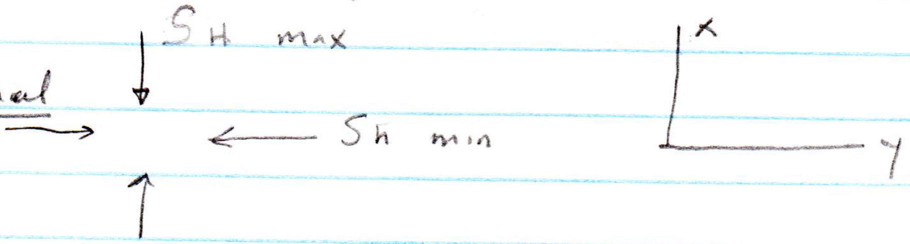
Stress Magnitudes from stress rotations

Interference of a regional stress field (1st order stress field resulting from plate boundary forces) by local stress rotations (2nd order stress fields) can be evaluated quantitatively. The local stress field is assumed to be uniaxial.

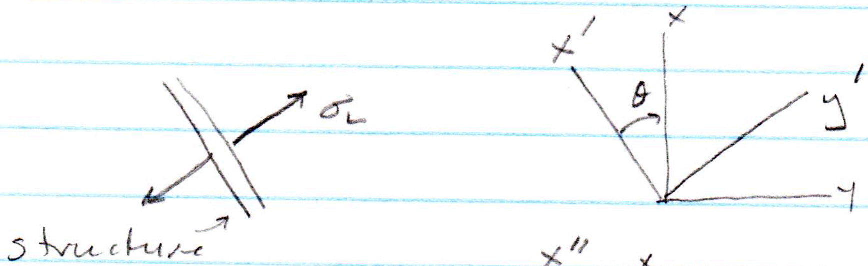
Interference depends upon the angle between the regional stress system and the local structure and the relative magnitudes of the regional and local stresses.

Reference coordinate system for the regional stress that coincides with the regional principal stress directions. So, there is no shear stress

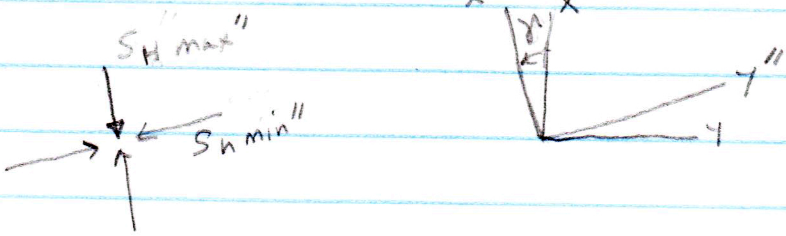
regional



local



resultant



Assume the regional stress field lies in the horizontal and vertical planes. Compression is positive; extension is negative
In the local stress field, x' is along the strike of the structure and the orthogonal uniaxial stress is along the y' axis

Magnitude of the local deviatoric stress is σ_L

for σ_L $\sigma_{x'} = 0$ $\sigma_L = \sigma_{y'}$

Can have a local vertical stress: $\sigma_{z'} = -\sigma_L$ for the case of a buoyancy force. This will not produce a reorientation of the horizontal stress but can change the relative stress magnitudes (stress regime). Say, changing from a strike-slip stress regime to normal faulting regime.

Horizontal stress rotations due to a local horizontal deviatoric compression or extension can be evaluated using simple tensor transformation in the horizontal plane (Sonder, 1990 paper)

Shear and normal stresses due to a local stress source in the reference coordinate system (x, y):

$$\tau_{xy} = -\frac{1}{2}(\sigma_{x'} - \sigma_{y'}) \sin 2\theta \qquad \text{shear}$$

$$= \frac{1}{2}\sigma_L \sin 2\theta$$

$$\sigma_x = \frac{1}{2}(\sigma_{x'} + \sigma_{y'}) + \frac{1}{2}(\sigma_{x'} - \sigma_{y'}) \cos 2\theta$$

$$= \frac{1}{2}\sigma_L (1 - \cos 2\theta)$$

$$\sigma_y = \frac{1}{2}(\sigma_{x'} + \sigma_{y'}) - \frac{1}{2}(\sigma_{x'} - \sigma_{y'}) \cos 2\theta$$

$$= \frac{1}{2}\sigma_L (1 + \cos 2\theta)$$

θ = angle between strike of the local structure and the maximum horizontal regional stress direction (angle between x' and x)

Now - superimpose the regional stresses and the local stresses in the reference regional coordinate system

$$\tau_{xy} = \frac{1}{2}\sigma_L \sin 2\theta$$

$$\sigma_x = S_{Hmax} + \frac{1}{2}\sigma_L (1 - 2\theta)$$

$$\sigma_y = S_{hmin} + \frac{1}{2}\sigma_L (1 + 2\theta)$$

new orientation of the resultant principal stress tensor is given by x'', y'', z

The orientation of this resultant coord. system in the reference, regional stress field is

$$\tan 2\gamma = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Now we can compute the amount of rotation of the regional stress field in the horizontal plane by substitution:

(1) $$\gamma = \frac{1}{2}\tan^{-1}\frac{\sin 2\theta}{(S_{H\,max} - S_{h\,min})/\sigma_L - 2\cos\theta}$$

for $S_{H\,max}$

$S_{h\,min}$ orientation is $\gamma + \frac{\pi}{2}$

here $\frac{(S_{H\,max} - S_{h\,min})}{\sigma_L} = r$

γ is the angle between x and x'' (regional coord system and resultant coord system)

