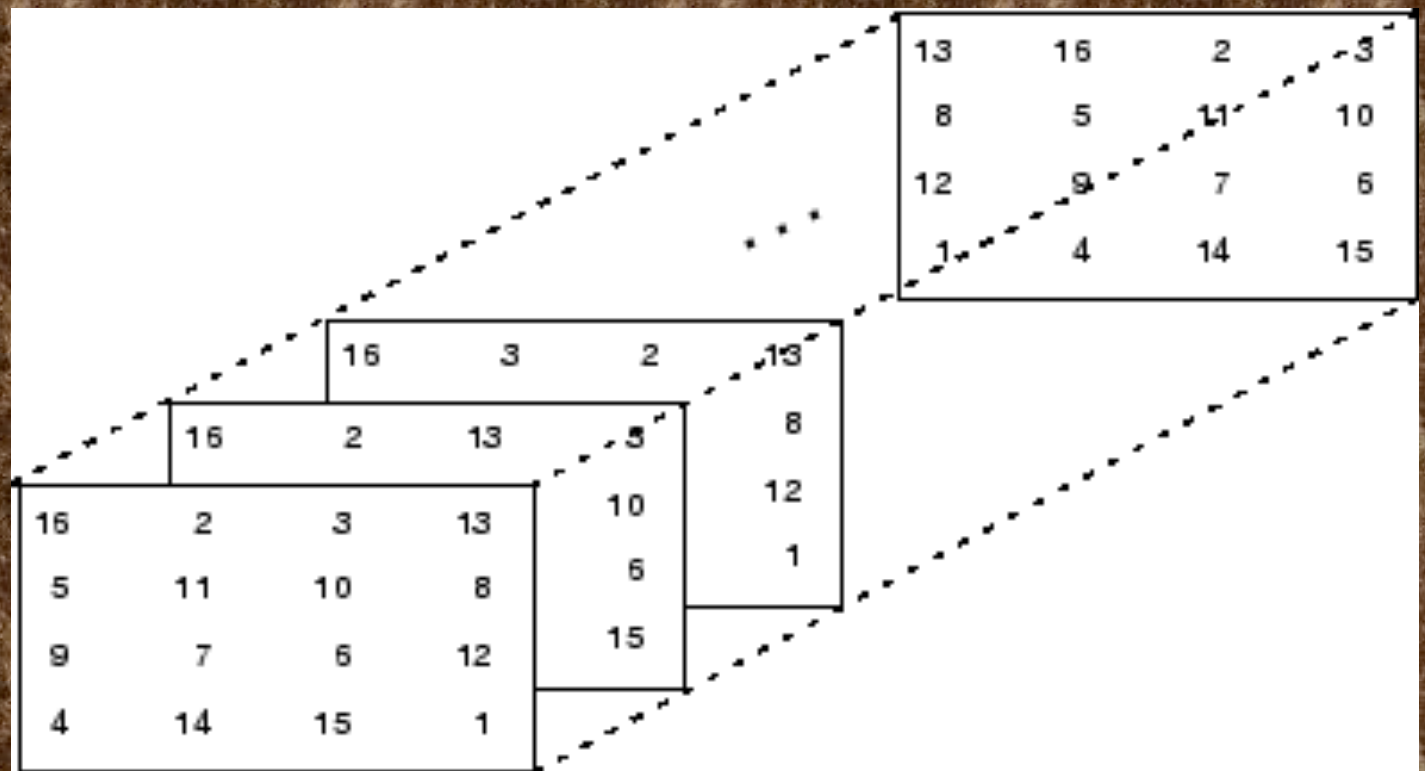


# Multidimensional Arrays

## Arrays with more than two subscripts

```
>>p = perms(1:4);  
>>A = magic(4);  
>>M = zeros(4,4,24);  
>>for k = 1:24  
M(:, :, k) = A(:, p(k, :));  
end
```



# Reshape command

Use to change the shape of matrices

```
>> x=[1 2 3 4 5 6 7 8]
```

```
x =
```

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|

```
>> x3d=reshape(x,2,2,2)
```

```
x3d(:,:,1) =
```

|   |   |
|---|---|
| 1 | 3 |
| 2 | 4 |

```
x3d(:,:,2) =
```

|   |   |
|---|---|
| 5 | 7 |
| 6 | 8 |

# Reshape command

```
>> x=[1 2;3 4;5 6; 7 8]
```

```
x =
```

|   |   |
|---|---|
| 1 | 2 |
| 3 | 4 |
| 5 | 6 |
| 7 | 8 |

```
>> x3d=reshape(x,2,2,2)
```

```
x3d(:, :, 1) =
```

|   |   |
|---|---|
| 1 | 5 |
| 3 | 7 |

```
x3d(:, :, 2) =
```

|   |   |
|---|---|
| 2 | 6 |
| 4 | 8 |

```
>> x=reshape(x,2,4)
```

```
x =
```

|   |   |   |   |
|---|---|---|---|
| 1 | 5 | 2 | 6 |
| 3 | 7 | 4 | 8 |

```
>>
```

# Building matrices by repeating parts

## repmat command

```
>> x=[1 2;3 4]
```

```
x =
```

```
1     2
3     4
```

```
>> xr=repmat(x,2,1)
```

```
xr =
```

```
1     2
3     4
1     2
3     4
```

```
>> xr=repmat(x,1,2)
```

```
xr =
```

```
1     2     1     2
3     4     3     4
```

```
>>
```



# Create constant matrix

```
>> val=pi
val =
    3.1416
>> siz=[2 2 2]
siz =
     2     2     2
>> x=repmat(val,siz)
x(:,:,1) =
    3.1416    3.1416
    3.1416    3.1416
x(:,:,2) =
    3.1416    3.1416
    3.1416    3.1416
>>
```

## Another way (seems more roundabout)

```
>> xx(prod(siz))=val
```

```
xx =
```

```
      0      0      0      0      0      0
0      3.1416
```

```
>> xx(:)=xx(end)
```

```
xx =
```

```
      3.1416      3.1416      3.1416      3.1416      3.1416      3.1416
3.1416      3.1416
```

```
>> xx=reshape(xx,siz)
```

```
xx(:, :, 1) =
```

```
      3.1416      3.1416
      3.1416      3.1416
```

```
xx(:, :, 2) =
```

```
      3.1416      3.1416
      3.1416      3.1416
```

## Another way (m, n and o have to be scalar variables)

```
>> m=2
m =
     2
>> n=2
n =
     2
>> o=2
o =
     2
>> x(m,n,o)=val
x(:, :, 1) =
    3.1416    3.1416
    3.1416    3.1416
x(:, :, 2) =
    3.1416    3.1416
    3.1416    3.1416
>>
```

Another way (val has to be a scalar variable, this syntax just populates the array with val)

```
>> x=val(ones(siz))
x(:, :, 1) =
    3.1416    3.1416
    3.1416    3.1416
x(:, :, 2) =
    3.1416    3.1416
    3.1416    3.1416
>>
```

Avoid using

```
X = val * ones(siz);
```

since it does unnecessary multiplications (versus just storing, above) and only works for classes for which the multiplication operator is defined.



