Earth Science Applications of Space Based Geodesy

DES-7355

Tu-Th 9:40-11:05

Seminar Room in 3892 Central Ave. (Long building)

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Office Hours – Wed 14:00-16:00 or if I'm in my office.

http://www.ceri.memphis.edu/people/smalley/ESCI7355/ESCI_7355_Applications_of_Space_Based_Geodesy.html

Class 3

So far we have

$$U(\vec{x}) = -\oint_c F(\vec{x}) \bullet d\vec{x}$$

Potential is negative of work, and

$$\vec{g}(\vec{x}) = -\nabla U(\vec{x})$$

Force is negative gradient of potential

(can also define with out the negatives on either).

We looked at case of uniform density sphere
(and hollow, uniform density, spherical shell)

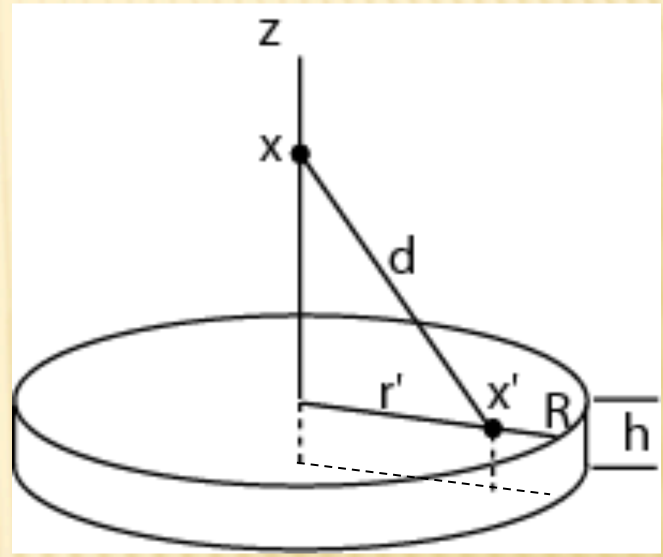
How about potential of shapes other than a sphere?

Potential for a thin disk

Use cylindrical coord

("natural" coordinate system)

Let the density be constant.



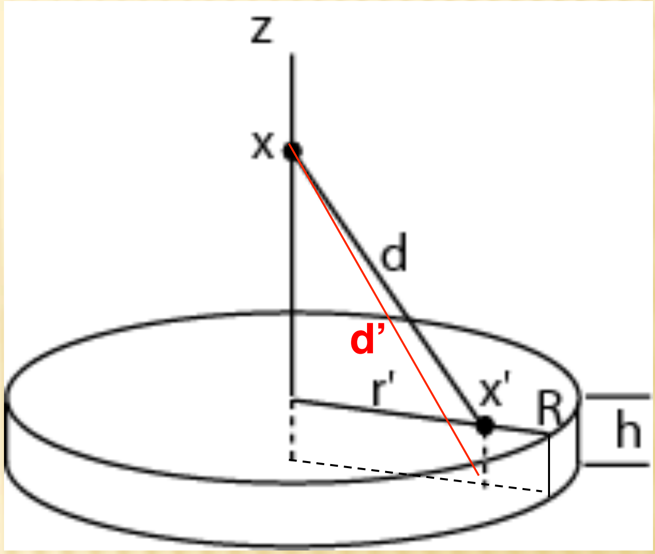
$$U(\vec{x}) = -G\int_V \frac{dm}{d} = G\rho\int_V \frac{dV}{d} = -G\rho\int_0^R\int_0^{2\pi}\int_0^h \frac{r'dr'd\varphi\, dz'}{d}$$

$$d = |\vec{x} - \vec{x}'| = \sqrt{(z - z')^2 + r'^2}$$

Potential for a thin disk

for a thin disk

$d \approx$ constant over 0 to h, so



$$d_{bottom} = |\vec{x} - \vec{x}'| = \sqrt{z^2 + r'^2}$$

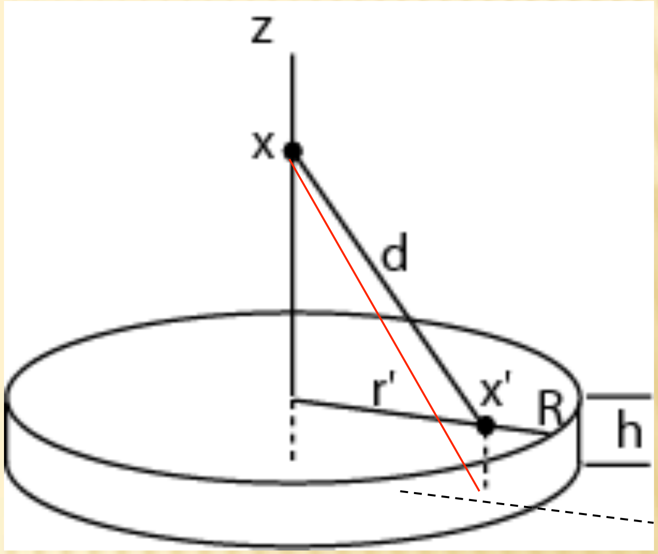
$$d_{top} = |\vec{x} - \vec{x}'| = \sqrt{(z-h)^2 + r'^2}$$

for $h << z$ $\qquad d_{bottom} \approx d_{top}$

and we can use

$$d \approx |\vec{x} - \vec{x}'| = \sqrt{z^2 + r'^2}$$

Potential for a thin disk



using

$$d \approx |\vec{x} - \vec{x}'| = \sqrt{z^2 + r'^2}$$

and letting $\sigma = \int_0^h \rho \, dz'$

where σ is the mass surface density

$$U(\vec{x}) = -G\rho \int_0^R \int_0^{2\pi} \int_0^h \frac{r' dr' d\varphi \, dz'}{d} \approx -G\sigma \int_0^{2\pi} d\varphi \int_0^R \frac{r'}{\sqrt{z^2 + r'^2}} dr'$$

