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Geodetic measurement of the local elastic response to the changing mass of water in Lago Laja, Chile

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Abstract

A geodetic station (ANTC) built in bedrock in southern Chile is undergoing non-steady vertical motion within a range of nearly 50 mm. These fluctuations are dominated by the earth's local elastic response to the changing weight of water in a reservoir located about 20 km away. There is also an annual periodic component of motion that is attributed to global and regional patterns of loading, as well as a steady tectonic signal. The local loading response constrains the average elastic structure of the area, and implies a stiffness that falls near the lower end of the range observed in rock samples.

Keywords: Elasticity; Loading; Deformation; Lake; Water; Geodetic

1. Introduction

Geodetically observed oscillations of the solid earth surface largely manifest earth's elastic response to annual and interannual changes in atmospheric and seafloor pressure, and, often more importantly, in the loads associated with snow, ice, surface and subsurface water (Mangiarotti et al., 2001; Blewitt et al., 2001; Dong et al., 2002; Heki, 2001). The rapid growth of geodetic reference networks of Global Positioning System (GPS) receivers raises the possibility of measuring this elastic response with sufficient spatial and temporal resolution and coverage to provide important new insights into local and regional mass fluxes associated with the hydrological cycle. But if we are to use the solid earth as an elastic balance with which we can weigh mass changes associated with seasonality and other short-term climate cycles then we must be able to calibrate the elastic response of a given area so as to provide this balance with a locally meaningful scale. We present here a case study from Chile that suggests that this calibration can be achieved by using continuous GPS stations to monitor crustal motion near lakes, such as hydroelectric reservoirs, that are characterized by large and carefully measured fluctuations in water volume.

Elastic loading of the solid earth occurs at global (Blewitt et al., 2001), regional (Heki, 2001) and local scales (this study). Lord Kelvin and George Darwin's studies of the solid earth tides in the nineteenth century led the former to conclude that "the earth has, on the whole, a rigidity greater than that of a solid globe of glass of the same dimensions; and perhaps greater than that of a globe of steel". Seismologists later established the existence of the liquid outer core,

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but confirmed that the overlying mantle and crust do behave as a stiff elastic solid, at least in the seismic frequency band. Space geodetic techniques have been used to infer the anelasticity (Q) of the earth's mantle at the main tidal frequencies, at the 14-month period of the Chandler wobble, and for the 18.6-year body tide (Smith and Dahlen, 1981; Ray et al., 2001; Wahr et al., 2003). While the frequency dependence of Q is still not completely understood, these studies indicate that at periods of $\sim$1 year mantle Q is sufficiently high (>100) that it is quite reasonable to ignore anelastic effects when modeling the deformation of the solid earth caused by surface loads of global or regional extent. Heki (2001), for instance, modeled snow and ice loading in Japan using a perfectly elastic model. Unlike the solid earth's response to tidal forcing, which is mainly sensitive to the rigidity of the lower mantle, the deformation produced by surface loads in an area the size of Japan is sensitive to the rheology of the lithosphere and upper mantle, including the asthenosphere.

We present a case study in which the surface load has dimensions of $<$50 km. We follow Heki (2001) in adopting a perfectly elastic model for the solid earth's response to this time variable load. Because our study area is located in a tectonically active region, and in the immediate vicinity of an active volcano, it might be objected that it is conceivable that the elastic response to surface loading may be modified by the existence of a mid or lower crustal magma chamber, or by a viscous component of deformation associated with creep of an unusually hot lower crust or a shallow asthenosphere characterized by unusually high degrees of partial melting. Nevertheless we begin our analysis by assuming a perfectly elastic response (i.e. that other modes of deformation can be neglected in a first approximation) and attempt to justify this assumption only a posteriori.

When considering loads of limited spatial extent it is often reasonable to ignore the curvature and topography of the earth, and to treat it as an elastic half-space (Love, 1929; Farrell, 1972). The relationship between the load and the associated deflection of the earth's surface is sensitive to the elastic properties of the subsurface in the general vicinity of the load. For a uniform half-space, the response to surface loading depends only on the geometry of the load and a pair of elastic constants (e.g. Young's modulus, Y, and Poisson's ratio, ν or, equivalently, the Lame parameters, λ and μ) that characterize the behavior of the linear, isotropic, elastic material composing the half-space (Turcotte and Schubert, 1982). It is also possible to invoke a vertically stratified earth model composed of one or more layers overlying a uniform half-space (Farrell, 1972).