## Another way

Below does not work (NaN not scalar variable, same with Inf)

```
x = NaN(ones(siz));
```

But following does work

(NaN can be scalar variable or function)

```
>> X = repmat(NaN, siz)
```

```
X(:, :, 1) =
```

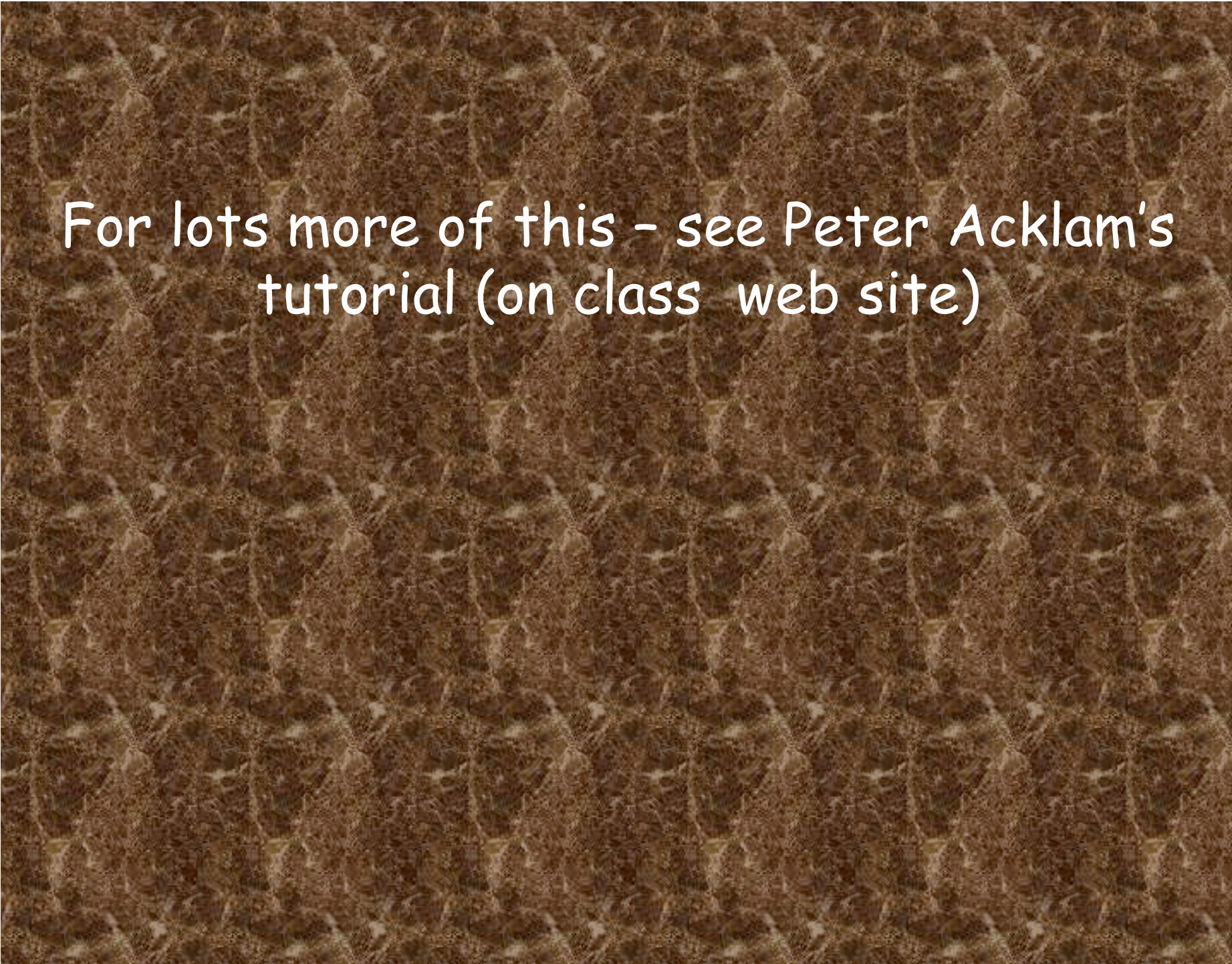
```
NaN NaN
```

```
NaN NaN
```

```
X(:, :, 2) =
```

```
NaN NaN
```

```
NaN NaN
```



For lots more of this - see Peter Acklam's  
tutorial (on class web site)

# Flipping vectors, matrices

```
>> a=[1 2;3 4]
```

```
a =
```

```
1 2
```

```
3 4
```

```
>> fliplr(a)
```

```
ans =
```

```
2 1
```

```
4 3
```

```
>> flipud(a)
```

```
ans =
```

```
3 4
```

```
1 2
```

```
>> a=[1 2 3;4 5 6]
```

```
a =
```

```
1 2 3
```

```
4 5 6
```

```
>> rot90(a)
```

```
ans =
```

```
3 6
```

```
2 5
```

```
1 4
```

```
>> a'
```

```
ans =
```

```
1 4
```

```
2 5
```

```
3 6
```

```
>> flipdim(a,1)
```

```
ans =
```

```
4 5 6
```

```
1 2 3
```



## How to represent "nothing"

Array = []  
String = ""

Useful for defining a name to be used on  
LHS.

Size and length are zero.



## Beyond simple array variables

Structures are variables that contain other variables. It is a way to organize data.

The different fields of a structure, can contain variables of different types, so if one gives the fields a meaningful name this becomes a great way to keep track of the data.

In MATLAB one can define a structure (as any other variable) as one goes.

# Structures

Like *nawk*, Matlab allows you create structures so that you may refer to elements of an array using textual *field designators*

```
S.name = 'Ed Plum';  
S.score = 83;  
S.grade = 'B+';
```

creates a scalar structure with three fields:

```
S =  
name: 'Ed Plum'  
score: 83  
grade: 'B+'
```

# Fields can be added one at a time

(a vector of the structure elements)

```
S(2).name = 'Toni Miller';  
S(2).score = 91;  
S(2).grade = 'A-';
```

## Or entire element added in single statement

```
S(3) = struct('name','Jerry Garcia',...  
'score',70,'grade','C')  
S =  
1x3 struct array with fields:  
Name  
Score  
Grade  
>>scores = [S.score]  
scores =  
83 91 70  
>>avg_score = sum(scores)/length(scores)  
avg_score =  
81.3333
```



Unfortunately structures don't behave as one might expect (hope?)

The following does not work.

```
>>avg_score = sum(S.score)/length(S.score)
```

You have to pull the vector you want to process out of the structure to use it (and make it a vector with the []).

```
>>scores = [S.score]
scores =
83 91 70
>>avg_score = sum(scores)/length(scores)
avg_score =
81.3333
```



# Example of structure and its use.

```
image.data=[1 2 3; 4 5 6; 7 8 5];  
image.date='13-Jan-2008';  
image.blank=NaN;  
image.ra=13.3212;  
image.dec=43.3455;
```

Address element of structure using  
structure name, decimal point, and element  
name.

```
image.date
```

Operate on the fields as you would with any  
variable of that particular type. Ex., to  
invert the data matrix

```
inv(image.data).
```

# Example for earthquake data

```
stn.name='mem';  
stn.lat=34.5'  
stn.lon=-89.5  
stn.elev=70;  
stn.inst='guralp cmg3'  
stn.p=15.673
```

## Pass structure by name of structure

```
some_fun(stn)
```

etc.

array of structures (and structure elements  
can be arrays).

Can be multidimensional.

```
stn(1).name='mem';  
stn(1).lat=34.5'  
stn(1).lon=-89.5  
stn(1).elev=70;  
stn(1).inst='guralp cmg3'  
stn(1).arrival(1)=15.673  
stn(1).arrival(2)=17.274  
stn(2).name='ceri';  
stn(2).lat=34.53'  
stn(2).lon=-89.57  
stn(2).elev=79;  
stn(2).inst='guralp cmg3'  
stn(2).arrival(1)=16.189  
stn(2).arrival(2)=19.923  
. . .
```



1111

$$S =$$

x: 1

S =

$$x: 1$$

n: 'ceri'

$$X =$$

2x2x2 struct array with fields:

X

n

$$X =$$

2x2x2 struct array with fields:

X

n

$$>> \mathbf{x}.\mathbf{x}$$

ans =

1

1

. . . 7 more times . . .

```
>> x.n
```

ans =

ceri

. . . 7 more times . . .

```
>> x(2,2,2)
```

ans =

 $x: 1$ 

n: 'ceri'



# Cell Arrays

multidimensional arrays whose elements are copies of other arrays.

cell arrays are created by enclosing a miscellaneous collection of things in curly braces, {}.