$$U(\vec{x}) = -2\pi \, G\sigma \int_0^R \frac{r'}{\sqrt{z^2 + r'^2}} dr'$$

Potential for a thin disk

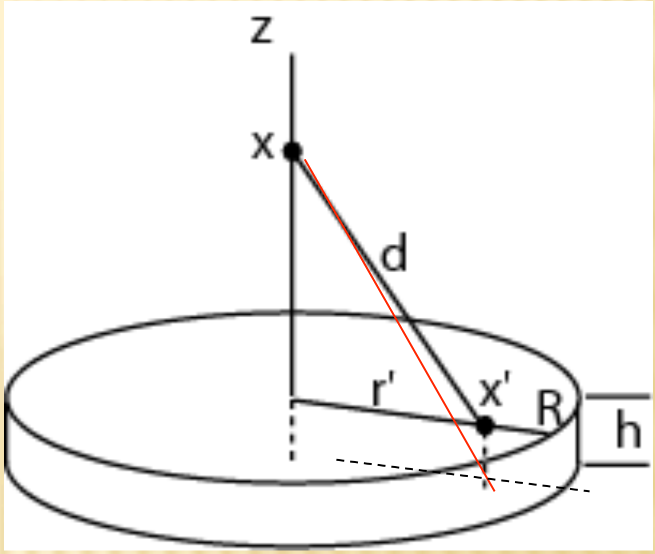
$$U(\vec{x}) = -2\pi G\sigma \int_0^R \frac{r'}{\sqrt{z^2 + r'^2}}\, dr'$$

substitute $u = z^2 + r'^2$, so $du = 2r'dr'$

$$U(\vec{x}) = -\pi G\sigma \int_{z^2}^{z^2+R^2} \frac{du}{\sqrt{u}}$$

$$U(\vec{x}) = -\pi G\sigma 2\sqrt{u}\Big|_{z^2}^{z^2+R^2} \quad \text{or} \quad = -2\pi G\sigma \sqrt{z^2 + r'^2}\Big|_0^R$$

$$\boxed{U(\vec{x}) = -2\pi G\sigma \left(\sqrt{z^2 + R^2} - z\right)}$$



Since we previously defined the potential to be the negative of the work to bring a test mass in from infinity, we would like U to be zero at one end and some finite value at the other.

Let U(z=0) =0.

$$U(\vec{x}, z=0) = -\left(2\pi G\sigma\left(\sqrt{z^2+R^2}-z\right)+C\right)=0$$

$$U(\vec{x}, z=0) = \sqrt{R^2} = R = -C$$

$$\text{so } U(\vec{x}) = -2\pi G\sigma\left(\sqrt{z^2+R^2}-\left(z+R\right)\right)$$

What about for an infinite thin sheet?

Problem - if we let R go to infinity – we get infinity.

$$\lim \xrightarrow{R\to\infty} U(\vec{x}) = -2\pi G\sigma\left(\sqrt{z^2+R^2}-z\right) = -\infty$$

Not good.

But remember that U only defined to a constant – so (we will see that) by judiciously assigning the constant we can "fix" the problem.

now take the limit $R \to \infty$

$$\lim \xrightarrow{R\to\infty} U(\vec{x}) = -2\pi G\sigma\left(\sqrt{z^2+R^2} - (z+R)\right) =$$

you can expand $\sqrt{z^2+R^2}$ and do it "right"

or just let $\sqrt{z^2+R^2} \to R$

$$\lim \xrightarrow{R\to\infty} \sqrt{z^2+R^2} - (z+R) \to R - (z+R) = -z$$

so for an infinite thin sheet

$$\boxed{U(\vec{x}) = 2\pi G\sigma z}$$

We can now find g(z) for the thin disk from

$$\vec{g}(z) = -\nabla U(\vec{x}) = -\nabla\left(2\pi\, G\sigma\, z\right)$$

$$\vec{g}(z) = -2\pi\, G\sigma\, \hat{z}$$

It is independent of z!

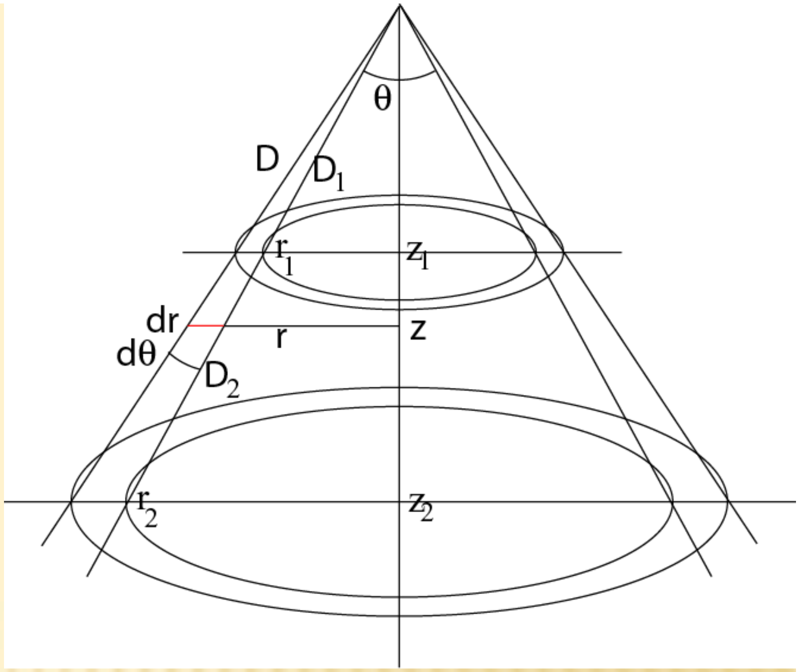
The gravity field is a constant in all space.

Direction – towards sheet.

