Earth Science Applications of Space Based Geodesy

DES-7355

Tu-Th 9:40-11:05

Seminar Room in 3892 Central Ave. (Long building)

Bob Smalley
Office: 3892 Central Ave, Room 103
678-4929
Office Hours – Wed 14:00-16:00 or if I'm in my office.

http://www.ceri.memphis.edu/people/smalley/ESCI7355/ESCI_7355_Applications_of_Space_Based_Geodesy.html

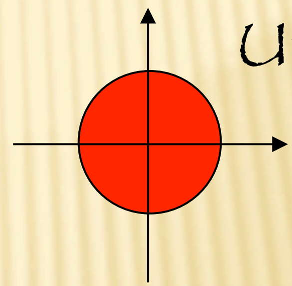
Class 9

Definition of vector norms

Vector Norms

L^2 (Euclidean) norm :

$$\|\mathbf{x}\|_2 = \sqrt{\sum_{i=1}^{n} |x_i|^2}$$

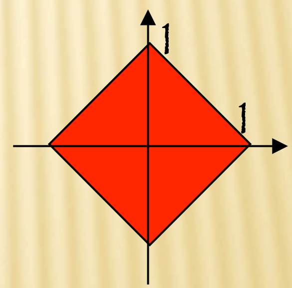


Unit circle

$$\|\mathbf{x}\|_2 < 1$$

L^1 norm :

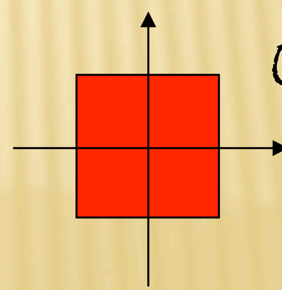
$$\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|$$



$$\|\mathbf{x}\|_1 < 1$$

∞ or "max" norm :

$$\|\mathbf{x}\|_\infty = \max_i |x_i|$$



Unit square

$$\|\mathbf{x}\|_\infty < 1$$

From lec3.gps

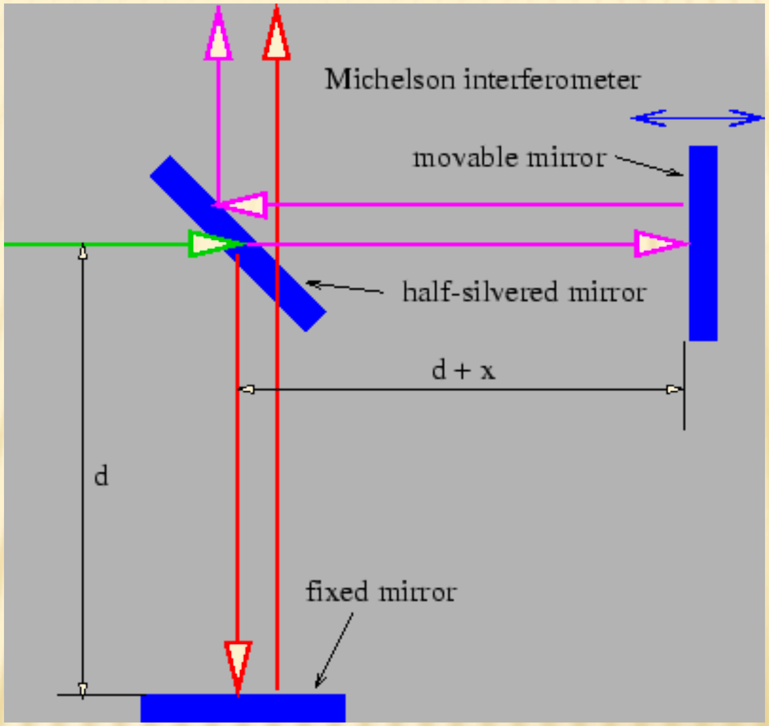
Interferemetry

The phase change comes from the change in distance (#wavelengths) between the two "rays"

(at constant velocity) change geometric distance traveled

(change in length of 1/2 wavelength causes π change in phase – and destructive interference)

Michelson Interferometer
Make two paths from same source
(for coherence, can do with white light!)



Can change geometric path length with movable mirror (eg mount on speaker).
Get interference "fringes" when recombine.

http://www.physics.nmt.edu/~raymond/classes/ph13xbook/node13.html

http://www.physics.uq.edu.au/people/mcintyre/applets/michelson/michelson.html

Note from animation

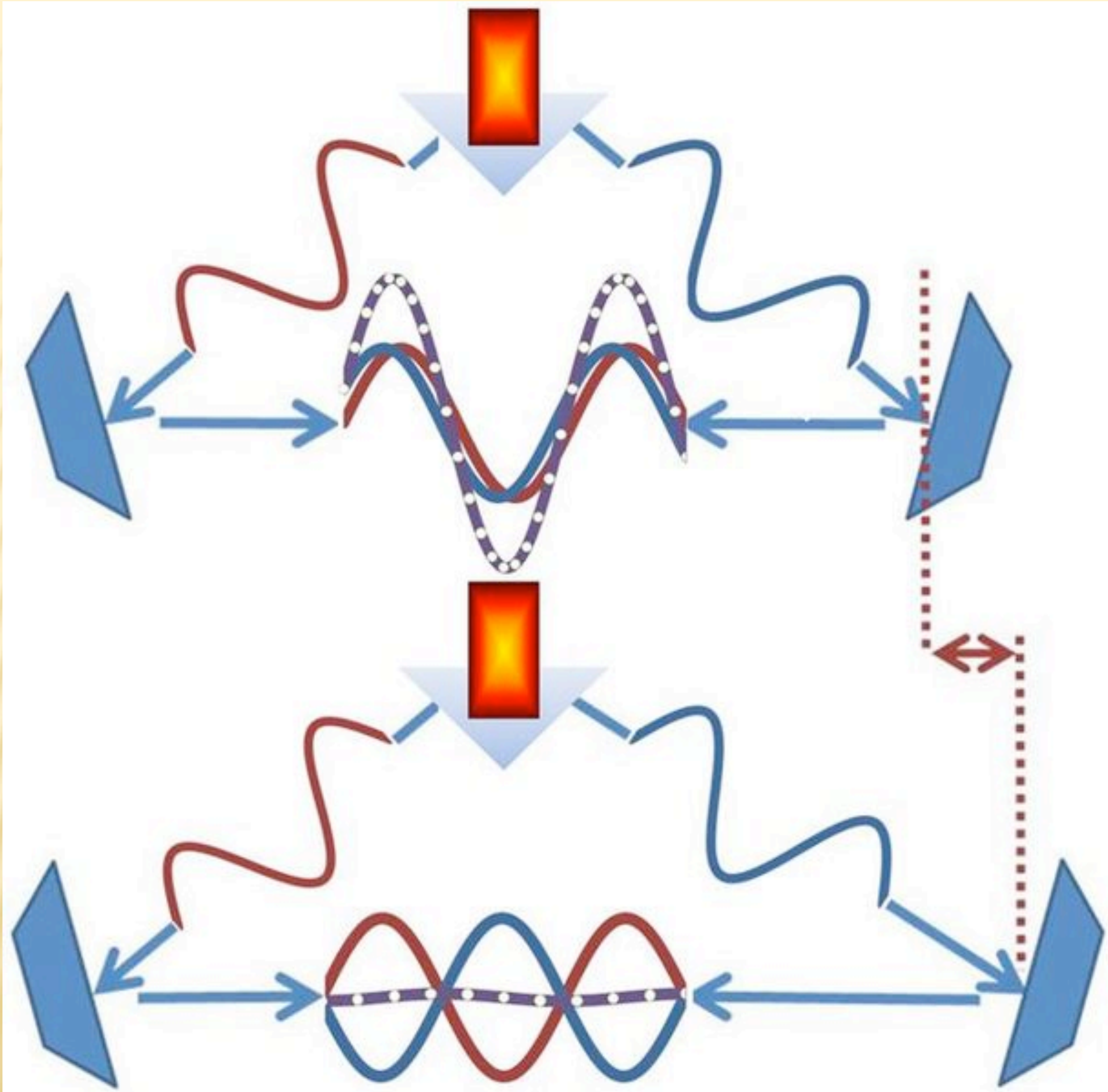
Can "integrate" (count continuously)

