data processing stage (s.):

\[
\Delta t_R = -\frac{2}{c^2} \sqrt{a\mu} \ e \ \sin E
\]
GPS control segment

- **Monitor stations:**
  - Monitor behavior of satellite orbit and clock, health of satellites
  - Uploads data to satellites according to orders from Master station.

- **Master station:**
  - Located at Falcon Air Force Base in Colorado Springs, Colorado
  - Calculates position and clock errors for each individual satellite based on information received from the monitor stations
  - Orders appropriate ground antennas to relay information back to satellites
  - Order maneuvers when necessary
  - Ensure clock synchronization = defines GPS time

- Computes and uploads **broadcast ephemerides** into the satellites
GPS broadcast ephemeris

- Distributed to users as part of the GPS signal in the navigation message included in the signal sent by the satellites.
- Following parameters are included:
  - Keplerian elements with periodic terms added to account for solar radiation and gravity perturbations.
  - Periodic terms are added for argument of perigee, geocentric distance and inclination.
  - Reference system is WGS84.
- Navigation message:
  - Updated every 2 hours
  - Considered valid from 2 hours before Time of Ephemeris (TOE) until 2 hours after TOE
  - Decoded by all GPS receivers from GPS signal
  - Distributed in ASCII format in Receiver Independent Exchange format (RINEX): [4-char][Day of year][Session].[yy]n (e.g. brdc0120.02n)

<table>
<thead>
<tr>
<th>svprn</th>
<th>satellite PRN number</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_0 )</td>
<td>mean anomaly</td>
</tr>
<tr>
<td>( t_{oe} )</td>
<td>time of ephemeris</td>
</tr>
<tr>
<td>( sqrt(a) )</td>
<td>( \sqrt{\text{semi-major axis}} )</td>
</tr>
<tr>
<td>( \Delta n )</td>
<td>variation of mean angular velocity</td>
</tr>
<tr>
<td>( e )</td>
<td>eccentricity</td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>argument of perigee</td>
</tr>
<tr>
<td>( i_0 )</td>
<td>inclination</td>
</tr>
<tr>
<td>( \dot{i} )</td>
<td>rate of inclination</td>
</tr>
<tr>
<td>( \Omega_0 )</td>
<td>right ascension</td>
</tr>
<tr>
<td>( \Omega \dot{\theta} )</td>
<td>rate of right ascension</td>
</tr>
<tr>
<td>( C_{wc} )</td>
<td>( C_{xc} )</td>
</tr>
</tbody>
</table>
### GPS broadcast ephemeris

#### Table A1: Navigation Message File - Data Record Description

<table>
<thead>
<tr>
<th>OBS. RECORD</th>
<th>DESCRIPTION</th>
<th>FORMAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRN / EPOCH / SV CLK</td>
<td>Satellite PRN number</td>
<td>12,</td>
</tr>
<tr>
<td></td>
<td>Epoch: Toc - Time of Clock (2 digits)</td>
<td>513,</td>
</tr>
<tr>
<td></td>
<td>year</td>
<td></td>
</tr>
<tr>
<td></td>
<td>month</td>
<td></td>
</tr>
<tr>
<td></td>
<td>day</td>
<td></td>
</tr>
<tr>
<td></td>
<td>hour</td>
<td></td>
</tr>
<tr>
<td></td>
<td>minute</td>
<td></td>
</tr>
<tr>
<td></td>
<td>second</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SV clock bias (seconds)</td>
<td>F5.1,</td>
</tr>
<tr>
<td></td>
<td>SV clock drift (sec/sec)</td>
<td>3D19.12</td>
</tr>
</tbody>
</table>

#### Table A2: Broadcast Orbit - 1

<table>
<thead>
<tr>
<th>Broadcast Orbit</th>
<th>Issue of Data, Ephemeris</th>
<th>3X,4019.12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crc (meters)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>delta n (radians/sec)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>h0 (radians)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SV data rate (sec/sec2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SV clock drift rate (sec/sec)</td>
<td></td>
</tr>
</tbody>
</table>

#### Table A3: Broadcast Orbit - 2

<table>
<thead>
<tr>
<th>Broadcast Orbit</th>
<th>Cuc (radians)</th>
<th>3X,4019.12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>e Eccentricity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cus (radians)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>sqrt(A) (radians)</td>
<td></td>
</tr>
</tbody>
</table>