Lets do another way -

Freshman physics approach

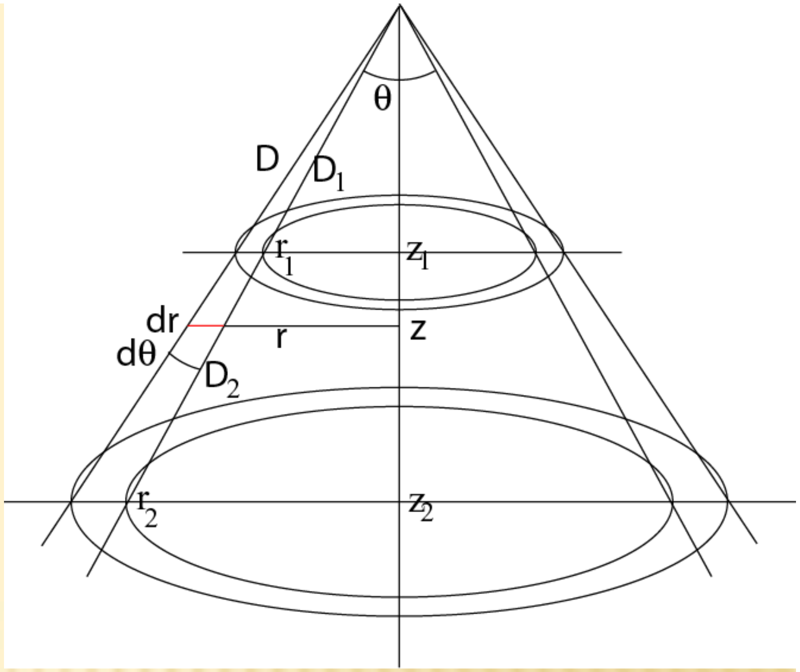
Find gravity due to a ring from θ to $\theta+d\theta$ for a distance z above the plane



Then sum the rings (integrate over θ)

Set up

as before use symmetry to simplify
Find gravity due to a ring at height z.

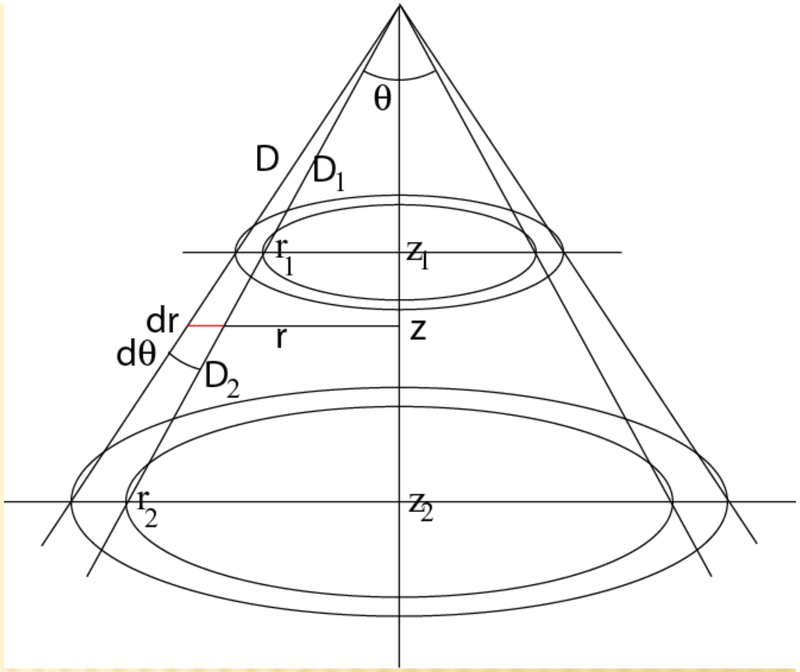


From symmetry, force is vertical only

So can look at magnitude (scalar) only

$$dg_{vert}(m(z,\theta)) = -\left(\frac{Gdm}{D^2}\right)\cos\theta$$

Set up



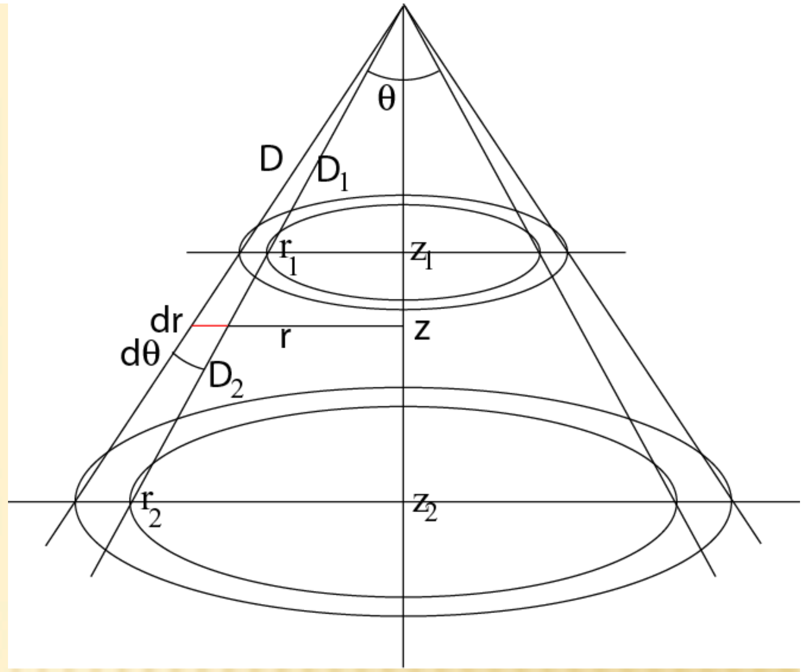
from geometry

$$dm(z,\theta) = 2\pi\, r dr \sigma$$

$$\cos\theta = \frac{z}{D}$$

so

$$dg_{vert}(m(z,\theta)) = -\left(\frac{Gdm}{D^2}\right)\cos\theta = -\left(\frac{G2\pi\, r dr\sigma}{D^2}\right)\frac{z}{D} = dg_{vert}(m(z,r))$$



$$dg_{vert}(m(z,r)) = -G2\pi z\,\sigma\left(\frac{rdr}{D^3}\right)$$

$$g_{vert} = -G2\pi z\,\sigma\int_0^\infty \frac{rdr}{D^3} = -G2\pi z\,\sigma\int_0^\infty \frac{rdr}{\left(z^2+r^2\right)^{\frac{3}{2}}}$$

For whole plane integrate over r

$$u = z^2 + r^2,\ du = 2rdr$$

$$g_{vert} = -G\pi z\,\sigma\int_z^\infty \frac{du}{u^{\frac{3}{2}}} = G\pi z\,\sigma 2u^{-\frac{1}{2}}\Big|_z^\infty = -G2\pi\,\sigma$$

Same as we got before.