the fringes and how they change,

but there is a certain ambiguity

(each set of fringes looks same as others)

[no "reference" fringe]



Another way to get phase change

Change the "optical path length" (e.g. change velocity)

What counts is number of "cycles" (wavelengths),
not geometric distance.

Change optical path length by changing index of refraction along path

(this is what happens to GPS in ionosphere and troposphere

– error for crustal motion,

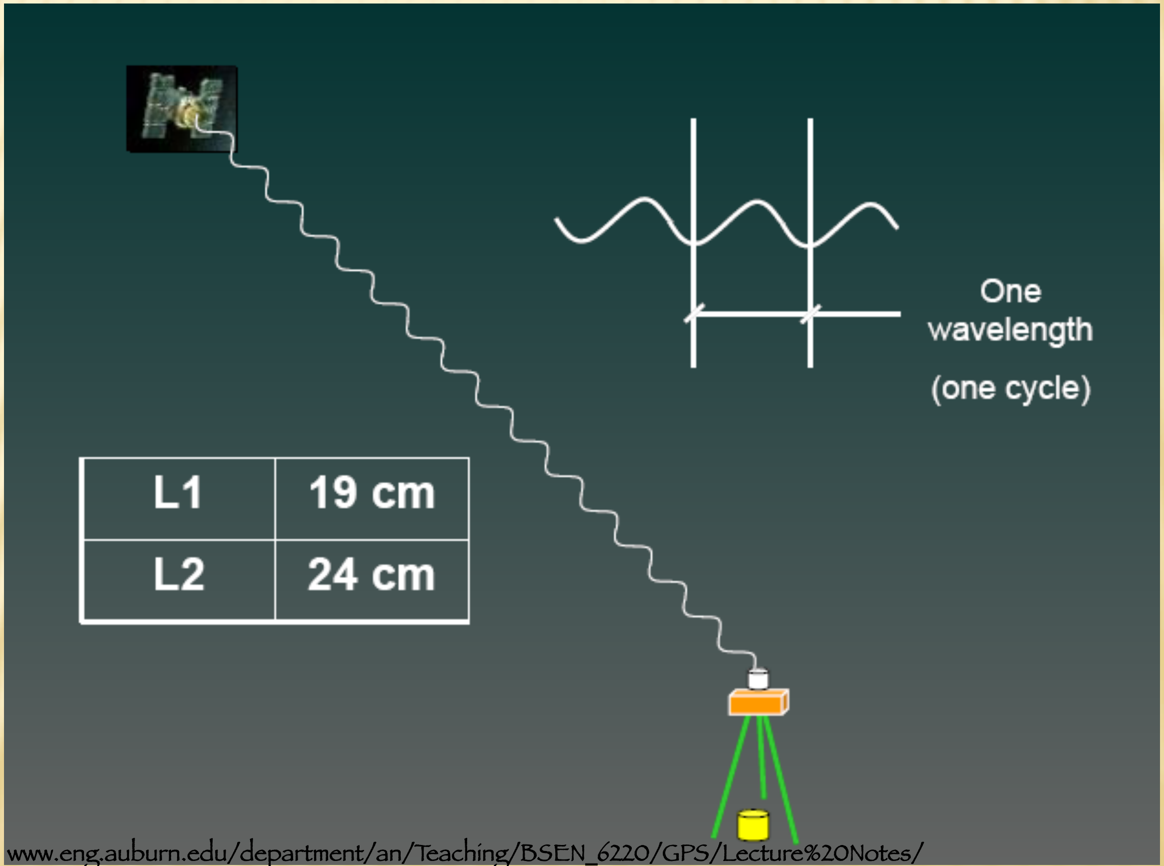
signal for ionospheric physics, weather, etc.)

GPS Carrier (beat) phase observable

(The word "beat" is usually not included in the "carrier phase observable" name, which can cause some confusion)

The key is to count radio wavelengths between satellites and receiver.

This number (the phase) is an integer plus a fraction.



L1	19 cm
L2	24 cm

www.eng.auburn.edu/department/an/Teaching/BSEN_6220/GPS/Lecture%20Notes/Carrier_Phase_GPS.pdf

Strang, http://www.siam.org/siamnews/general/gps.htm

Phase measurements

One can convert phase to distance by multiplying by the wavelength

(so phase measurements are another way to measure the distance from the satellite to the receiver)

The wavelengths of the carrier waves are very short –

Approximately

19cm for L1 and

24cm for L2 –

compared to the C/A (~300m, Global positioning system: theory and applications, Volume 1; Volume 163, By Bradford W. Parkinson, James J. Spilker) and P code chip lengths.

http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap3/323.htm

Phase measurements

Phase can be measured to about 1% of λ (3°)