The curly braces are also used with subscripts to access the contents of various cells.

```
>>C = {A      sum(A)      prod(prod(A)) }  
[4x4 double] [1x4 double] [20922789888000]
```

to retrieve a cell from a cell array

C{1} -> A, the magic square

C{2} -> row vector of the sum of the columns of A

C{3} -> 16

Important distinction with respect to other programming languages -

cell arrays contain copies of other arrays,  
not pointers to those arrays.



## Cell Arrays vs Multidimensional Arrays

You can use three-dimensional arrays to store a sequence of matrices of the *same size*.



Cell arrays can be used to store a sequence of matrices of *different sizes*.



## Characters and Text

Matlab treats text like a character vector


Enter text into MATLAB using single quotes.

```
>>s = 'Hello'
```

essentially, *s* is now a 1 x 5 array with each element equal to a character: H,e,l,l,o

Characters are stored as numbers using ASCII coding with the type *char*






```
a = double(s)
a =
72    101    108    108    111
```

Because characters are stored as numbers,  
you can convert numeric vectors to their  
ASCII characters, if the character exists

```
s=char(a)
```



Printable ASCII characters go from 32 to  
127

| Char  | Dec | Oct  | Hex  | Char | Dec | Oct  | Hex  | Char | Dec | Oct  | Hex  | Char  | Dec | Oct  | Hex  |
|-------|-----|------|------|------|-----|------|------|------|-----|------|------|-------|-----|------|------|
| (nul) | 0   | 0000 | 0x00 | (sp) | 32  | 0040 | 0x20 | @    | 64  | 0100 | 0x40 | `     | 96  | 0140 | 0x60 |
| (soh) | 1   | 0001 | 0x01 | !    | 33  | 0041 | 0x21 | A    | 65  | 0101 | 0x41 | a     | 97  | 0141 | 0x61 |
| (stx) | 2   | 0002 | 0x02 | "    | 34  | 0042 | 0x22 | B    | 66  | 0102 | 0x42 | b     | 98  | 0142 | 0x62 |
| (etx) | 3   | 0003 | 0x03 | #    | 35  | 0043 | 0x23 | C    | 67  | 0103 | 0x43 | c     | 99  | 0143 | 0x63 |
| (eot) | 4   | 0004 | 0x04 | \$   | 36  | 0044 | 0x24 | D    | 68  | 0104 | 0x44 | d     | 100 | 0144 | 0x64 |
| (enq) | 5   | 0005 | 0x05 | %    | 37  | 0045 | 0x25 | E    | 69  | 0105 | 0x45 | e     | 101 | 0145 | 0x65 |
| (ack) | 6   | 0006 | 0x06 | &    | 38  | 0046 | 0x26 | F    | 70  | 0106 | 0x46 | f     | 102 | 0146 | 0x66 |
| (bel) | 7   | 0007 | 0x07 | '    | 39  | 0047 | 0x27 | G    | 71  | 0107 | 0x47 | g     | 103 | 0147 | 0x67 |
| (bs)  | 8   | 0010 | 0x08 | (    | 40  | 0050 | 0x28 | H    | 72  | 0110 | 0x48 | h     | 104 | 0150 | 0x68 |
| (ht)  | 9   | 0011 | 0x09 | )    | 41  | 0051 | 0x29 | I    | 73  | 0111 | 0x49 | i     | 105 | 0151 | 0x69 |
| (nl)  | 10  | 0012 | 0x0a | *    | 42  | 0052 | 0x2a | J    | 74  | 0112 | 0x4a | j     | 106 | 0152 | 0x6a |
| (vt)  | 11  | 0013 | 0x0b | +    | 43  | 0053 | 0x2b | K    | 75  | 0113 | 0x4b | k     | 107 | 0153 | 0x6b |
| (np)  | 12  | 0014 | 0x0c | ,    | 44  | 0054 | 0x2c | L    | 76  | 0114 | 0x4c | l     | 108 | 0154 | 0x6c |
| (cr)  | 13  | 0015 | 0x0d | -    | 45  | 0055 | 0x2d | M    | 77  | 0115 | 0x4d | m     | 109 | 0155 | 0x6d |
| (so)  | 14  | 0016 | 0x0e | .    | 46  | 0056 | 0x2e | N    | 78  | 0116 | 0x4e | n     | 110 | 0156 | 0x6e |
| (si)  | 15  | 0017 | 0x0f | /    | 47  | 0057 | 0x2f | O    | 79  | 0117 | 0x4f | o     | 111 | 0157 | 0x6f |
| (dle) | 16  | 0020 | 0x10 | 0    | 48  | 0060 | 0x30 | P    | 80  | 0120 | 0x50 | p     | 112 | 0160 | 0x70 |
| (dc1) | 17  | 0021 | 0x11 | 1    | 49  | 0061 | 0x31 | Q    | 81  | 0121 | 0x51 | q     | 113 | 0161 | 0x71 |
| (dc2) | 18  | 0022 | 0x12 | 2    | 50  | 0062 | 0x32 | R    | 82  | 0122 | 0x52 | r     | 114 | 0162 | 0x72 |
| (dc3) | 19  | 0023 | 0x13 | 3    | 51  | 0063 | 0x33 | S    | 83  | 0123 | 0x53 | s     | 115 | 0163 | 0x73 |
| (dc4) | 20  | 0024 | 0x14 | 4    | 52  | 0064 | 0x34 | T    | 84  | 0124 | 0x54 | t     | 116 | 0164 | 0x74 |
| (nak) | 21  | 0025 | 0x15 | 5    | 53  | 0065 | 0x35 | U    | 85  | 0125 | 0x55 | u     | 117 | 0165 | 0x75 |
| (syn) | 22  | 0026 | 0x16 | 6    | 54  | 0066 | 0x36 | V    | 86  | 0126 | 0x56 | v     | 118 | 0166 | 0x76 |
| (etb) | 23  | 0027 | 0x17 | 7    | 55  | 0067 | 0x37 | W    | 87  | 0127 | 0x57 | w     | 119 | 0167 | 0x77 |
| (can) | 24  | 0030 | 0x18 | 8    | 56  | 0070 | 0x38 | X    | 88  | 0130 | 0x58 | x     | 120 | 0170 | 0x78 |
| (em)  | 25  | 0031 | 0x19 | 9    | 57  | 0071 | 0x39 | Y    | 89  | 0131 | 0x59 | y     | 121 | 0171 | 0x79 |
| (sub) | 26  | 0032 | 0x1a | :    | 58  | 0072 | 0x3a | Z    | 90  | 0132 | 0x5a | z     | 122 | 0172 | 0x7a |
| (esc) | 27  | 0033 | 0x1b | ;    | 59  | 0073 | 0x3b | [    | 91  | 0133 | 0x5b | {     | 123 | 0173 | 0x7b |
| (fs)  | 28  | 0034 | 0x1c | <    | 60  | 0074 | 0x3c | \    | 92  | 0134 | 0x5c |       | 124 | 0174 | 0x7c |
| (gs)  | 29  | 0035 | 0x1d | =    | 61  | 0075 | 0x3d | ]    | 93  | 0135 | 0x5d | }     | 125 | 0175 | 0x7d |
| (rs)  | 30  | 0036 | 0x1e | >    | 62  | 0076 | 0x3e | ^    | 94  | 0136 | 0x5e | ~     | 126 | 0176 | 0x7e |
| (us)  | 31  | 0037 | 0x1f | ?    | 63  | 0077 | 0x3f | _    | 95  | 0137 | 0x5f | (del) | 127 | 0177 | 0x7f |



To manipulate a body of text with lines of different lengths, you have two choices

- a padded character array
- a cell array of strings.