Look at Figure 6 next page

plot amount of horizontal stress rotation γ as a function of the orientation of the structure θ for various ratios of the regional horizontal stress difference to the local stress $\frac{(S_{H\,max} - S_{h\,min})}{\sigma_L}$

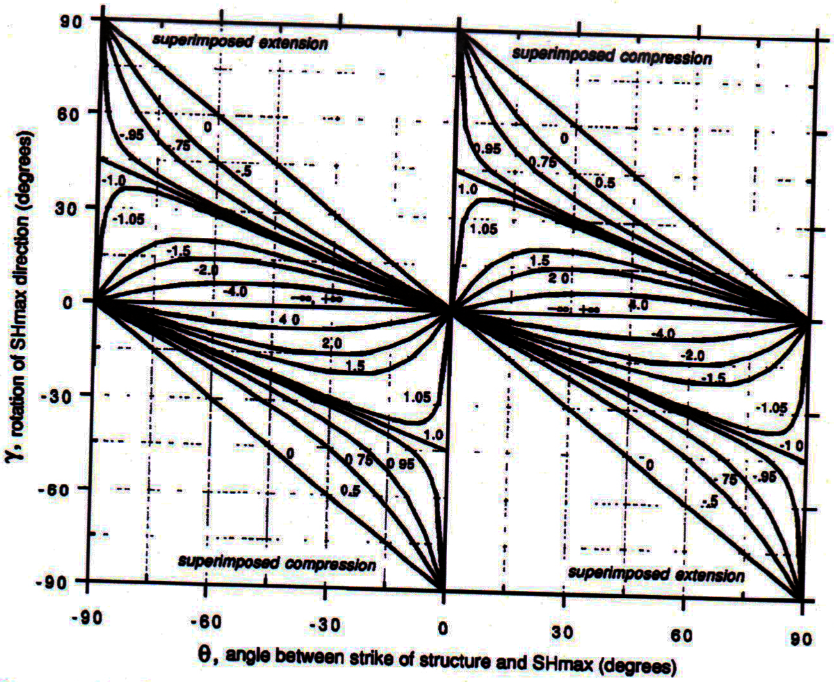


Fig. 6. Stress rotation of regional horizontal stresses (γ) as a function of θ, the angle between the strike of the local feature producing the horizontal uniaxial compression or extension and the regional S_{Hmax} direction, computed from equation (8). Numbers on curves refer to values of the ratio of regional horizontal stress differences to magnitude of local uniaxial stress, $(S_{Hmax} - S_{hmin})/\sigma_L$; positive values indicate superimposed uniaxial compression and negative values indicate superimposed uniaxial extension.

from Zoback 1992

Sense of rotation depends upon the orientation of the structure and whether the local stress is compressive $(S_{H\,max} - S_{h\,min})/\sigma_L > 0$ $r > 0$ or extensional $(S_{H\,max} - S_{h\,min})/\sigma_L < 0$ $r < 0$

So, for a very small local stress field $r \to \infty$ and there is no rotation
If the local stress field dominates $r \to 0$ and the regional field rotates into the alignment with the local stress field

for $r \cong 2$ the maximum possible rotation is only about 15° this is roughly the detection threshold using regional stress orientations in the world stress map data base

Thus, to be resolved, the local uniaxial stress must be > ½ the regional horizontal stress difference

The amount of rotation is inversely proportional to the difference of the two horizontal stress magnitudes
Thus the larger the stress difference, the smaller the rotation
For a strike slip faulting regime horizontal stress difference $S_{Hmax} - S_{hmin} = S_1 - S_3$

For a regional thrust or normal faulting regime
$$S_{Hmax} - S_{hmin} = \underset{\text{(thrust)}}{S_1 - S_2} \quad \text{or} \quad \underset{\text{(normal)}}{S_2 - S_3}$$

So would expect more possible rotation in a thrust or normal stress regime than in a strike-slip regime.

Mazzotti + Townend

maximum horizontal compressive stress

S_{HB} from boreholes
S_{HS} from focal mechanisms

odd Charlevoix, Lower St. Lawrence
Central Va

$S_{HB} - S_{HS}$ 30° to 50° amount of rotation

Assume local stress is $\perp$ to the trend of the Iapetus Rift (NE-SW)

Solve for σ_L by solving for r.

$$r = \frac{\sin 2\theta}{\tan 2\gamma} + \cos 2\theta$$ (from 1 page 4)

example $\theta = 15°$ $\gamma = 44°$ lower St. Lawrence

$r = .88$ or ~ 88% of the stress difference

What is S_{HB}? regional stress

at 8 km coef of friction $\mu = 0.6$ to 1.0

hydrostatic pore pressure $\lambda = 0.4$

here $R = (S_1 - S_2)/(S_2 - S_3)$

see that $R = 0.6$ peak

Let $\mu = .8$

$\lambda = .4$

$R = .6$

$\rho = 2700\ kg/m^3$

then can determine the differential stress in the horizontal plane at 8 km (see Appendix A Mazzotti + Townend)

~ 160 MPa reverse faulting

250 MPa strike slip

so need a large S_H to produce rotation

can reduce using low coeff of friction $\mu = .1$

or lithostatic pore pressure $\lambda = .9$

then down to 20 to 40 MPa