This gives a precision of

~2 mm for L1

~2.4 mm for L2

Phase measurements

this means that carrier phase can be measured to

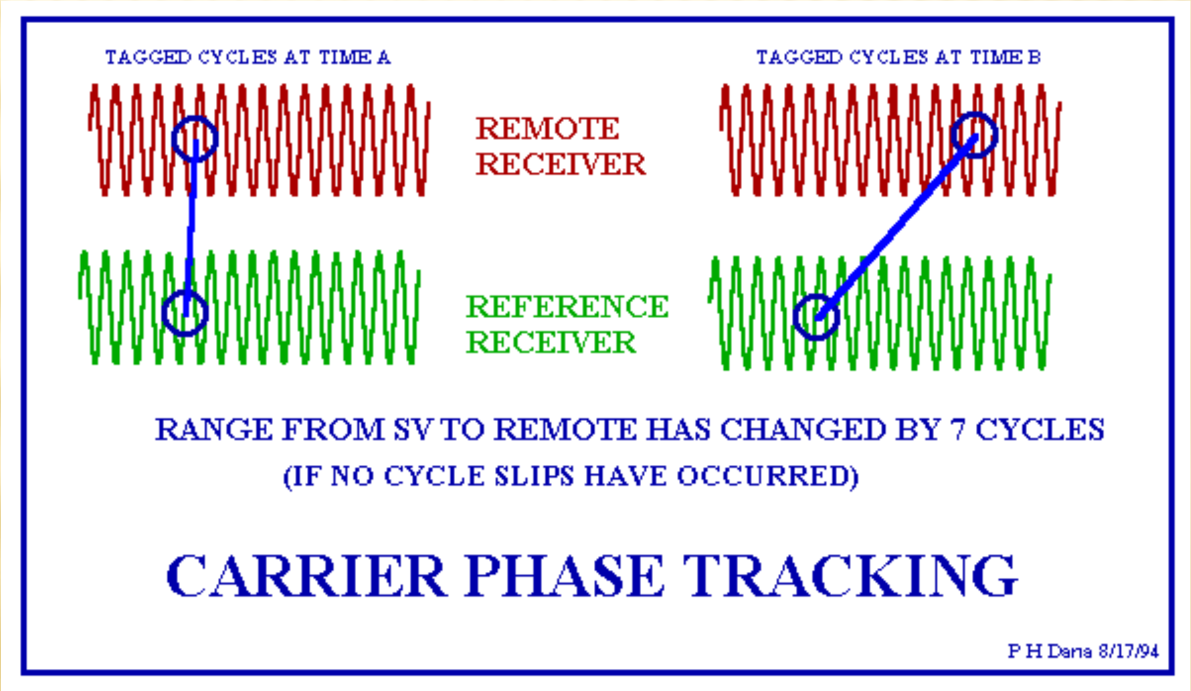
millimeter precision

compared with a few meters for C/A code measurements (and several decimeters for P code measurements).

http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap3/323.htm

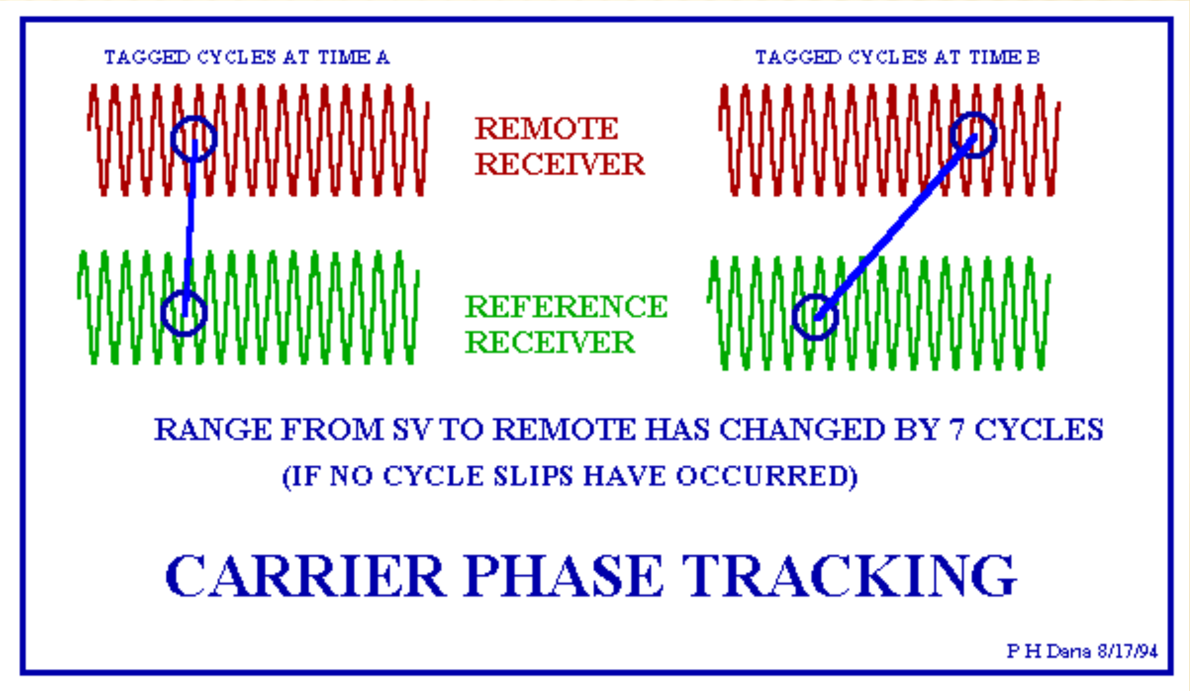
Tracking carrier phase signals, however, provides no time of transmission information.

The carrier signals, while modulated with time tagged binary codes, carry no time-tags that distinguish one cycle from another.



Dana, http://www.colorado.edu/geography/gcraft/notes/gps/gps_f.html

The measurements used in carrier phase tracking are differences in carrier phase cycles and fractions of cycles over time.

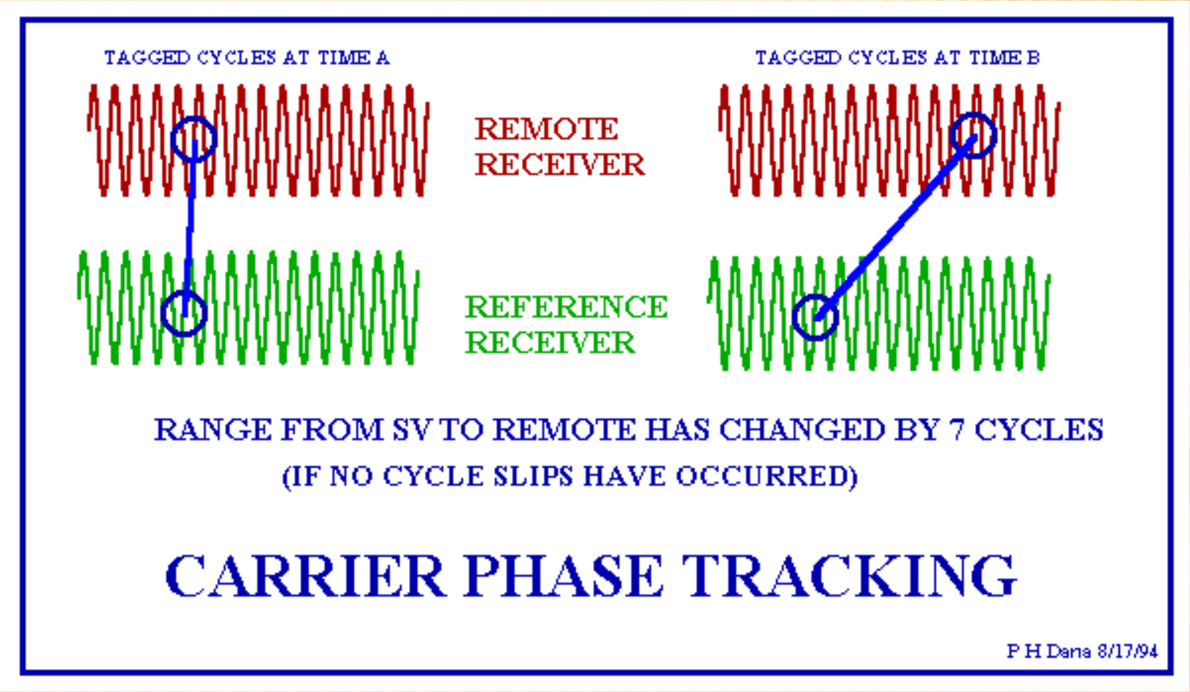


Dana, http://www.colorado.edu/geography/gcraft/notes/gps/gps_f.html

Unfortunately

phase measurement is "ambiguous" as it cannot discriminate one (either L1 or L2) cycle from another