When creating a character array, each row of the array must be the same length.

The `char` function pads with spaces to create equal rows

```
S = char('A','rolling','stone','gathers','momentum.')
```

produces a 5-by-9 character array:

```
S =  
A  
rolling  
stone  
gathers  
momentum.
```



You don't have to worry about this with a cell array


```
C = {'A'; 'rolling'; 'stone'; 'gathers'; 'momentum.'}
```

You can convert a padded character array to a cell array of strings with

```
C = cellstr(S)
```

and reverse the process with

```
S = char(C)
```



To create a character array from one of the text fields in a structure (name, for example), call the char function on the comma-separated list produced by S.name:

```
>>names = char(S.name)
names =
Ed Plum
Toni Miller
Jerry Garcia
```

## Checking for special elements (NaN, Inf)

`isnan(a)` Returns 1 for every NaN in array `a`.

`isinf(a)` Returns 1 for every Inf in array `a`.

`isfinite(a)` Returns 1 for every finite number (not a (Nan or Inf)) in array `a`.

`isreal(a)` Returns 1 for every non-complex number array `a`.



Using special elements to your advantage.

Since NaNs propagate through calculations (answer is NaN if there is a NaN somewhere in the calculation), it is sometimes useful to throw NaNs out of operations like taking the mean.

(A handy trick to ignore stuff you don't want while you continue calculating.)

So the function that identifies NaNs can be very useful:

```
ix=find(~isnan(a));  
m=mean(a(ix));
```

this finds all values that are not NaNs and averages them.



help

Built into matlab

help "command"

To get help on the command "command"



# Problem when you don't know the name of the command

Just type "help"

```
>> help
```

```
HELP topics:
```

|                  |  |
|------------------|--|
| Documents/MATLAB | - (No table of contents file)                  |
| matlab/general   | - General purpose commands.                    |
| matlab/ops       | - Operators and special characters.            |
| matlab/lang      | - Programming language constructs.             |
| matlab/elmat     | - Elementary matrices and matrix manipulation. |
| matlab/randfun   | - Random matrices and random streams.          |

Lists topics of help available



# Then to get contents of topics type help "topic"

```
>> help elmat
```

Elementary matrices and matrix manipulation.

Elementary matrices.

zeros - Zeros array.

ones - Ones array.

eye - Identity matrix.

repmat - Replicate and tile array.

linspace - Linearly spaced vector.

logspace - Logarithmically spaced vector.

freqspace - Frequency spacing for frequency response.

meshgrid - X and Y arrays for 3-D plots.

accumarray - Construct an array with accumulation.

: - Regularly spaced vector and index into

matrix.

Basic array information.

size - Size of array.

# Help on individual command

```
>> help zeros
```

ZEROS Zeros array.

ZEROS(N) is an N-by-N matrix of zeros.

ZEROS(M,N) or ZEROS([M,N]) is an M-by-N matrix of zeros.

ZEROS(M,N,P,...) or ZEROS([M N P ...]) is an M-by-N-by-P-by-... array of zeros.

ZEROS(SIZE(A)) is the same size as A and all zeros.

ZEROS with no arguments is the scalar 0.

ZEROS(M,N,...,CLASSNAME) or ZEROS([M,N,...],CLASSNAME) is an

M-by-N-by-... array of zeros of class CLASSNAME.

Note: The size inputs M, N, and P... should be nonnegative integers.

Negative integers are treated as 0.

Example:

```
x = zeros(2,3,'int8');
```

See also eye, ones.

Reference page in Help browser

doc zeros

Some unix commands (pwd, ls, ???) "work" in matlab (they are actually matlab commands)

```
a=pwd;  
b=ls;
```

Some matlab commands have the same names as unix commands, but are not the same

"cat" is a matlab command that concatenates matrices (not files)

Matlab does not pass things it does not understand to the OS to see if they are OS commands.



# MATLAB vectorized high level language

Requires change in programming style  
(if one already knows a non-vectorized  
programming language such as Fortran, C,  
Pascal, Basic, etc.)

Vectorized languages allow operations over  
arrays using simple syntax, essentially the  
same syntax one would use to operate over  
scalars.  
(looks like math again.)

## What is vectorization? (with respect to matlab)

Vectorization is the process of writing code for MATLAB that uses matrix operations or other fast builtin functions instead of using for loops.

The benefits of doing this are usually sizeable.

The reason for this is that MATLAB is an interpreted language. Function calls have very high overhead, and indexing operations (inherent in a loop operation) are not particularly fast.

# Loop versus vectorized version of same code.

## New commands "tic" and "toc" - time the execution of the code between them.

```
>> a=rand(1000);  
>> tic;b=a*a;toc  
Elapsed time is 0.229464 seconds.
```

```
>> tic;for k=1:1000,for l=1:1000,c(k,l)=0;for m=1:1000,  
c(k,l)=c(k,l)+a(k,m)*a(m,l);end, end, end, toc  
Elapsed time is 22.369451 seconds.
```

```
>> whos
```

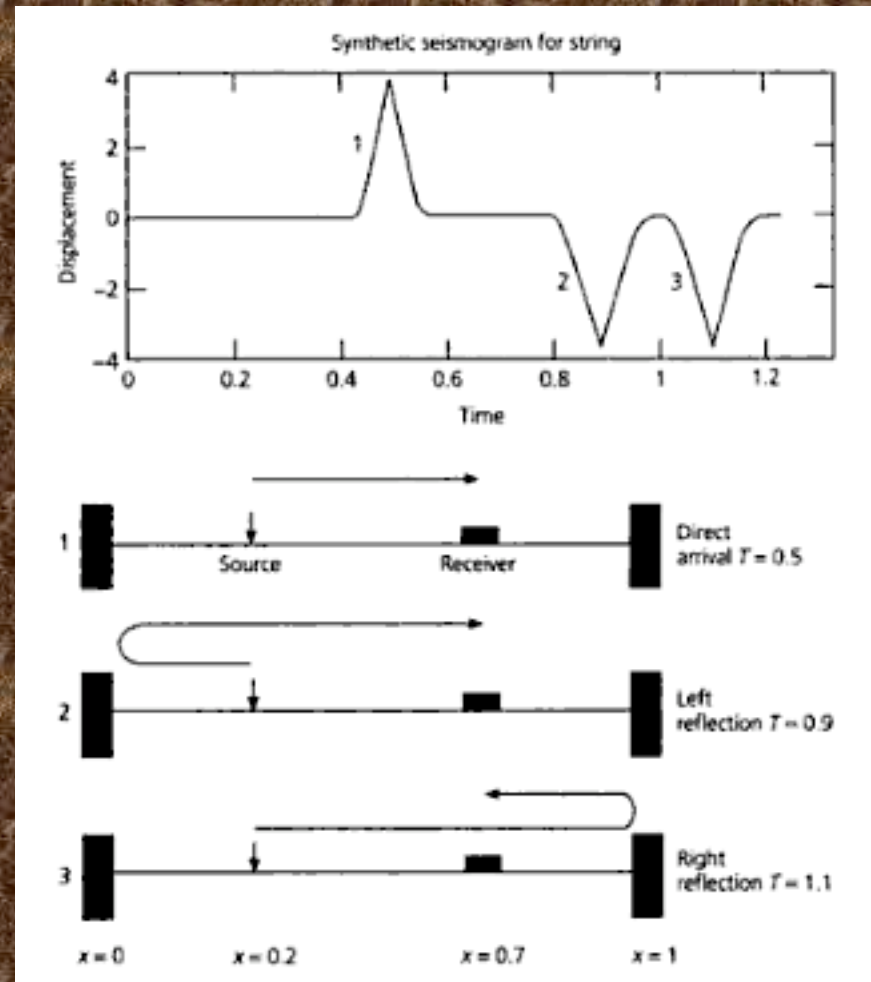
| Name | Size      | Bytes   | Class  | Attributes |
|------|-----------|---------|--------|------------|
| a    | 1000x1000 | 8000000 | double |            |
| b    | 1000x1000 | 8000000 | double |            |
| c    | 1000x1000 | 8000000 | double |            |
| k    | 1x1       | 8       | double |            |
| l    | 1x1       | 8       | double |            |
| m    | 1x1       | 8       | double |            |

```
>> max(max(b-c))  
ans =  
9.6634e-13
```