Seismologists have developed standard models for the elastic structure of the earth, but these models were achieved by massive lateral averaging, often to the point of addressing only the radial variations in earth structure (Dziewonski and Anderson, 1981), or characterizing lateral variability only in terms of continental versus oceanic settings. In addition, the vertical resolution of most whole earth mechanical models is, not unreasonably, rather limited. Elastic loading phenomena can be sensitive to the details of near surface structure (Marthelot et al., 1980). Laboratory measurements of the elastic moduli in actual rock samples exhibit surprising levels of variability (often much more than an order of magnitude), even for rocks of the same general type (Birch, 1966; Kulhawy, 1975). Clearly it could be misleading to assume standard or conventional values for the elastic parameters in order to model surface loading processes in a specific location. Indeed, this danger is underlined by the results of this study.

2. Geodetic observations of surface loading in Antuco, Chile

Our interest in the vertical deflections attending hydrological loading of the earth's surface began several years ago when we recognized that one of the continuous GPS stations we had established in the Southern Andes, primarily for the purpose of studying active tectonic processes, was undergoing unusually large vertical fluctuations which were strongly anti-correlated with the water level in a nearby reservoir (Fig. 1). This station (ANTC) is located near the village of Antuco and about 20 km from Lago Laja, one of the largest reservoirs in Chile (Fig. 2a). This lake (which is also known as Laguna de la Laja) came into existence after an eruption of Volcán Antuco constructed a natural dam across an adjacent river valley. Because Lago Laja is used to generate hydroelectricity its water level is continuously monitored, and

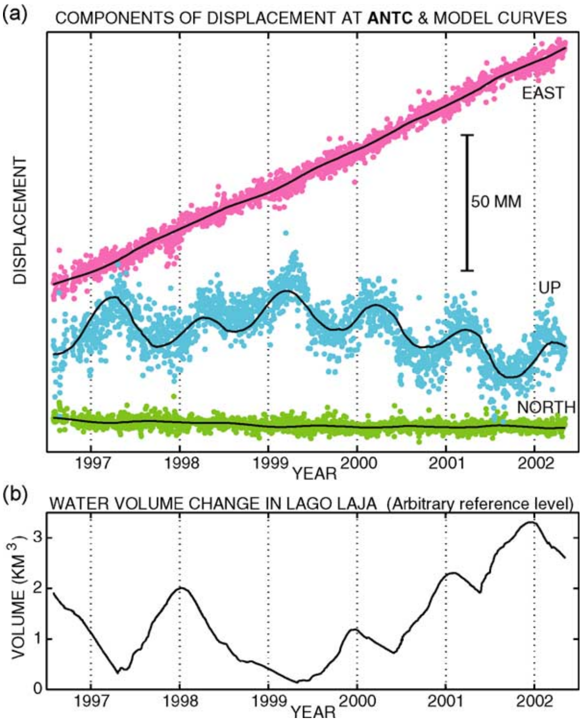

Fig. 1. Crustal motion at continuous GPS station ANTC near Antuco, Chile, and the water volume history for Lago Laja, a reservoir located about 20 km from ANTC (see Fig. 2). (a) The daily site position time series for ANTC manifests the motion of this station relative to the stable core of the South American plate. The black curves represent a simple model for each component of displacement (Table 1). (b) The volume of water in Lago Laja relative to an arbitrary reference surface.