(they all "look" the same).



http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap3/323.htm

In other words, time-of-transmission information for the signal cannot be imprinted onto the carrier wave as is done using PRN codes

(this would be possible only if the PRN code frequency was the same as the carrier wave,

rather than 154 or 120 times lower – and longer - in the case of the P code,

and 1540 or 1200 times lower – and longer - for the C/A code).

http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap3/323.htm

The basic phase measurement is therefore in the range

0° to 360°

(or 0 to 2π)

http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap3/323.htm

Phase measurements review:

Phase measurement PRECISE

But AMBIGUOUS

Another complication -

Phase measurements have to be corrected for

propagation effects

(several to 10's of meters) to benefit from the increased precision

The key is to count radio wavelengths between satellites and receiver.

This number (the phase) is an integer plus a fraction.

The integer part (called the ambiguity) is the tricky problem.

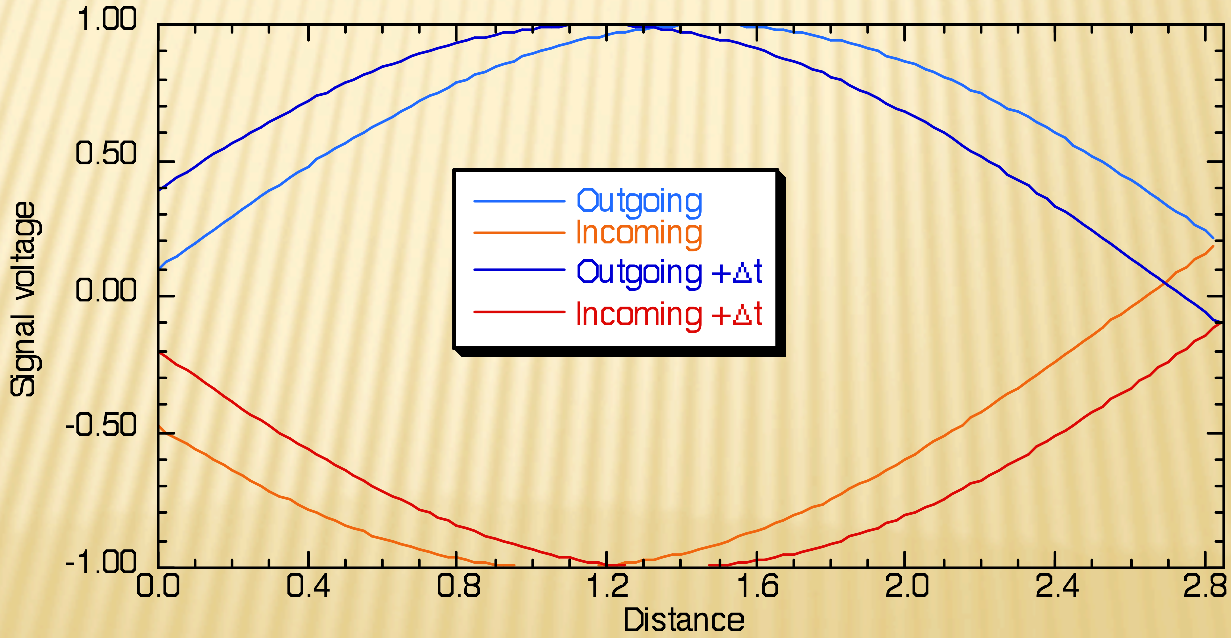
It has to be right, because one missing wavelength means an error of 19 cm or 24 cm (the satellite transmits on two frequencies).

Strang, http://www.siam.org/siamnews/general/gps.htm

Difference of phase measurement at two points

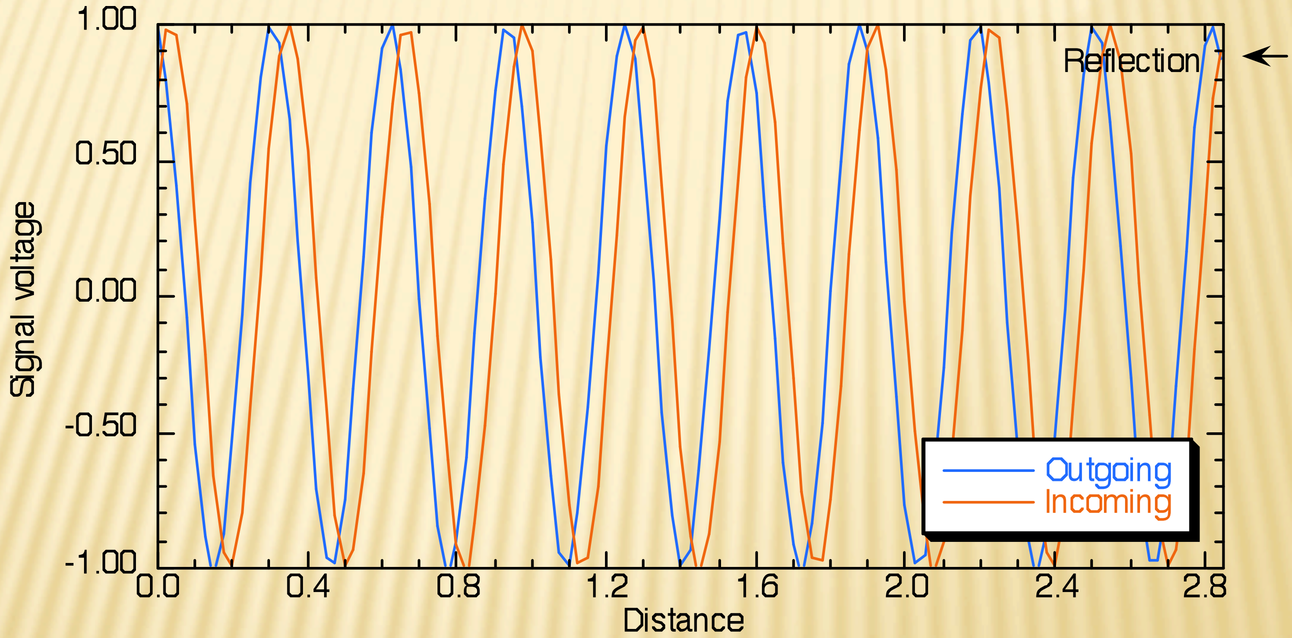
(stays constant with time and depends on distance [for stationary source])

Low frequency



Higher frequency.

Phase difference still says something about distance but how to know number of cycles?



Note that the phase is not constant for fixed positions of the transmitter and receiver.