Factor 100 difference in time for multiplication of  $10^6 \times 10^6$  matrix!



# Vectorization of synthetic seismogram example from Stein and Wyession, Intro to Seismology and Earth Structure.



$$u(x,t) = \sum_{n=1}^{\infty} \sin(n\pi x / L) \sin(n\pi x_s / L) \cos(\omega_n t) \exp[-(\omega_n \tau / 4)]$$

This is just the Fourier transform for a standing wave

$$u(x,t) = \sum_{n=1}^{\infty} \sin(n\pi x_s / L) \sin(n\pi x / L) \cos(\omega_n t) \exp[-(\omega_n \tau / 4)]$$

(Note:  $\omega_n = n * \omega_0$ )

$$u(x,t) = \sum_{n=1}^{\infty} \left( \sin(n\pi x_s / L) \exp[-(\omega_n \tau / 4)] \right) \sin(n\pi x / L) \cos(\omega_n t)$$

Wt - no dependence on x or t

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x / L) \cos(\omega_n t)$$

Standing wave from 2 opposite direction traveling waves

$$u(x,t) = \sum_{n=1}^{\infty} a'_n \left[ \cos(n\pi x / L + \omega_n t) + \cos(n\pi x / L - \omega_n t) \right]$$

# Look at the basic element of Fourier series, weighted sum of sin and cos functions

(look at cos only to see how works).

$$u(t_m) = \frac{a_0}{2} + \sum_{n=1}^N a_n \cos(\omega_n t_m)$$

$$u(t_m) = \frac{a_0}{2} + (a_1 \ a_2 \ a_3 \ \cdots \ a_n) \bullet (\cos(\omega_1 t_m) \ \cos(\omega_2 t_m) \ \cos(\omega_3 t_m) \ \cdots \ \cos(\omega_n t_m))$$

$$u(t_m) = \frac{a_0}{2} + (a_1 \ a_2 \ a_3 \ \cdots \ a_n) \begin{pmatrix} \cos(\omega_1 t_m) \\ \cos(\omega_2 t_m) \\ \cos(\omega_3 t_m) \\ \vdots \\ \cos(\omega_n t_m) \end{pmatrix} = \frac{a_0}{2} + (\cos(\omega_1 t_m) \ \cos(\omega_2 t_m) \ \cos(\omega_3 t_m) \ \cdots \ \cos(\omega_n t_m)) \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix}$$

$$\vec{u}(t_m : t_{m+k}) = \frac{a_0}{2} + \begin{pmatrix} \cos(\omega_1 t_m) & \cos(\omega_2 t_m) & \cos(\omega_3 t_m) & \cdots & \cos(\omega_n t_m) \\ \cos(\omega_1 t_{m+1}) & \cos(\omega_2 t_{m+1}) & \cos(\omega_3 t_{m+1}) & \cdots & \cos(\omega_n t_{m+1}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \cos(\omega_1 t_{m+k}) & \cos(\omega_2 t_{m+k}) & \cos(\omega_3 t_{m+k}) & \cdots & \cos(\omega_n t_{m+k}) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix}$$

$$\vec{u}(t_m : t_{m+k}) = \frac{a_0}{2} + \vec{W} \vec{a}$$



# Look at the basic Fourier series

constant time, weighted sum of cosines at different frequencies at that time

$$\bar{u}(t_m : t_{m+k}) = \frac{a_0}{2} + \begin{pmatrix} \cos(\omega_1 t_m) & \cos(\omega_2 t_m) & \cos(\omega_3 t_m) & \cdots & \cos(\omega_n t_m) \\ \cos(\omega_1 t_{m+1}) & \cos(\omega_2 t_{m+1}) & \cos(\omega_3 t_{m+1}) & \cdots & \cos(\omega_n t_{m+1}) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cos(\omega_1 t_{m+k}) & \cos(\omega_2 t_{m+k}) & \cos(\omega_3 t_{m+k}) & \cdots & \cos(\omega_n t_{m+k}) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \cdots \\ a_n \end{pmatrix}$$

$$\bar{u}(t_m : t_{m+k}) = \frac{a_0}{2} + \vec{W} \vec{a}$$

constant frequency cosine as function of time  
(basis functions)

Is multiplication of a matrix (with cosines as functions of frequency - across - and time - down) times a vector with the Fourier series weights.

We have just vectorized the equations!

Even though this is a major improvement over doing this with for loops, and is clear conceptually, it is still not computable as it takes  $O(N^2)$  operations (and therefore time) to do it. This is OK for small  $N$ , but quickly gets out of hand.

Fourier analysis is typically done using the Fast Fourier transform algorithm - which is  $O(N \log N)$ .



Fourier decomposition.  
"Basis" functions are  
the sine and cosine  
functions.

Notice that first sine  
term is all zeros (so  
don't really need it) and  
last sine term (not  
shown) is same as last  
cosine term, just  
shifted one - so will only  
need one of these).