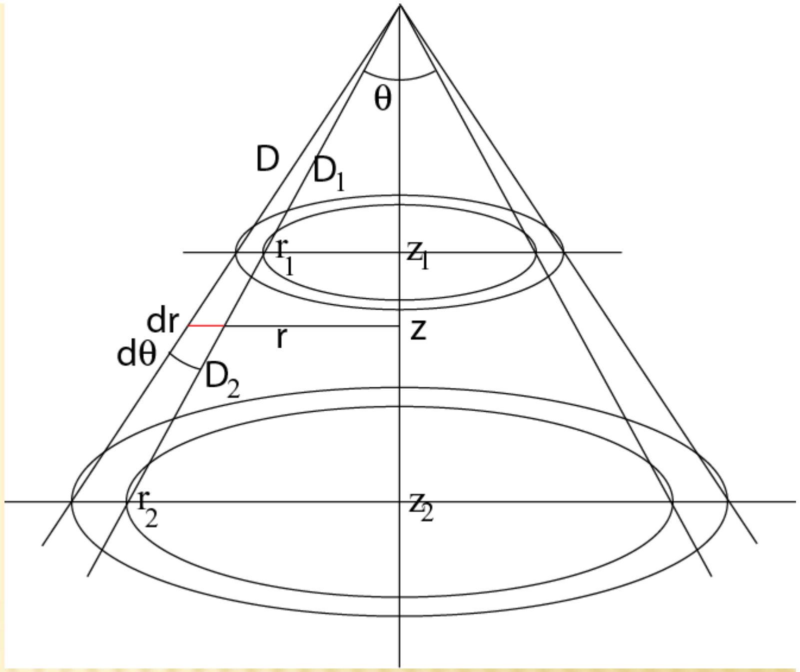
In words

As one changes one's distance with respect to the thin sheet

- the amount of mass in the ring with a fixed width of <u>angle</u> dθ goes as D^2

- but the force due the mass in the ring goes as $1/D^2$

(they both have the same functional form)

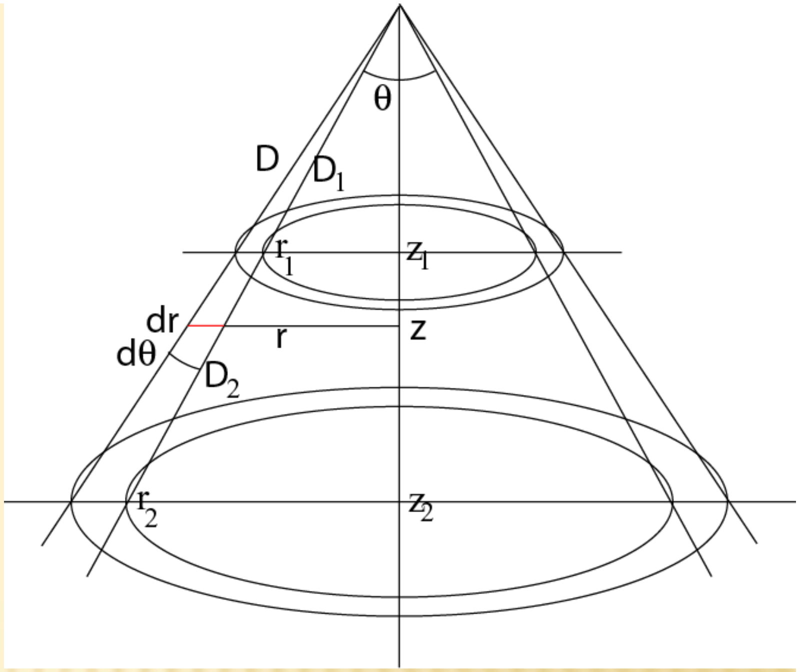


$$\vec{g}(\vec{x}) = \vec{g}(z) = -2\pi G\sigma\,\hat{z}$$

--- so the distance dependence cancels out!!

Interpretation:

There is no "scale length"



$$\boxed{\vec{g}(\vec{x}) = \vec{g}(z) = -2\pi G\sigma \hat{z}}$$

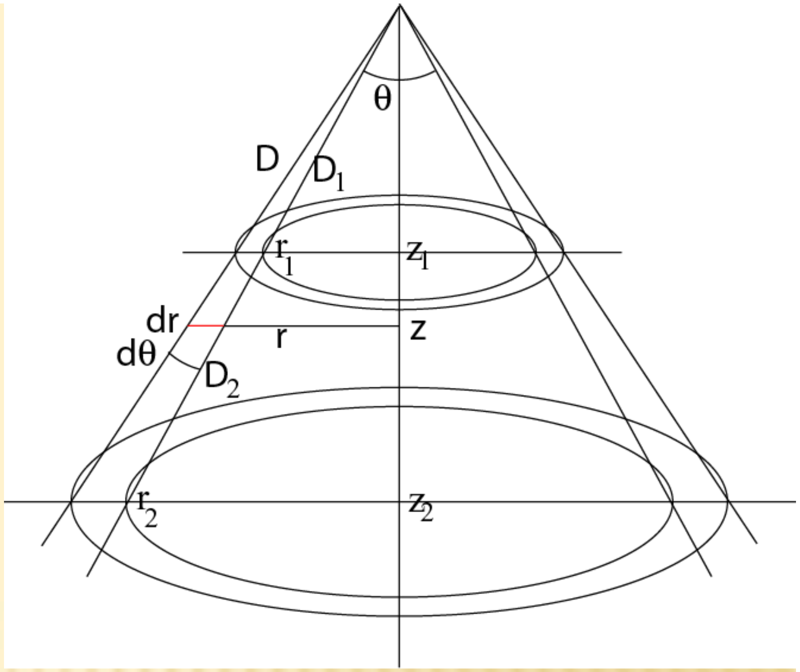
No matter your position (horizontally – but this we get from simple symmetry, or more important - vertically) – the plane "looks" the same.