The rate of phase change, and therefore the frequency (frequency is rate of change of phase), is constant in this case.

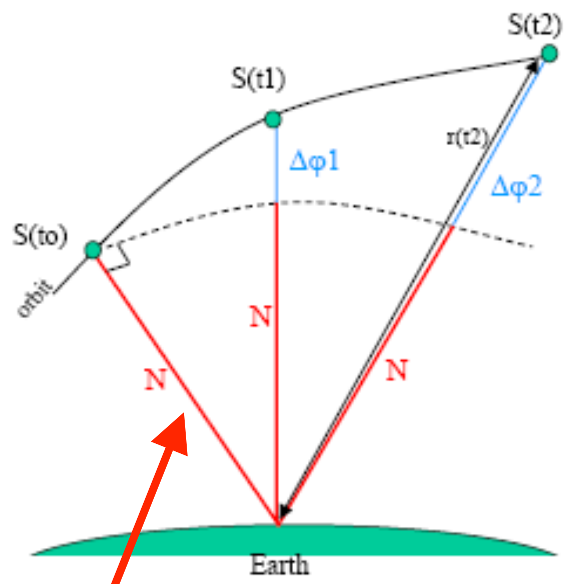
Moving transmitters and receivers cause the rate of phase change to vary, and therefore the frequency to vary --- a Doppler shift.

Ambiguity

We can keep track of phase once we lock onto it.

Phase measurements

- When a satellite is locked (at t_o), the GPS receiver starts tracking the incoming phase
- It counts the (real) number of phases as a function of time = $\Delta\varphi\ (t)$
- But the initial number of phases N at t_o is unknown…
- However, if no loss of lock, N is constant over an orbit arc



But can't tell how many whole cycles/wavelengths there are between satellite and receiver

From E. Calais

Determining this integer is like swimming laps in a pool

after an hour, the fractional part is obvious, but it is easy to forget the number of laps completed.

Strang, http://www.siam.org/siamnews/general/gps.htm

You could estimate it by dividing total swim time by approximate lap time.

For a short swim, the integer is probably reliable.

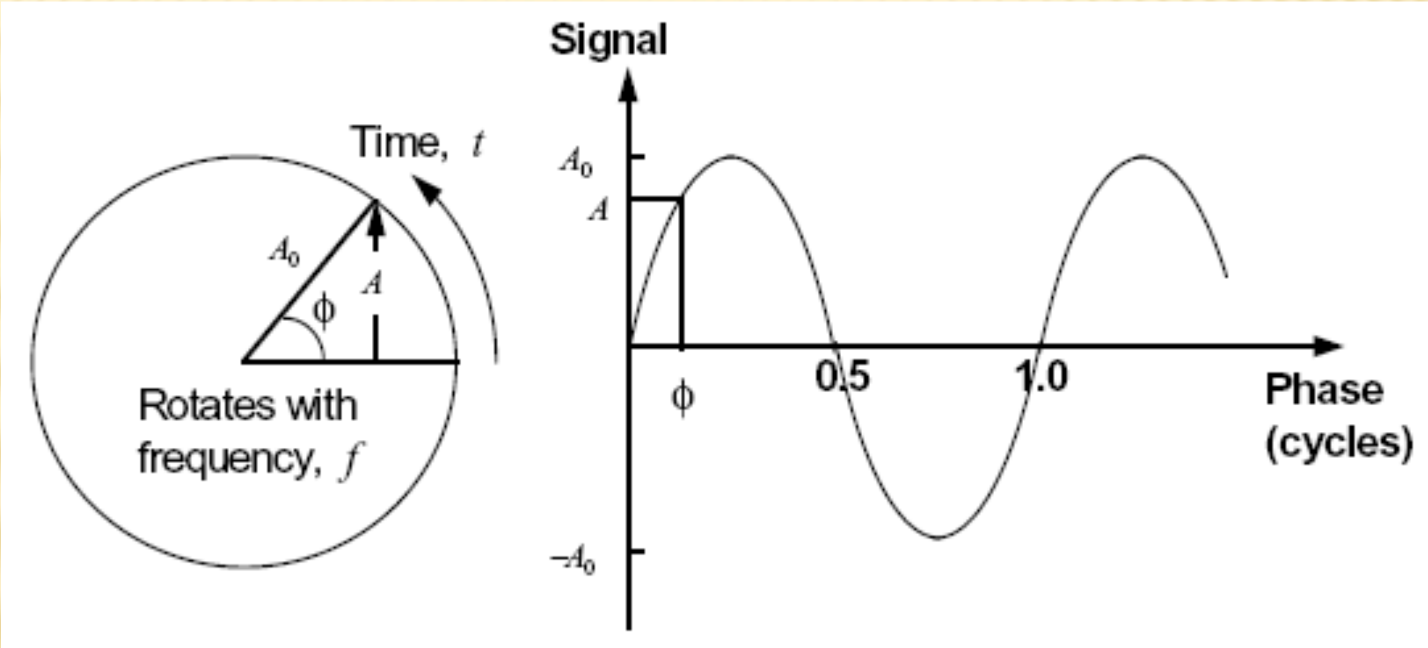
But the longer you swim, the greater the variance in the ratio.

In GPS, the longer the baseline between receivers, the harder it is to find this whole number.

Strang, http://www.siam.org/siamnews/general/gps.htm

Phase, frequency and Clock time

Phase is angle of rotation



Unit is cycles

Note is ambiguous by whole "rotations"

Concept of time

(or at least keeping track of it)

based on periodic "motion"

Day – rotation of earth on own axis
Year – rotation of earth around sun
Quartz crystal (or atomic) oscillations
Etc.

Phase is "%" of period.
But does not count whole periods.

Need way to convert phase to time units.

write

$$T(t) = k(\phi(t) - \phi_0)$$

Where

T(t) is time according to our clock at (some "absolute" time) t

$\phi_0 = \phi(t{=}0)$ is the time origin (our clock reads 0 at ϕ_0)

k is the calibration constant converting cycles to seconds

Frequency

Expressed as cycles-per-second
(SI unit is actually Hertz)