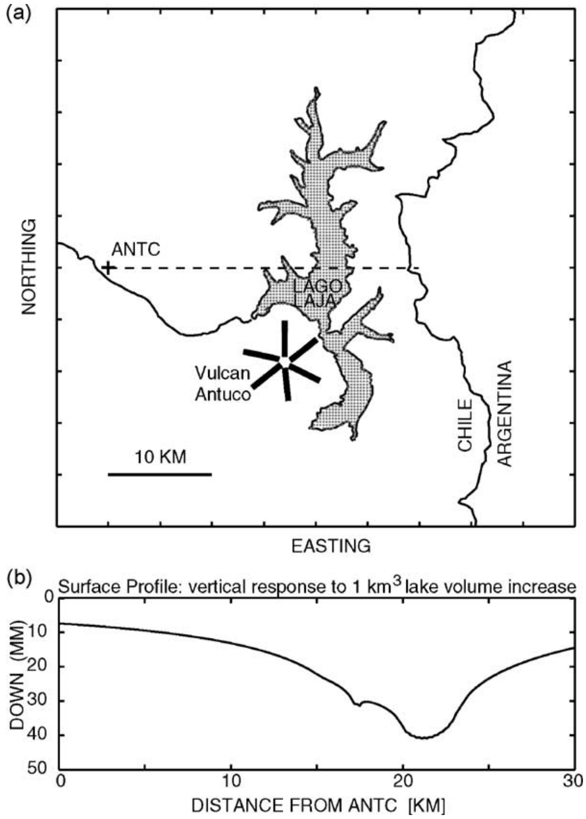

Fig. 2. A map showing the geometry of the loading problem and the elastic model. (a) The GPS station ANTC, the active Volcán Antuco and Lago Laja, all located in Southern Chile close to the Argentine border. The dots in the lake represent the centers of a suite of circles, organized in a regular grid, used to model the load represented by a lake water volume change of 1 km^3. Each of these circular loads has a radius equal to one half of the grid spacing. (b) The vertical motion of the surface produced in response to this incremental load, for the profile indicated by the dashed line in (a).

the relationship between water level and water volume has been established through a careful topographic analysis.

The geodetic time series at ANTC (Fig. 1) is expressed in a reference frame attached to the stable core of the South American plate (Kendrick et al., 2001). This time series is composed of daily geodetic solutions in which (to the best of our abilities) the influence of all astronomical tides, including the pole tide (Dong et al., 2002), has been removed. The motion of ANTC has a steady secular component associated with interseismic deformation of the leading edge of the South American plate in response to locking of the main plate boundary (Bevis et al., 2001). Superimposed on this steady velocity is a fluctuation most strongly developed in the vertical component of motion (Fig. 1a). This vertical fluctuation is clearly anti-correlated in time with the change in the volume of water in Lago Laja (Fig. 1b), and therefore with the changing weight of the lake.

In order to quantify the influence of the lake, we assume that the east (E), north (N) and upward (U) components of displacement at ANTC can each be expressed as the superposition of three modes of deformation: (i) a steady, linear displacement with time constituting the interseismic (tectonic) velocity, (ii) a time-varying displacement which is strictly

Table 1
Best fit model for the east, north and upward components of displacement at station ANTC, for the simplest class of model in which the periodic displacement component at ANTC (nominally associated with global and regional loading) consists of a single harmonic with a periodicity of 1 year

	East	North	Upward
Lake loading (mm/km^3)	1.02 ± 0.08	0.59 ± 0.05	−7.37 ± 0.22
Tectonic velocity (mm per year)	15.08 ± 0.04	−0.66 ± 0.03	0.03 ± 0.12
Periodic component			
Period = 1 year			
Amplitude SIN (mm)	−0.13 ± 0.08	−0.16 ± 0.05	5.47 ± 0.21
Amplitude COS (mm)	−0.56 ± 0.08	−0.39 ± 0.05	2.69 ± 0.22
Total amplitude (mm)	0.57	0.42	6.09
Phase (°)	−167.3	−158.0	63.8
RMS misfit (mm)	2.37	1.57	6.41

This harmonic is constructed from a sine and a cosine term, so as to allow arbitrary phase. All uncertainties represent 1-sigma standard errors.

proportional to the volume, V, of water in the lake, and (iii) a time-varying displacement representing the response at ANTC to global and regional patterns of environmental loading (as well as any local loading other than that produced by the lake). Because there have not, until very recently, been any other continuous GPS stations located in this part of Chile, we cannot estimate this third mode of deformation using direct observations, and so, in order to perform a preliminary analysis, we resort to the assumption that it can be approximated using an empirical periodic function. Our initial analysis assumes a particularly simple form for this function: a pure sinusoid with a period of 1 year. The parameters or coefficients of this composite model were obtained by a least squares analysis, and the best fitting curves are shown in Fig. 1a, and described in Table 1. Only the E and U components of displacement at ANTC contain large signals. The E component is dominated by a large tectonic signal (∼15 mm per year), whereas the U component is dominated by the combination of a lake loading signal (7.4 mm/km^3, producing a total variation of ∼23 mm of displacement) and a smaller periodic component of motion (∼12 mm peak-to-peak amplitude). This last component is largely responsible for the apparent phase shift between the U and the lake level time series (Fig. 1).