(We will run into this result again - disguised as Bouguer's formula)

Remembering back to our selection of a constant – I did not say why we picked

$U(z=0)=0$

(for the case of the earth we use $U(z=\text{infinity})=0$)



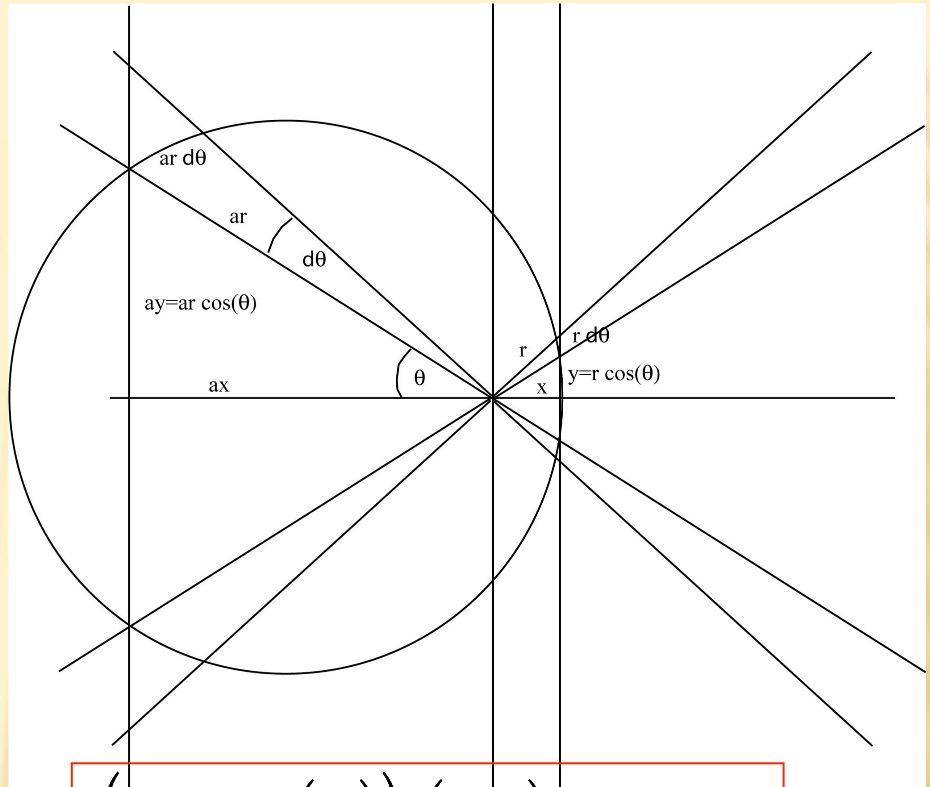
$$\boxed{\vec{g}(\vec{x}) = \vec{g}(z) = -2\pi G\sigma\,\hat{z}}$$

Since g is a constant, it will take an infinite amount of work to move from 0 to infinity (or back in).

Of course an infinite plane would have an infinite total mass and is not physically possible. A physically realizable g field has to fall off with distance.

Apply same technique to gravity inside a spherical shell.

Find same effect – mass goes as r^2, g goes as $1/r^2$ – effects cancel.



ring on left (similar triangles)

$$-\frac{\left(2\pi\, ar\cos(\theta)\right)\left(ar\, d\theta\right) dr\cos\theta}{(ar)^2} = \boxed{-\left(2\pi\cos(\theta)\right)\left(d\theta\right) dr\cos\theta}$$

No r dependence, same magnitude, opposite directions.

ring on right

$$\frac{\left(2\pi\, r\cos(\theta)\right)\left(r\, d\theta\right) dr\cos\theta}{(r)^2} = \boxed{\left(2\pi\cos(\theta)\right)\left(d\theta\right) dr\cos\theta}$$

What would you get if you were inside an infinite cylinder?