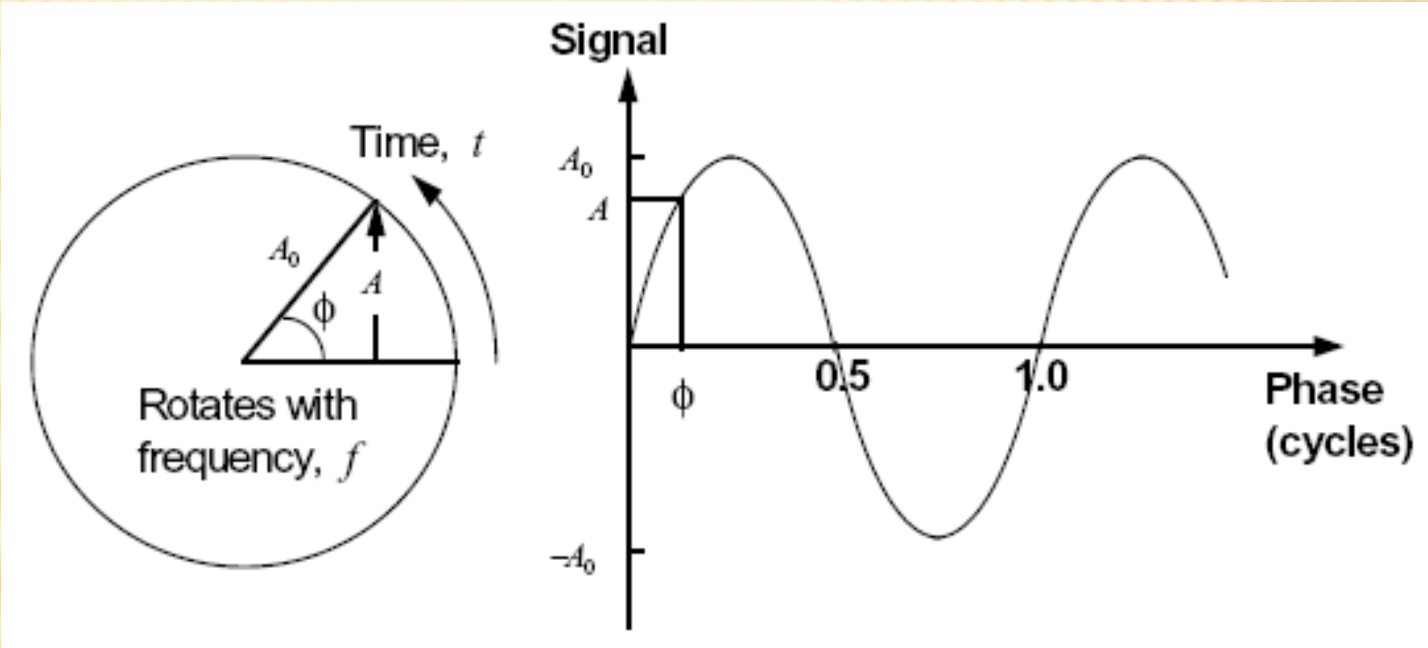
Assumes rotation rate is constant

Better definition – rate of change of phase with respect to time

$$f = \frac{d\phi(t)}{dt}$$

$$f = \frac{d\phi(t)}{dt} = \text{constant}$$

pure sine/cosine



Phase changes linearly with time

We will treat

-- Phase as the fundamental quantity

-- Frequency as the derived quantity or dependent variable

Basis for "ideal" clock

Constant frequency

$$\phi_{ideal} = f_0 t + \phi_0$$

$$T_{ideal} = k f_0 t$$

$$\phi_{ideal} = f_0 t + \phi_0$$
$$T_{ideal} = k f_0 t$$

This suggests that

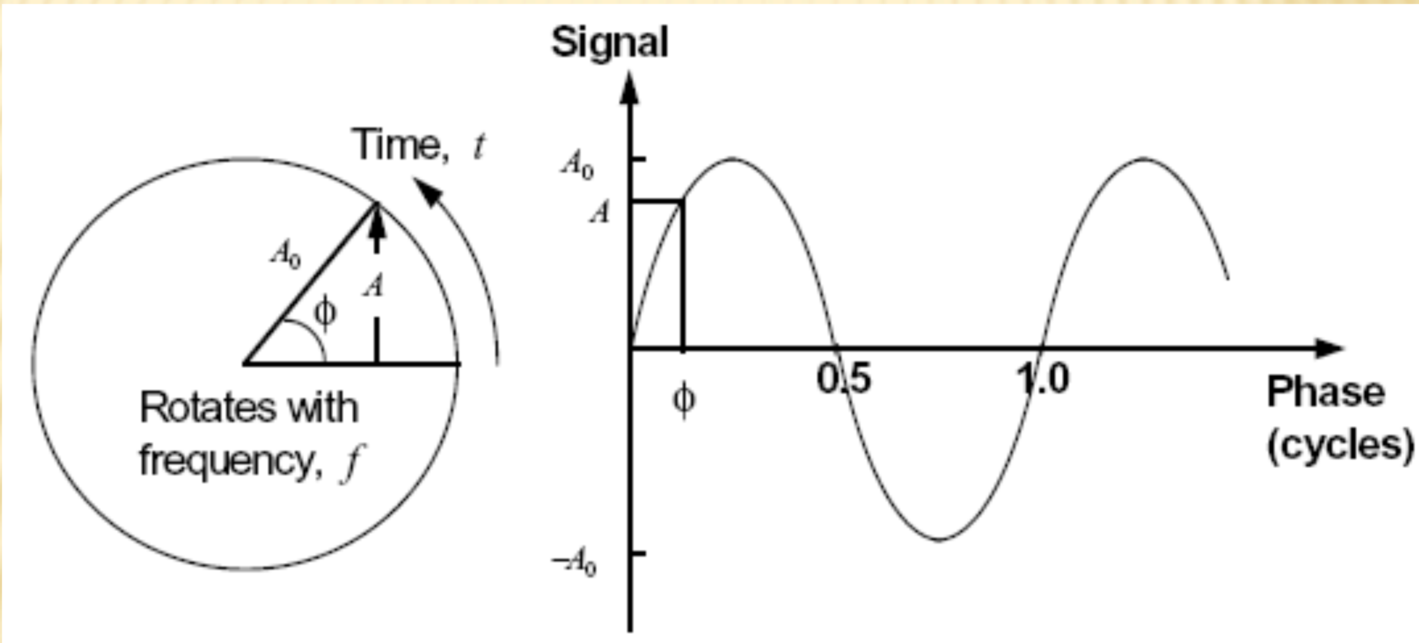
$k=1/f_0$

so

$$T(t) = \frac{\left(\phi(t) - \phi_0\right)}{f_0}$$

So we can describe the signal below as

$$A(t) = A_0 \sin\left(2\pi\phi(t)\right)$$



If one measures *A(t)* one can determine *ϕ(t)*

Signal for ideal clock

$$A_{ideal}(t) = A_0 \sin\left(2\pi\phi_{ideal}(t)\right)$$

$$A_{ideal}(t) = A_0 \sin\left(2\pi\left(f_0 t + \phi_0\right)\right)$$

$$A_{ideal}(t) = A_0 \cos\left(2\pi\phi_0\right)\sin\left(2\pi f_0 t\right) + A_0 \sin\left(2\pi\phi_0\right)\cos\left(2\pi f_0 t\right)$$

$$A_{ideal}(t) = A_0^S \sin\left(\omega_0 t\right) + A_0^C \cos\left(\omega_0 t\right)$$