#### Table A4: Broadcast Orbit - 3

<table>
<thead>
<tr>
<th>Broadcast Orbit</th>
<th>Epoch Time of Ephemeris (sec of GPS week)</th>
<th>3X,4019.12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Clc (radians)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>omega (radians)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cic (radians)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>sqrt(A) (radians)</td>
<td></td>
</tr>
</tbody>
</table>

#### Table A5: Broadcast Orbit - 4

<table>
<thead>
<tr>
<th>Broadcast Orbit</th>
<th>IODE Issue of Data, Ephemeris</th>
<th>3X,4019.12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SV clock drift rate (sec/sec)</td>
<td></td>
</tr>
</tbody>
</table>

#### Table A6: Broadcast Orbit - 5

<table>
<thead>
<tr>
<th>Broadcast Orbit</th>
<th>IODE (radians/sec)</th>
<th>3X,4019.12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Codes on l2 channel</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GPS Week # (to go with TOE)</td>
<td></td>
</tr>
</tbody>
</table>

#### Table A7: Broadcast Orbit - 6

<table>
<thead>
<tr>
<th>Broadcast Orbit</th>
<th>SV accuracy (meters)</th>
<th>3X,4019.12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SV health (NED only)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TGD (seconds)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TODE Issue of Data, Clock</td>
<td></td>
</tr>
</tbody>
</table>

#### Table A8: Broadcast Orbit - 7

<table>
<thead>
<tr>
<th>Broadcast Orbit</th>
<th>Transmission time of message (sec of GPS week, derived e.g. from 1-count in Hand Over Word (HON))</th>
<th>3X,4019.12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>spare</td>
<td></td>
</tr>
<tr>
<td></td>
<td>spare</td>
<td></td>
</tr>
</tbody>
</table>

---

Example code:

```python
# Example code for reading and processing GPS broadcast ephemeris data
```

---

**Note:** The above code is a placeholder for the actual implementation of reading and processing GPS broadcast ephemeris data.
GPS broadcast ephemeris

- Accuracy of broadcast ephemeris:
  - ~ 10 m
  - Can be degraded by the DoD to 300 m
  - \( \delta s/s = \delta r/r \Rightarrow \) if \( \delta r = 200 \) m (\( r = 20,000 \) km):
    - \( s = 100 \) km \( \Rightarrow \) \( \delta r = 1 \) m, not sufficient for geophysical applications
    - \( s = 1 \) km \( \Rightarrow \) \( \delta r = 1 \) cm, sufficient for surveying

- Broadcast ephemerides are not accurate enough for most geophysical applications.

- Requirements for geophysical applications (mm – cm level):
  - Accurate orbits: e.g. 10 cm orbit error \( \Rightarrow \) 0.5 mm error on a 100 km baseline
  - Independent from the DoD
IGS: The International GNSS Service for Geodynamics

- International service of the IAG (International Association of Geodesy)
- Coordinates data archiving and processing of a global control network of >350 dual-frequency permanent GPS stations
- Test campaign in 1992, routine operations since 1994
- Provides precise GPS products, in particular satellite orbits

Map of IGS station distribution - http://igsccb.jpl.nasa.gov/
IGS orbits

- Core network of globally distributed, high-quality, continuous GPS stations
- Data processed by IGS analysis centers
- Analysis center coordinator produces weighted average = final IGS orbit
- Process takes 2 weeks (to ensure best station distribution and solution quality)
- IGS also provides satellite clock corrections
- Other products:
  - Rapid
  - Predicted

<table>
<thead>
<tr>
<th>Orbits Type</th>
<th>Accuracy/clock accuracy</th>
<th>Latency</th>
<th>Updates</th>
<th>Sample interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broadcast</td>
<td>~260 cm/~7 ns</td>
<td>Real-time</td>
<td>daily</td>
<td></td>
</tr>
<tr>
<td>Final</td>
<td>&lt; 5 cm/0.1 ns</td>
<td>14 days</td>
<td>Weekly</td>
<td>15 min</td>
</tr>
<tr>
<td>Rapid</td>
<td>5 cm/0.2 ns</td>
<td>17 hours</td>
<td>Daily</td>
<td>15 min</td>
</tr>
<tr>
<td>Predicted (ultra-rapid)</td>
<td>~10 cm/~7 ns</td>
<td>Real-time</td>
<td>Twice daily</td>
<td>15 min</td>
</tr>
</tbody>
</table>

Graph courtesy Analysis Coordinator G. Gendt, GFZ Potsdam
Orbit comparison: IGS - broadcast

Comparison igs11484.sp3 - epgga2.010