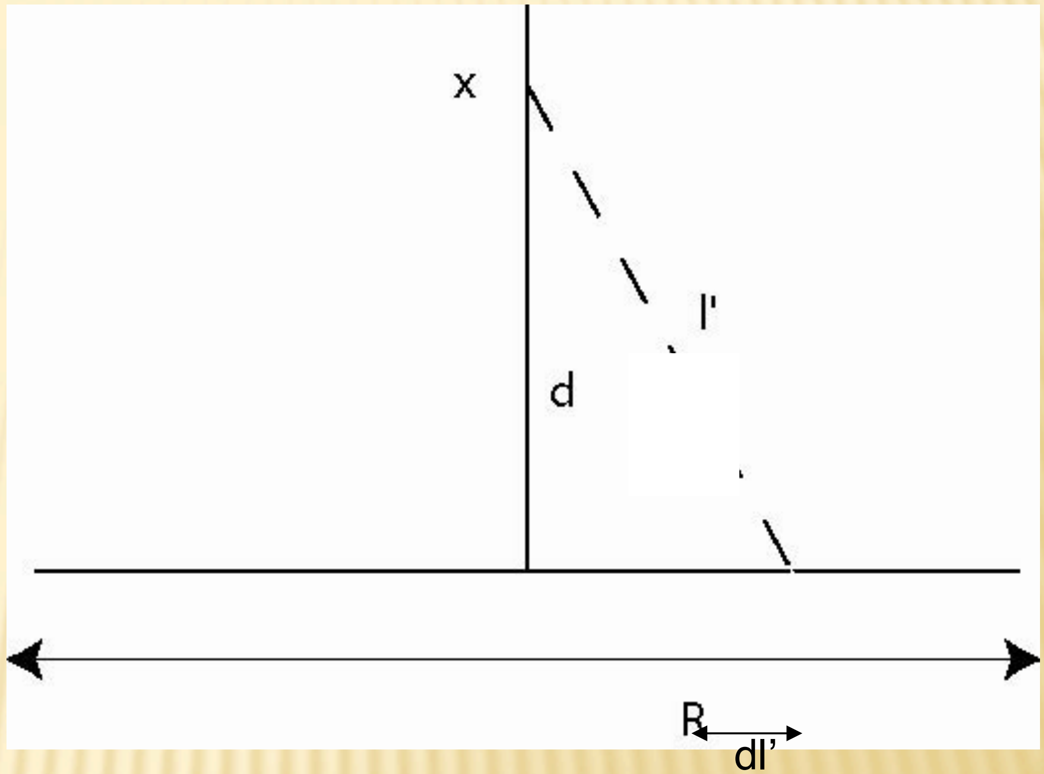
Potential and force for a line

$$dm = \rho\, dl' dA$$

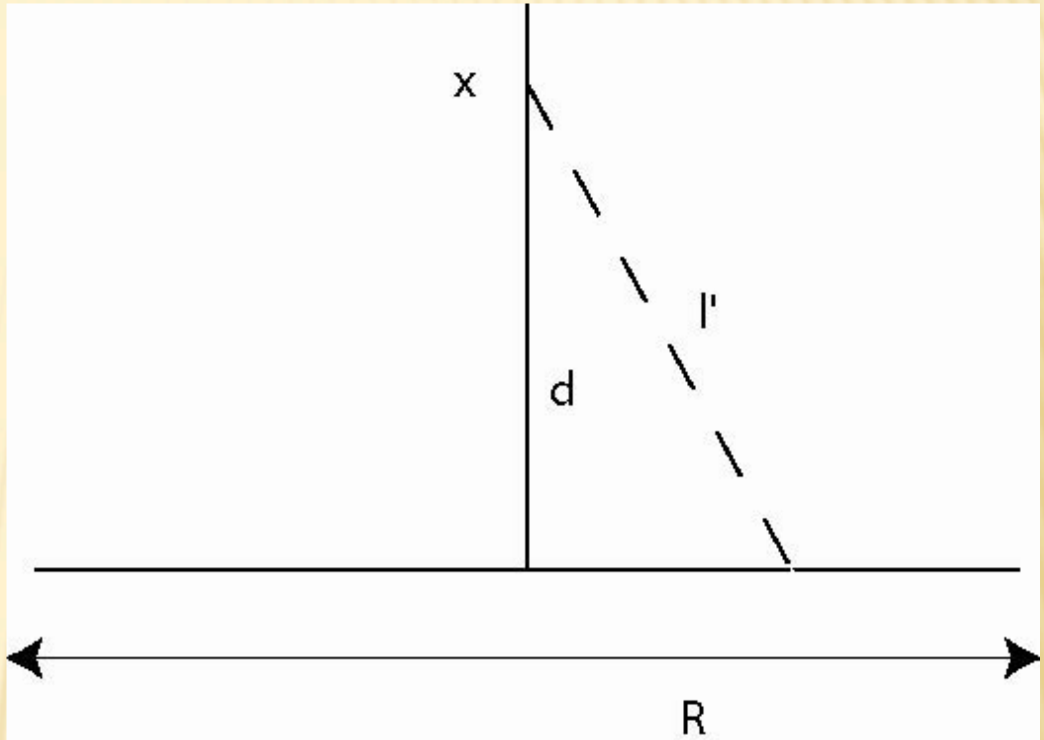
$$U(\vec{x}) = G\int_{-\frac{R}{2}}^{\frac{R}{2}} dl' \int_A \frac{\rho\, dA}{|x - x'|}$$

$$\lambda = \int_A \rho\, dA$$

$$U(\vec{x}) = G\lambda \int_{-\frac{R}{2}}^{\frac{R}{2}} \frac{dl'}{\sqrt{d^2 + l'^2}}$$



Potential and force for a line



$$U(\vec{x}) = G\lambda \int_{-\frac{R}{2}}^{\frac{R}{2}} \frac{dl'}{\sqrt{d^2 + l'^2}}$$