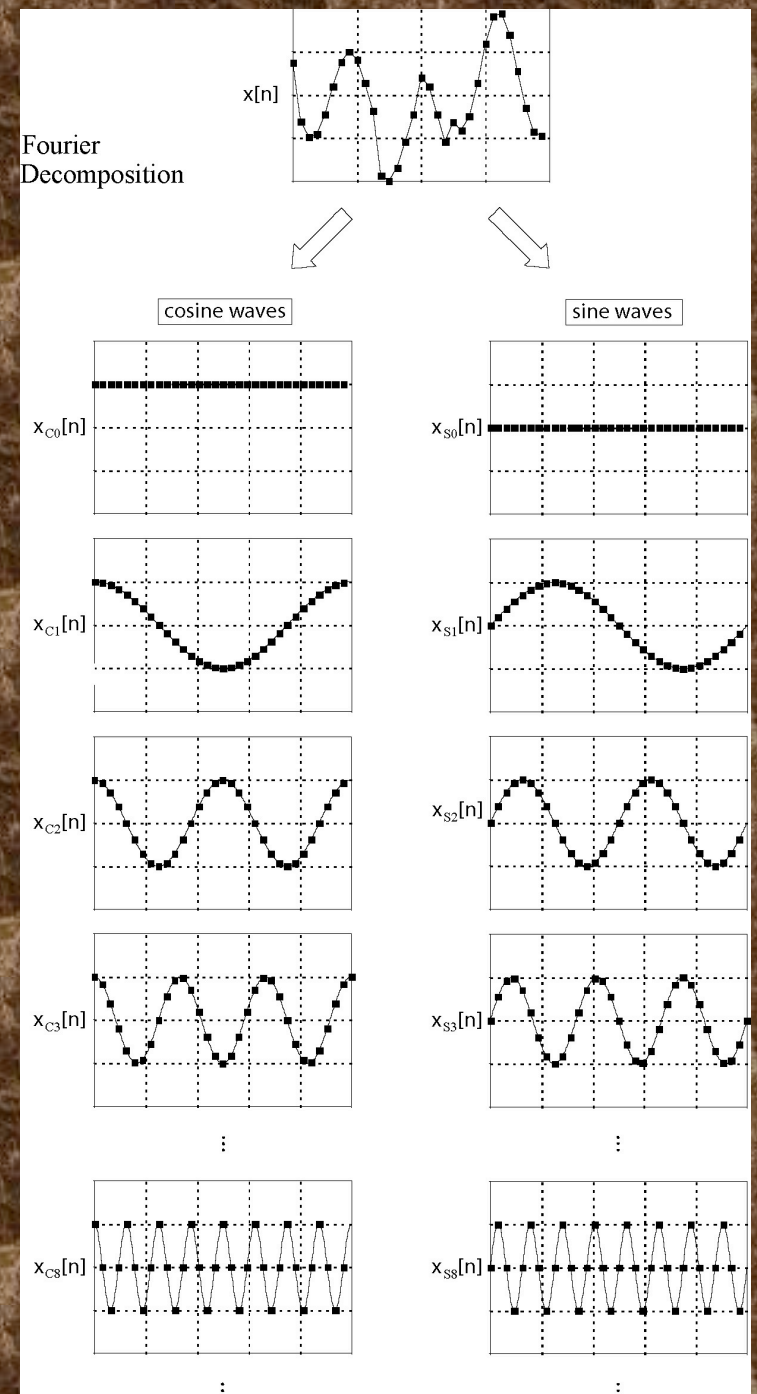


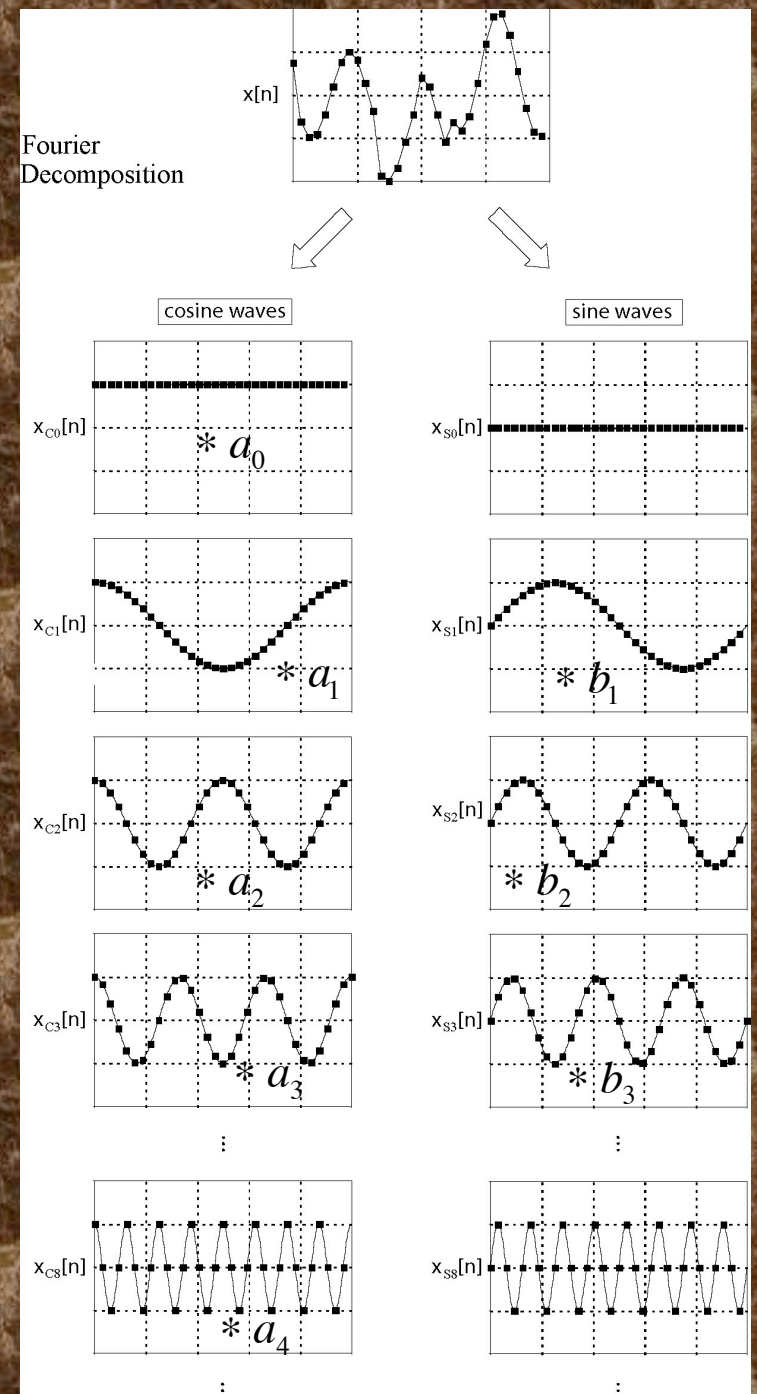
Figure from Smith

# Fourier transform

$$u(t_m) = \frac{a_0}{2} + \sum_{n=1}^N a_n \cos(\omega_n t_m) + \sum_{n=1}^N b_n \sin(\omega_n t_m)$$

The Fast Fourier Transform (FFT) depends on noticing that there is a lot of repetition in the calculations - each higher frequency basis function can be made by selecting points from the  $w_0$  function. The weight value is multiplied by the same basis function value an increasing number of times as  $w$  increases.

Figure from Smith





# FFT

$$u(t_m) = \frac{a_0}{2} + \sum_{n=1}^N a_n \cos(\omega_n t_m) + \sum_{n=1}^N b_n \sin(\omega_n t_m)$$

The FFT basically does each unique multiplication only once, stores it, and then does the bookkeeping to add them all up correctly.

The points in the trace at the top are made from vertical sums of the weighted points at the same time in the cos and sin traces in the bottom.