Signal for real clock

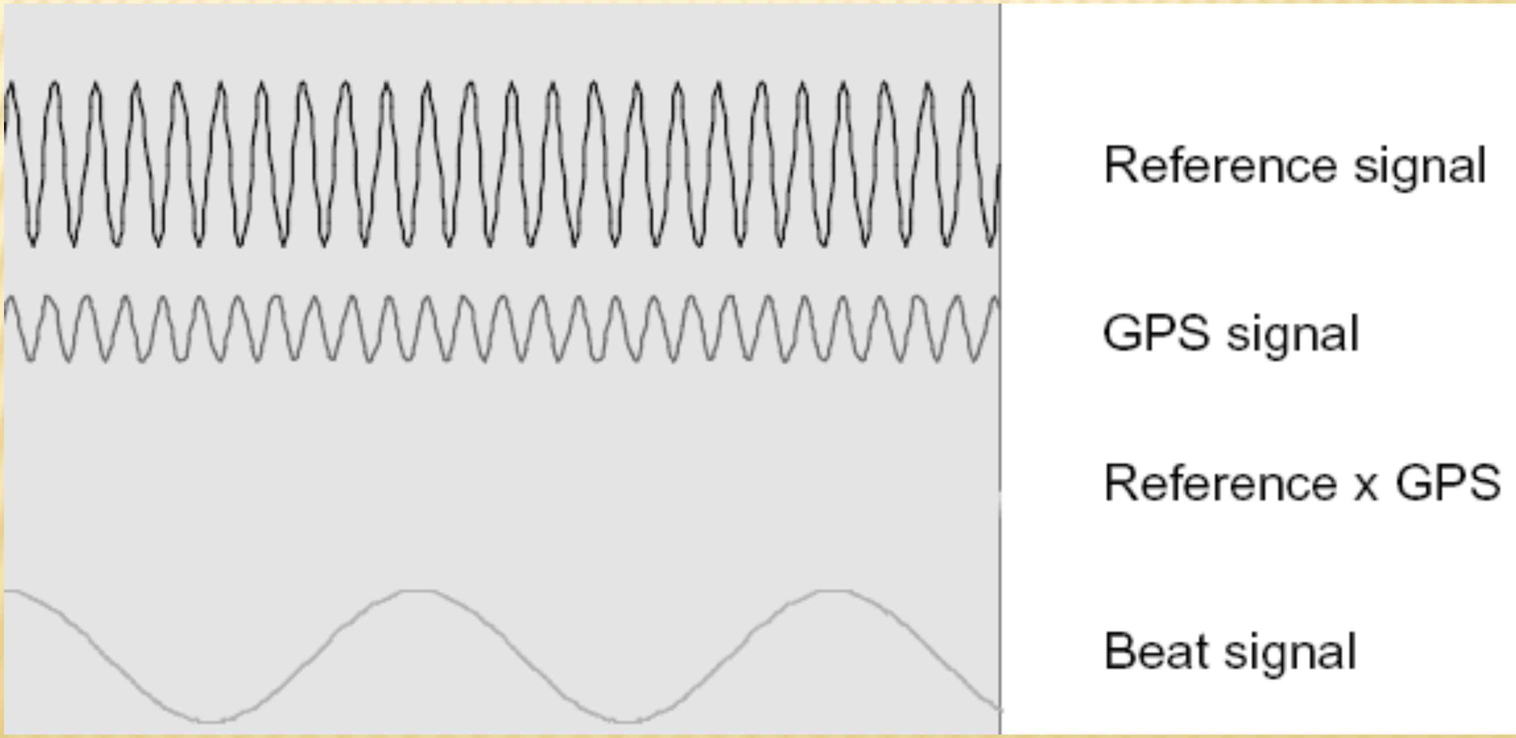
$$A_{real}(T) = A_0^S \sin\left(\omega_0 T\right) + A_0^C \cos\left(\omega_0 T\right)$$

GPS signal of this form PLUS "modulation" by + or – 1.

To "receive" a GPS signal the

received signal (whose frequency has been shifted by the Doppler effect – more later) is <u>mixed</u> with a receiver generated copy of the signal

producing a beat due to the difference in frequency

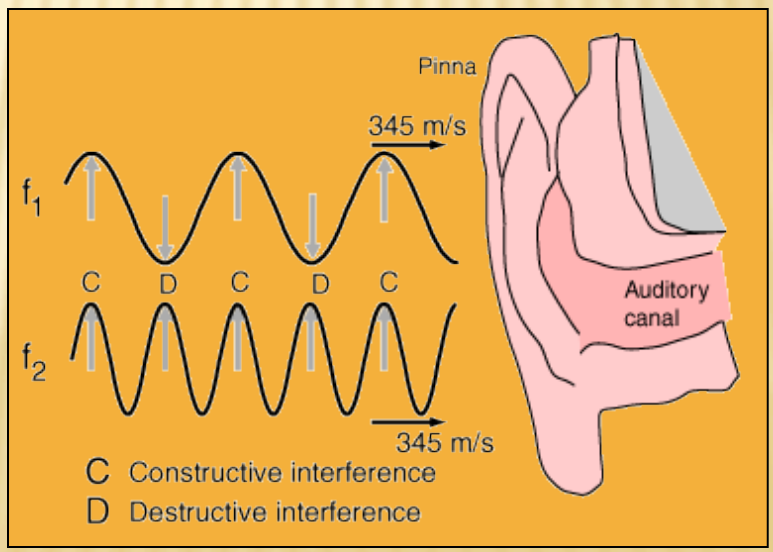


Blewitt, Basics of GPS in "Geodetic Applications of GPS"

When two sound waves of different frequency approach your ear, the alternating constructive and destructive interference causes the sound to be alternatively soft and loud

-a phenomenon which is called "beating" or producing beats.

-The beat frequency is equal to the absolute value of the difference in frequency of the two waves.



http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html

Beat Frequencies in Sound

The sound of a beat frequency or beat wave is a <u>fluctuating volume</u> caused when you add two sound waves of slightly different frequencies together.

If the frequencies of the sound waves are close enough together, you can hear a relatively slow variation in the volume of the sound.

A good example of this can be heard using two tuning forks that are a few frequencies apart. (or in a twin engine airplane when the engines are not "synched" = you hear a "wa-wa-wa-wa-… noise)

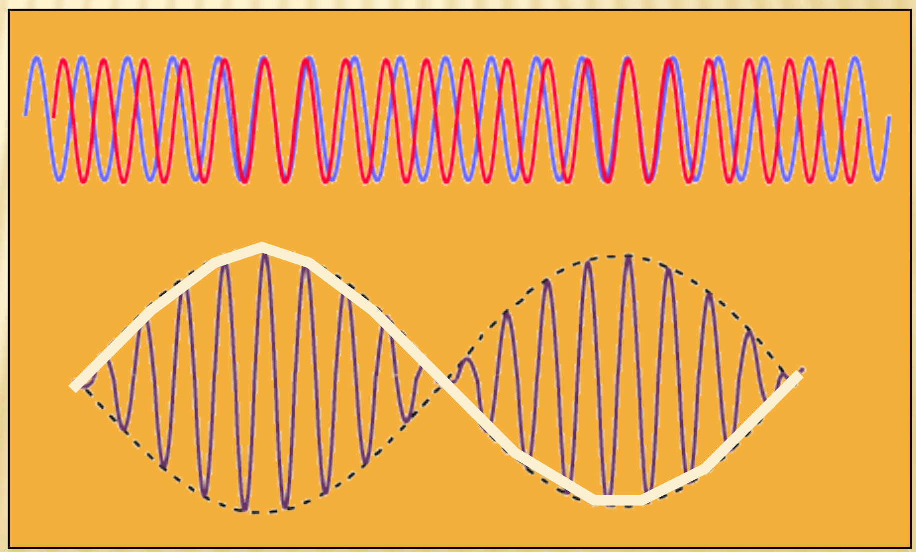
http://www.school-for-champions.com/science/sound_beat.htm