$$U(\vec{x}) = 2G\lambda \ln\left(\frac{R + \sqrt{R^2 + 4d^2}}{2d}\right)$$

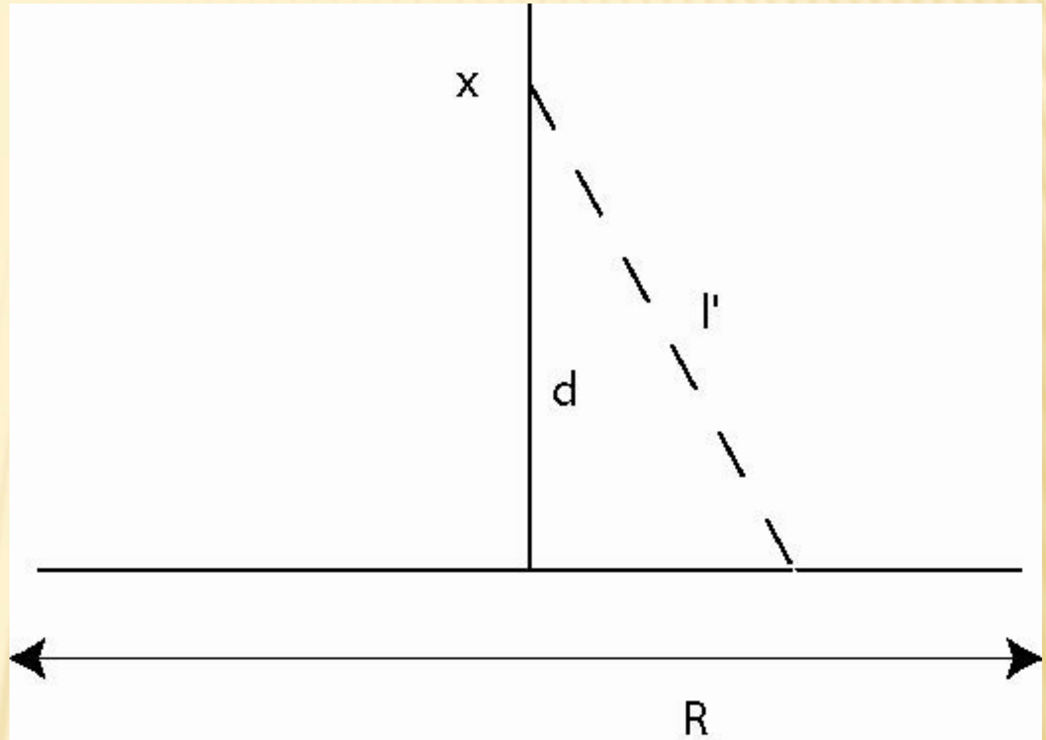
Let R go to infininty – problem – blows up

Fix (again) by adding appropriate constant.

Potential and force for a line

Add a constant

(no d dependence)



$$U(\vec{x}) = 2G\lambda \ln\left(\frac{R+\sqrt{R^2+4d^2}}{2d}\right) - 2G\lambda \ln(R)$$

$$U(\vec{x}) = 2G\lambda \ln\left(\frac{R+\sqrt{R^2+4d^2}}{2dR}\right)$$

Notice that this puts an R in the denominator to cancel the pesky R's in the numerator.

Potential and force for a line

Let R go to infininty again

$$U(\vec{x}) = 2G\lambda \ln\left(\frac{R+\sqrt{R^2+4d^2}}{2dR}\right) \xrightarrow{R\to\infty} 2G\lambda \ln\left(\frac{1}{d}\right) = -2G\lambda \ln(d)$$

And for g

$$\vec{g}(\vec{x}) = -\nabla U(\vec{x}) = -2G\lambda \nabla \ln(d)$$

$$\vec{g}(\vec{x}) = \frac{-2G\lambda}{d}\hat{d}$$

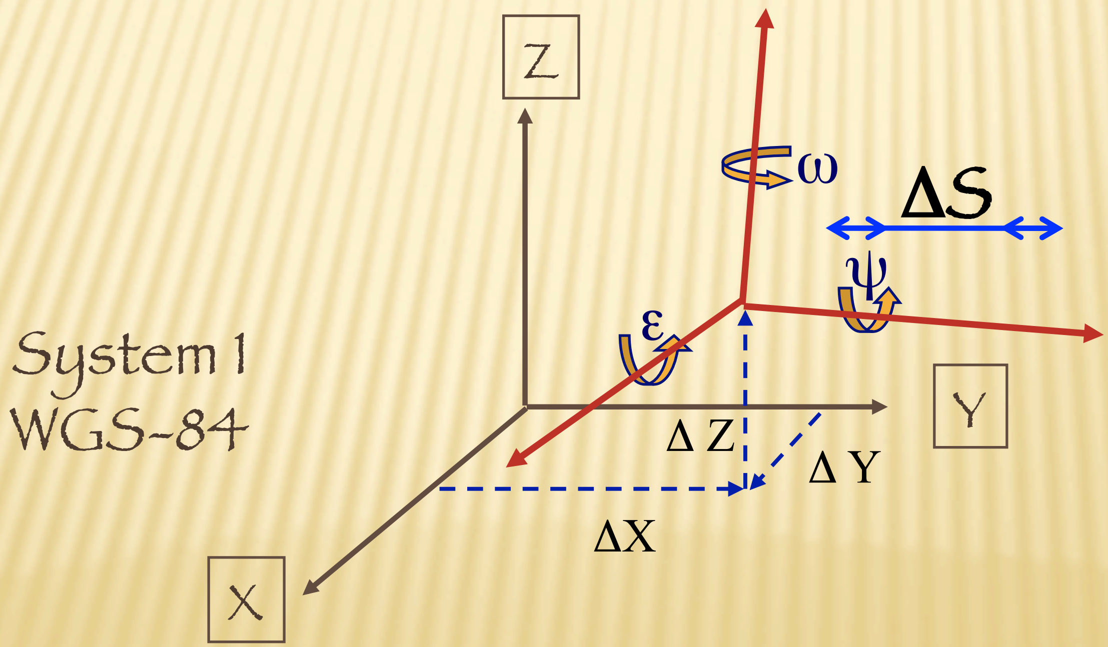
Notice that g is infinite on the line (not a problem – infinite line also not realizable)

Coordinate Systems

- New issues - Effects that need to be considered for accuracies that are achievable with GPS.
  - Coordinate systems on a deformable Earth.
  - Ability to determine polar motion and changes in the rotation rate of the Earth
  - Rotations and translations between coordinate systems.

Differences Between Horizontal Datums

- The two ellipsoid centers called Δ X, Δ Y, Δ Z
- The rotation about the X,Y, and Z axes in seconds of arc
- The difference in size between the two ellipsoids
- Scale Change of the Survey Control Network ΔS



This means, in practice, a given geographical position described as latitude and longitude but without a specified datum can actually indicate different physical locations on the earth. A physical location can have as many geographical positions as there are datums.

For example, the position of Hornby Light at South Head varies according to whether the Australian Geodetic Datum 1966 (AGD66) or the World Geodetic System 1984 datum (WGS84). The following diagram shows the WGS84 and AGD66 positions of Hornby Light on an extract of chart Aus 201, which is a WGS84 chart. The difference in positions represents a distance on the ground of 204 metres (~1 cable).