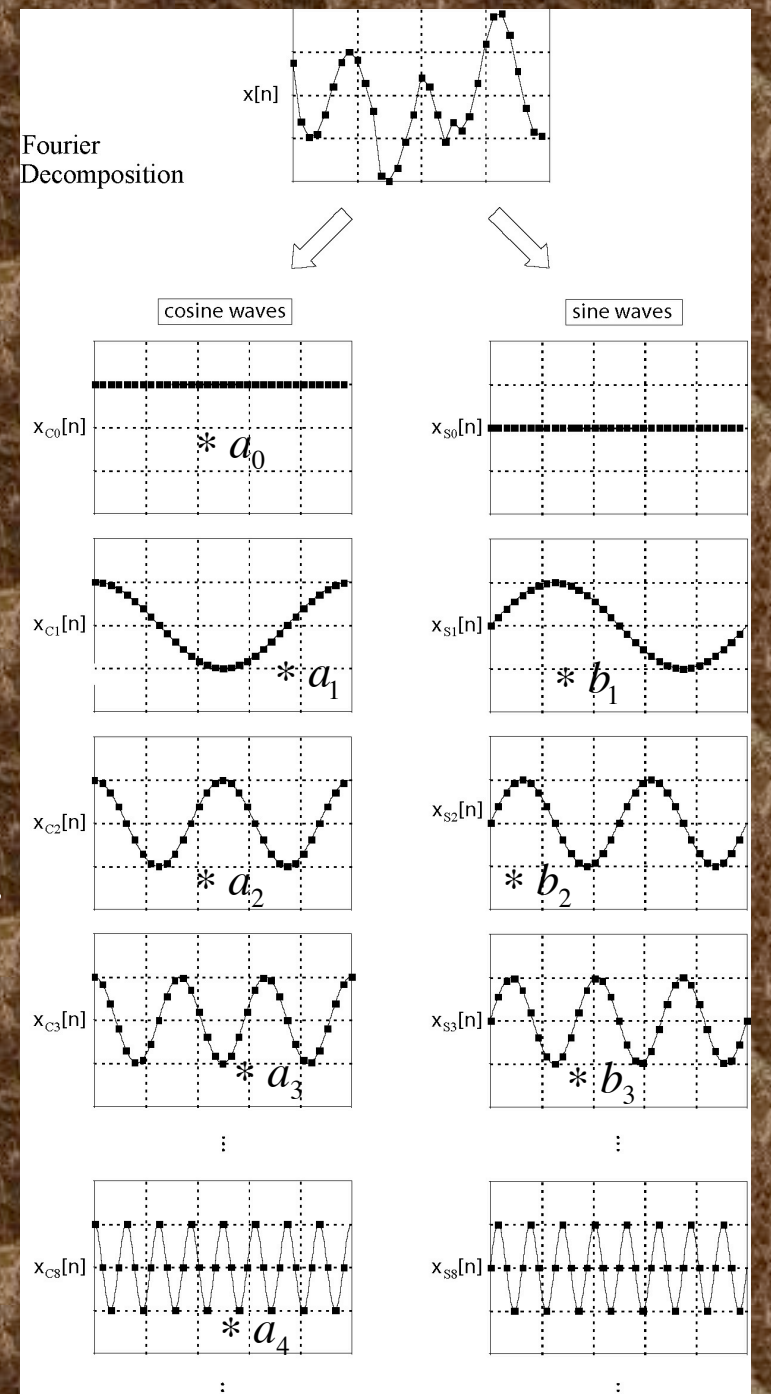


Figure from Smith



```

C SYNTHETIC SEISMOGRAM FOR HOMOGENEOUS STRING
C DISPLACEMENT U AS FUNCTION OF TIME T
C CALCULATED BY NORMAL MODE SUMMATION
  DIMENSION U(200)
  PI = 3.1415927
C
C PARAMETERS (NORMALLY WOULD COME FROM INPUT)
C STRING LENGTH (M)
  ALNGTH = 1.0
C VELOCITY (M/S)
  C = 1.0
C NUMBER OF MODES
  NMODE = 200
C SOURCE POSITION (M)
  XSRC = 0.2
C RECEIVER POSITION (M)
  XRCVR = 0.7
C SEISMOGRAM TIME DURATION (S)
  TDURAT = 1.25
C NUMBER TIME STEPS
  NTSTEP = 50
C TIME STEP (S)
  DT = TDURAT/NTSTEP
C SOURCE SHAPE TERM
  TAU = .02
C
C LIST PARAMETERS
  WRITE (6,3000)
3000 FORMAT('SYNTHETIC SEISMOGRAM FOR STRING')
  WRITE (6,3001) NMODE
3001 FORMAT('NUMBER OF MODES', I6)
  WRITE (6,3002) ALNGTH, C
3002 FORMAT('LENGTH (M)' F7.3, 'VELOCITY,
  X (M/S)', F7.3)
  WRITE (6,3003) XSRC, XRCVR
3003 FORMAT('POSITION (M): SOURCE', F7.3,
  X 'RECEIVER', F7.3)
  WRITE (6,3004) TDURAT, NTSTEP
3004 FORMAT('SEISMOGRAM DURATION (S)', F7.3,
  X I6, 'TIME STEPS')
  WRITE (6,3005) TAU
3005 FORMAT('SOURCE SHAPE TERM', F7.3)
C
C INITIALIZE DISPLACEMENT
  DO 5 I = 1, NTSTEP
    U(I) = 0.0
  5 CONTINUE
C
C OUTER LOOP OVER MODES
  DO 10 N = 1, NMODE
    ANPIAL = N*PI/ALNGTH

```

```

C SPACE TERMS: SOURCE AND RECEIVER
    SXS = SIN(ANPIAL*XSRC)
    SXR = SIN(ANPIAL*XRCVR)
C MODE FREQUENCY
    WN = N*PI*C/ALNGTH
C TIME INDEPENDENT TERMS
    DMP = (TAU*WN)**2
    SCALE = EXP(-DMP/4.)
    SPACE = SXS*SXR*SCALE
C
C INNER LOOP OVER TIME STEPS
    DO 15 J = 1, NTSTEP
      T = DT*(J - 1)
      CWT = COS(WN*T)
C COMPUTE DISPLACEMENT
      U(J) = U(J) + CWT*SPACE
15 CONTINUE
10 CONTINUE
C
C OUTPUT SEISMOGRAM FOR LATER PLOTTING
  WRITE (6, 3101) (U(J), J = 1, NTSTEP)
3101 FORMAT (7F10.4)
  STOP
  END

```

Traditional programming with nested loops.

Related to the details of the math (as if you were doing it by hand).

```

%synthetic seismogram for homogeneous string, u(t)
%calculated by normal mode summation
%string length
alngth=1;
%velocity m/sec
c=1.0;
%number modes
nmode=200;
%source position
xsrc=0.2;
%receiver position
xrcvr=0.7;
%seismogram time duration
tdurat=1.25;
%number time steps
nstep=50;
%time step
dt=tdurat/nstep;
%source shape term
tau=0.02;
fprintf('%s\n','synthetic seismogram for string')
fprintf('%s %0.5g\n','number modes', nmode)
fprintf('%s %0.5g %0.5g\n','length and velocity', alngth, c)
fprintf('%s %0.5g %0.5g\n','posn src and rcvr',xsrc,xrcvr)
fprintf('%s %0.5g %0.5g %0.5g\n','durn, time steps, del
t',tdurat,nstep,dt)
fprintf('%s %0.5g\n','source shape', tau)
%initialize displacement
for cnt=1:nstep
    u(cnt)=0;
end
for k=1:nstep
    t(k)=dt*(k-1);
end
%outer loop over modes
for n=1:nmode
    anpial=n*pi/alngth;
    %space terms - src & rcvr
    sxs=sin(anpial*xsrc);
    sxr=sin(anpial*xrcvr);
    %mode freq
    wn=n*pi*c/alngth;
    %time indep terms
    dmp=(tau*wn)^2;
    scale=exp(-dmp/4);
    space=sxs*sxr*scale;
    %inner loop over time steps
    for k=1:nstep
        %
        t=dt*(k-1);
        %
        cwt=cos(wn*t);
        cwt=cos(wn*t(k));
        %compute disp
        u(k)=u(k)+cwt*space;
    end
end
plot(t,u)

```

```

%synthetic seismogram for homogeneous
string, u(t)
%calculated by normal mode summation
%string length
alngth=1;
%velocity m/sec
c=1.0;
%number modes
nmode=200;
%source position
xsrc=0.2;
%receiver position
xrcvr=0.7;
%seismogram time duration
tdurat=1.25;
%number time steps
nstep=50;
%time step
dt=tdurat/nstep;
%source shape term
tau=0.02;
fprintf('%s\n','synthetic seismogram for
string')
fprintf('%s %0.5g\n','number modes',
nmode)
fprintf('%s %0.5g %0.5g\n','length and
velocity', alngth, c)
fprintf('%s %0.5g %0.5g\n','posn src and
rcvr',xsrc,xrcvr)
fprintf('%s %0.5g %0.5g %0.5g\n','durn,
time steps, del t',tdurat,nstep,dt)