We performed a suite of similar analyses in which we assumed that the periodic term could be expressed as a linear combination of sinusoids of differing frequency (and with adjustable amplitude and phase) in order to explore annually periodic functions of varying shape. These models produced very little improvement in the fit of the model to the GPS observations, and as a result we prefer the simpler model (Table 1) already described.

If we examine the loading coefficients associated with the E, N and U components of motion, we see that they all appear to be statistically significant in their difference from zero (Table 1). Nevertheless, because the loading coefficients for the E and N components are small in terms of their absolute size, we are concerned that systematic errors associated with the GPS analysis (which can produce slowly-changing biases of magnitude ∼1 mm in the horizontal, and ∼3 mm in the vertical) could make this statistical interpretation misleading. The RMS misfits associated with our models for the E, N and U components (Fig. 1) are 2.37, 1.57 and 6.41 mm, respectively. If we eliminate the lake loading component of the model, and keep only the linear and periodic components, then the RMS misfits are increased to 2.46, 1.63, and 8.06 mm, respectively (corresponding to changes of about 4% in the horizontal, and about 26% in the vertical). This indicates that the lake loading term makes very little difference to our ability to model the horizontal components of motion at ANTC. Furthermore, even though the ratio of horizontal to vertical displacement magnitude at ANTC (due to lake loading) depends on the unknown value of Poisson's ratio, for any value of the elastic constants the east component of displacement should be very much larger than the north component of displacement, since the lake is located well to the east of ANTC and has roughly equal areas north and south of ANTC. Since the E and N loading coefficients (Table 1) are similar in magnitude, we conclude that one or both of these estimates are corrupted by systematic error and are not credible. However, we are confident that we are clearly resolving a loading signal in the vertical time series. The fit of the model curve to the U time series in Fig. 1 is impressive given that this model enjoys only five degrees of freedom. For the rest of this study, we restrict our attention to the vertical loading signal observed at ANTC.

3. A uniform elastic half-space model

We have found that for each cubic kilometer of water added to Lago Laja the ground at ANTC is displaced downwards by about 7.4 mm. We model this effect using a uniform elastic half-space model based on the analytical solution for the surface displacement field induced by a circular load (Love, 1929; Farrell, 1972). We approximate the incremental lake load using a suite of 1280 circular loads (Fig. 2a), each applying the same uniform pressure to the surface of the half-space, and exerting a net force equal to that associated with 1 km^3 of water in the lake. We compute the surface displacement associated with each circular loading element, and find the total vertical displacement field by linear superposition. The vertical displacement field depends solely on the geometry of the lake, and on a single constant $C = (1 - \nu^2)/Y$, which depends only on Poisson's ratio, ν, and Young's modulus, Y, for the half-space. Any combination of Y and ν that leaves C unchanged will produce an identical vertical surface displacement field in response to a given surface load. In order to match the observed rate of displacement at ANTC, we require $C = (5.20 \pm 0.16) \times 10^{-11}\ \mathrm{Pa}^{-1}$. We illustrate the resulting displacement field along a profile starting at ANTC and extending 30 km to the east (Fig. 2b). Our simple elastic model suggests that close to the middle of the lake the ground depresses about 40 mm for each cubic kilometer of water added to the lake (implying total vertical range of more than 12 cm between late 1996 and early 2002).