Beats are caused by the interference of two waves at the same point in space.

$$\cos(2\pi f_1) + \cos(2\pi f_2) = 2A\cos\left(2\pi\frac{f_1 - f_2}{2}\right)\cos\left(2\pi\frac{f_1 + f_2}{2}\right)$$

$$f_{beat} = \left|\frac{f_1 - f_2}{2}\right|$$

Beat -- Frequency of minimia, which happens twice per cycle.



http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html

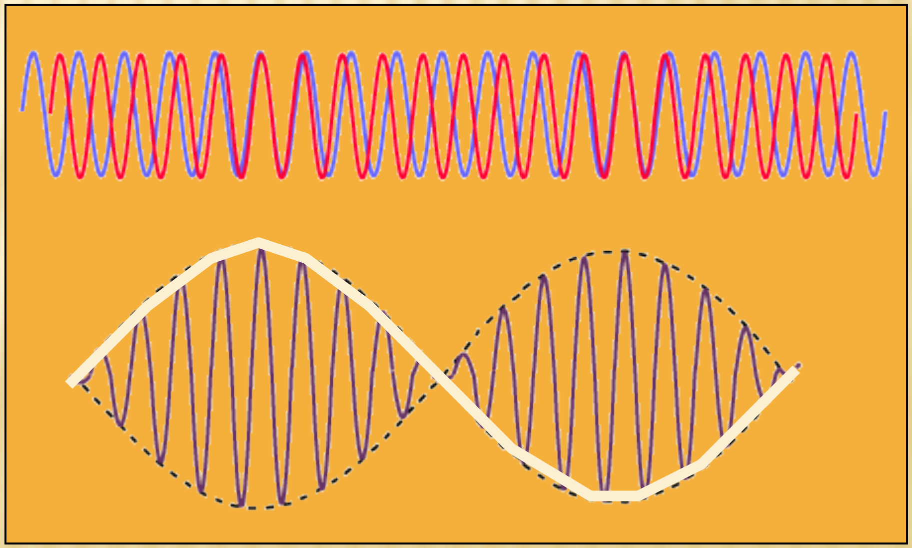
Note the frequencies are

half the difference and

the average of the original frequencies.

$$\cos(2\pi f_1) + \cos(2\pi f_2) = 2A\cos\left(2\pi\frac{f_1 - f_2}{2}\right)\cos\left(2\pi\frac{f_1 + f_2}{2}\right)$$

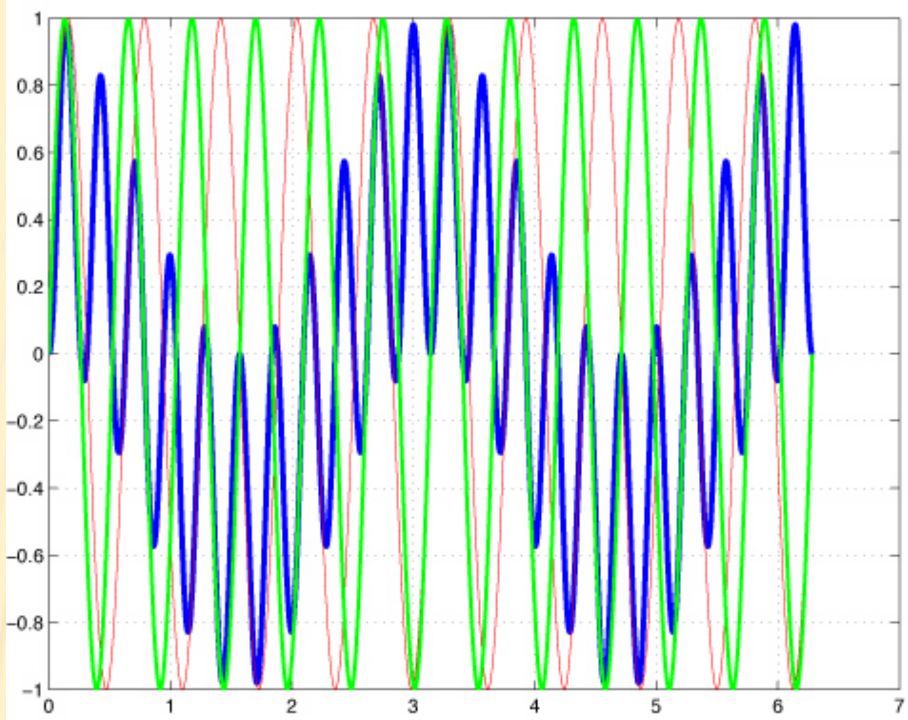
Different than multiplying (mixing) the two frequencies.



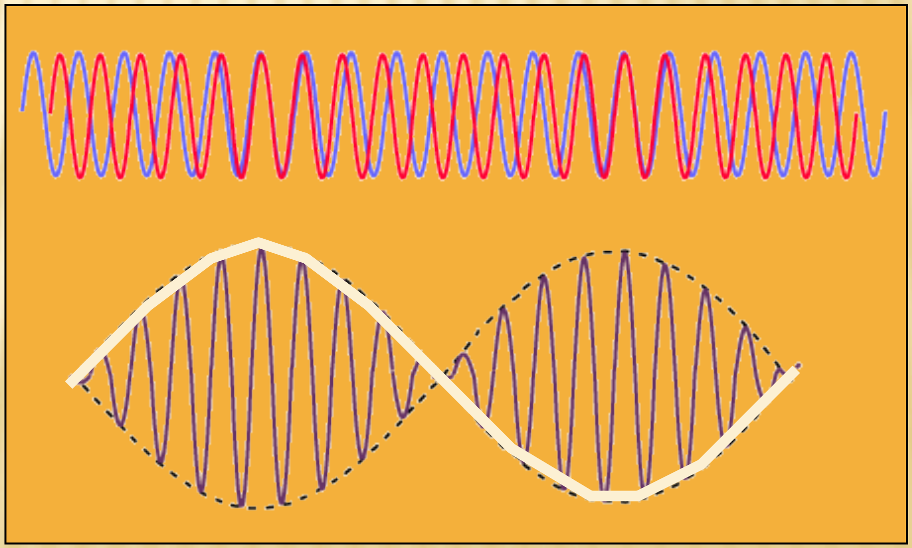
http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html

Product (mix)

(get sum and difference, not half of them)



sum



http://hyperphysics.phy-astr.gsu.edu/hbase/sound/beat.html

http://webphysics.ph.msstate.edu/jc/library/15-11/index.html

Set up to see phase vel and group vel opposite sign

(package goes one way, waves inside go other)

λ = 24 and 22, v = 5 and 3 respectively

http://www.geneseo.edu/~freeman/animations/phase_versus_group_velocity.htm