```

```

fprintf('%s %0.5g\n','source shape',
tau)
%initialize displacement
for cnt=1:nstep
    u(cnt)=0;
end
for k=1:nstep
    t(k)=dt*(k-1);
end
%outer loop over modes
for n=1:nmode
    anpial=n*pi/alngth;
%space terms - src & rcvr
    sxs=sin(anpial*xsrc);
    sxr=sin(anpial*xrcvr);
%mode freq
    wn=n*pi*c/alngth;
%time indep terms
    dmp=(tau*wn)^2;
    scale=exp(-dmp/4);
    space=sxs*sxr*scale;
%inner loop oner time steps
    for k=1:nstep
        % t=dt*(k-1);
        % cwt=cos(wn*t);
        cwt=cos(wn*t(k));
%compute disp
        u(k)=u(k)+cwt*space;
    end
end
plot(t,u)

```

Slightly  
cleaned up  
version of  
Fortran  
program in  
Stein and  
Wyss  
"translated  
" to Matlab.

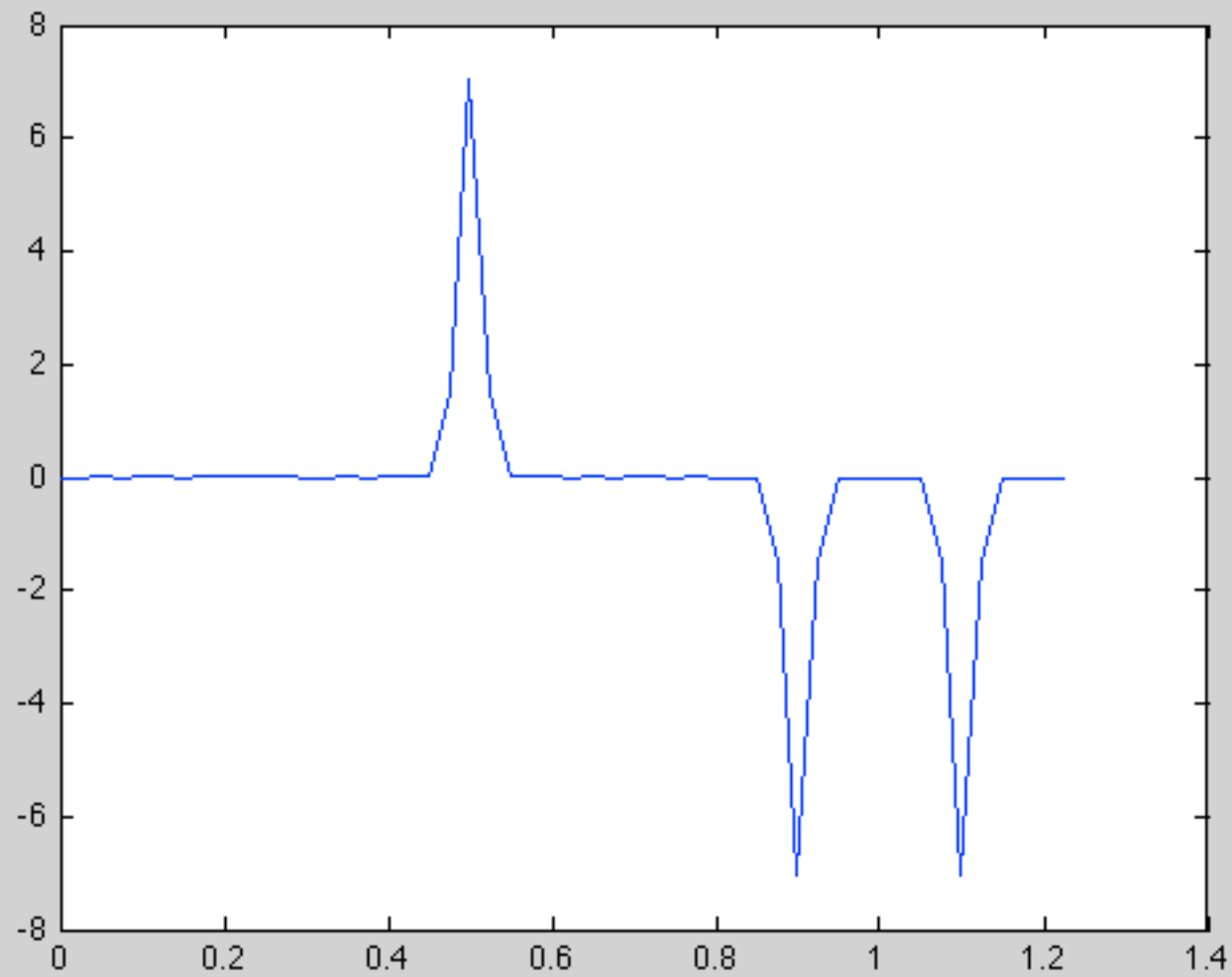


>> whos

| Name   | Size | Bytes | Class  | Attributes |
|--------|------|-------|--------|------------|
| alngth | 1x1  | 8     | double |            |
| anpial | 1x1  | 8     | double |            |
| c      | 1x1  | 8     | double |            |
| cnt    | 1x1  | 8     | double |            |
| cwt    | 1x1  | 8     | double |            |
| dmp    | 1x1  | 8     | double |            |
| dt     | 1x1  | 8     | double |            |
| k      | 1x1  | 8     | double |            |
| n      | 1x1  | 8     | double |            |
| nmode  | 1x1  | 8     | double |            |
| nstep  | 1x1  | 8     | double |            |
| scale  | 1x1  | 8     | double |            |
| space  | 1x1  | 8     | double |            |
| sxr    | 1x1  | 8     | double |            |
| sxs    | 1x1  | 8     | double |            |
| t      | 1x1  | 8     | double |            |
| tau    | 1x1  | 8     | double |            |
| tdurat | 1x1  | 8     | double |            |
| u      | 1x50 | 400   | double |            |
| wn     | 1x1  | 8     | double |            |
| xrcvr  | 1x1  | 8     | double |            |
| xsrc   | 1x1  | 8     | double |            |



Synthetic seismogram produced by Matlab  
code on previous slide.



```

% number of time samples M
% points
% source position xs (meters)
% speed c (meters/sec)
% length L (meters)
% number of modes N
% source pulse duration Tau
% (sec)
% length of seismogram T (sec)

M=50;
xs=0.25;
c=1;
L=1;
N=200;
Tau=0.02;
T=1.25;

%time vector, 1 row by M
% columns
%start, step, stop
dt=T/M;
t=0:dt:T-dt;

% receiver posn

xr = 0.7;

%stein actually starts at mode
% 1
%freq vector from 0 to n*pi*c/L
%, 1 row by N columns
wn=linspace(1,N,N);
wn=wn*pi*c/L;

%time independent terms - modes
%- 1xN vector (row vector)
timeindep=sin(wn*xr).*sin(wn*xs)
.*exp(-(wn*Tau).^2/4);

%time dependent terms -
%time*freqs = MxN matrix
timedep=cos(t'*wn);

%use matrix * vector multiply
%to do "loop"
%(MxN)times(Nx1)=(Mx1) (column
%vector)
seism=timedep*timeindep';

plot(t,seism)

```

Same problem in  
Matlab after  
vectorization (is  
mostly comments!)

Get same figure as before.