As noted above, we cannot resolve the separate values of Y and ν from the vertical loading signal measured at ANTC, but we can infer a curve that describes

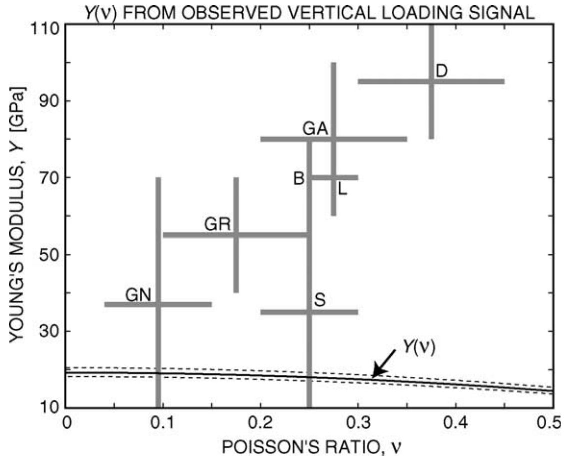

Fig. 3. The solid black curve, $Y(\nu)$, shows the possible combinations of Young's modulus, Y, and Poisson's ratio, ν, that are consistent with our elastic loading model and the relationship between water volume changes in Lago Laja and the vertical displacement measured at ANTC. The thin dashed curves represent the nominal 95% confidence interval associated with $Y(\nu)$. The vertical and horizontal grey lines indicate the typical ranges in Y and ν associated with various rock types, according to Turcotte and Schubert (1982). The abbreviations are: S: sandstone, L: limestone, GN: gneiss, B: basalt, GR: granite, D: diabase, GA: gabbro.

all pairs of values for Y and ν that are consistent with our observations. This curve is shown in Fig. 3, along with the characteristic ranges of Y and ν for various rock types, as determined by laboratory measurements (Turcotte and Schubert, 1982). We see that no matter what value we assume for ν, the effective value for Y in the vicinity of Lago Laja appears to fall near the bottom of the range established by laboratory measurements.

4. Interpreting low apparent stiffness

In this section, we seek to explain the low elastic stiffness inferred for the earth's crust beneath Lago Laja and Antuco—does this result indicate possible shortcomings in our measurement technique or in our mathematical model? We first consider the possibility that our determination of the loading coefficient (7.4 mm/km^3) is somehow biased, and that this has led to a biased constraint on the crustal stiffness of the area. The water entering the lake is largely derived from the melting of ice and snow, and ignoring the reduction of this hydrologic load in the watershed of the lake, while inferring the response associated with filling of the lake, will cause us to underestimate the magnitude of the lake loading coefficient at ANTC. However, this bias would cause us to *overestimate* the stiffness of the crust (i.e. Y), whereas we seek a systematic error producing the opposite effect. Furthermore, because we are already modelling a large part of the local snow and ice load signal in the harmonic component of our model (Table 1), and because the remaining anharmonic component of local snow and ice loading would have a very different form (structure) than the lake volume curve over a nearly 6-year period, the magnitude of this bias is probably very small. Some simple numerical experiments suggest that it is not likely to exceed a few percent.

A more likely explanation for the low elastic stiffness implied by our numerical model is that we invoked a *uniform* elastic half-space, and, even at the scale of the lake, the elastic structure of the earth is more complicated than this. It is likely that the actual value of Y varies in space—especially with depth, and that our estimate for Y really represents a spatially-weighted average value. Because poorly consolidated sediments in the shallow subsurface tend to be less stiff than competent rock at greater depth, these materials would reduce the average value of Y below that normally associated with the elastic lithosphere and with most of the competent rock samples used in laboratory determinations of elastic moduli. This interpretation is supported by studies of local ocean loading signals recorded by tiltmeters installed near coastlines. These tidal signals have amplitudes as large as several microradians very close to the water's edge but decay rapidly inland. This localization of the land surface's tilt response to nearby changes in seafloor pressure has led several workers to infer a near surface layer of very low rigidity (e.g. Takahashi, 1929; Marthelot et al., 1980).

There are a few additional complications associated with the special setting of Lago Laja. It is possible that the crust in this area is more compliant (i.e. less stiff) than in most continental regions due to the thermal anomalies associated with active volcanism and because of the presence of numerous fractures and faults.

5. Searching for a phase lag associated with anelasticity

We now return to the issue of anelasticity mentioned in the introduction, and seek to justify our adoption of a purely elastic model on grounds somewhat broader than Occam's razor and the minimal nature of our observational geometry (only one clearly resolved component of motion at a single point). Although it is not possible to characterize the influence of a volcanic magma chamber or more diffuse viscoelastic behavior of the subsurface without knowing, among other things, the geometry associated with such rheology, any significant level of energy dissipation in the system must manifest itself as a phase or time lag between the temporal variation in the surface load (i.e. the driving force) and the associated crustal deformation or response (cf. Ray et al., 2001). Accordingly we generalize our numerical model (more specifically, that part associated with the first coefficient in Table 1) so that the crustal displacement at ANTC at time t is proportional not to the volume of lake water at time t, but to the volume at time $(t - \tau)$. That is, the time development of crustal displacement is delayed relative to that of the load by a time lag, τ. We restrict τ to positive values to respect the principal of causality, i.e. to avoid the physical absurdity of framing a model in which effect precedes cause. We search over a range of values for the time lag, and for each value of τ we repeat our least squares analysis to obtain the best fit between our generalized model and the vertical motion time series at ANTC. We plot the RMS misfit between model and data as a function of τ in Fig. 4. We see that the RMS misfit monotonically decreases as τ is reduced to zero. We find no evidence for a time lag in the system.

This finding is hardly a definitive basis for rejecting the possibility of local anelastic effects. Our model is, after all, a relatively crude one, particularly in the way that it handles the composite global and regional loading signal, which is not likely, in reality, to be strictly periodic. Another objection is that there may be a *spectrum* of time lags associated with different driving frequencies, whereas we have assumed a single value for τ in each of the scenarios used to produce Fig. 4. This objection is less weighty because the lake volume time series is relatively narrow band and is clearly dominated by a 1 year periodicity, and thus

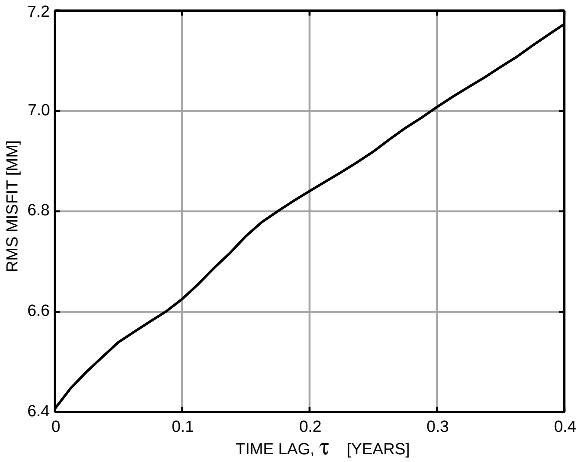

Fig. 4. The RMS misfit between the vertical displacement time series at ANTC and a generalized loading model in which the displacement at ANTC at time t is proportional to the volume in the lake at some earlier time $(t - \tau)$. Causality requires the time lag τ to be positive. The best fitting model (described in Table 1 and Fig. 1) occurs when the response time $\tau = 0$.

our search over a range of discrete time lags ought to capture at least some of the response delay even if not all of it. We find no sign at all of a (causal) time lag in the system, and while our analysis is rather simplistic, it leaves us with no compelling reason to suppose that a purely elastic model of the loading process is not adequate as a first order description of surface deformation.

6. Discussion

To further develop our understanding of the elastic loading process near Lago Laja, we will need to measure the surface displacement history at several geodetic stations and not just at ANTC. Given more stations we will be able to utilize more complicated elastic models such as a relatively compliant layer over a half-space, or (given enough stations) even a multilayered elastic model. There is very little point in developing such models when one is restricted to a single station. By measuring the displacement time series much closer to the lake it should be possible to resolve the loading coefficients for the horizontal components of displacement in addition to the vertical component. In principle, this can help us separate the influence of the two elastic parameters (Y and ν). By adding at least one station 20–40 km west of ANTC (i.e. farther from the lake), we should be able to estimate the global and regional loading signals directly and better separate them from the local loading signals associated with the lake.

It is desirable to perform similar measurements near other bodies of water which, like Lago Laja, undergo large variations in water volume, in order to compare the effective elastic structure inferred in different tectonic settings and using loads of different spatial scales.

This study suggests that continuous GPS stations can be used to study local and regional mass fluxes associated with the hydrological cycle, and that observations made near lakes and major rivers that undergo large and carefully monitored fluctuation in water volume could be used to calibrate this measurement system. However, this new tool for studying the hydrological cycle comes with a significant geophysical price tag—we are going to have to probe the shallow elastic structure of each study area in some detail in order to ensure that our measurements are properly interpreted.

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