WGS84 33° 50.014' S 151° 16.860' E
ADG66 33° 50.109' S 151° 16.791' E

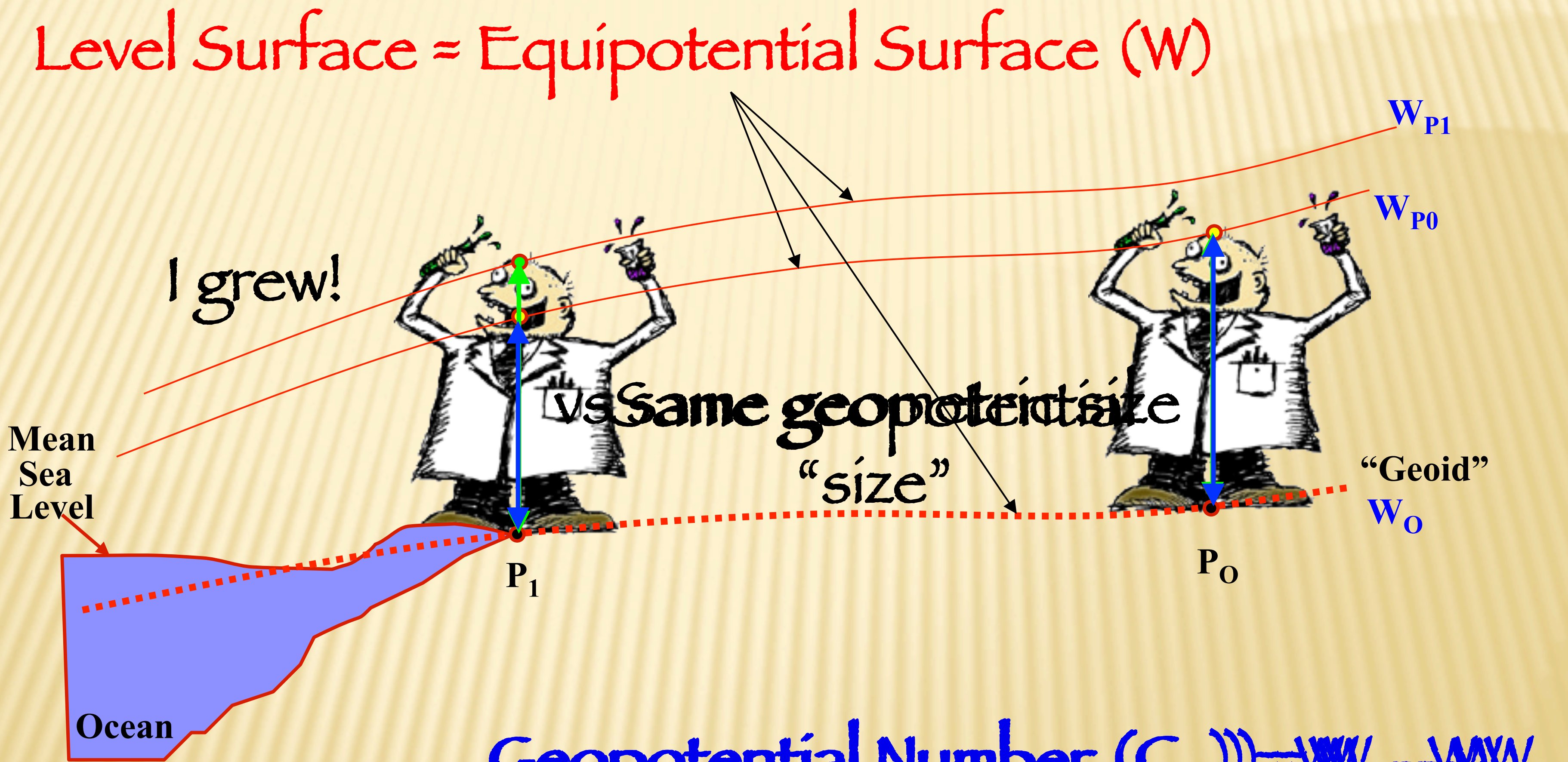
Fact sheet: Positions and horizontal datums on paper and electronic charts
Australian Maritime Safety Authority
http://www.amsa.gov.au/Shipping_Safety/Navigation_Safety/Positions_and_horizontal_datums_on_paper_and_electronic_charts/index.asp

Geometric vs Potential based coordinate systems

- The basic problem is "realization": Until distance measurements to earth-orbiting satellites and galactic-based distance measurements, it was not possible to actually implement (realize) the simple geometric type measurement system.

- But water can run up-hill!

Level Surfaces – Geopotential Number



Geopotential Number (C_P) = $W_P - W_O$

From Pearson, NGS, http://matcmadison.edu/civiltech/htmod/PowerPoint/HM-Primer_files/frame.htm

http://www.noeticart.com/clipart/madscic.gif

Geometric vs Potential based coordinate systems

- The origin of a potential based physical system was hard to define because determining the position of the center of mass of the Earth was difficult before the development of Earth-orbiting artificial satellites.
- The difference between astronomical (physical) and geodetic latitude and longitude is called "deflection of the vertical"

Geocentric relationship to XYZ

- One of the advantages of geocentric is that the relationship to XYZ is easy. R is taken to be radius of the sphere and H the height above this radius

Problems with Geocentric

- If the radius of the Earth is taken as b (the smallest radius), then H_c for a site at sea-level on the equator would be 21km (compare with Mt. Everest 28,000feet~8.5km).
- Geocentric quantities are never used in any large scale maps and geocentric heights are never used.

Relationship between ellipsoidal coordinates and XYZ.

This conversion is more complex than for the spherical case.

$$X = (N + h_g)\cos(\phi_g)\cos(\lambda_g)$$

$$Y = (N + h_g)\cos(\phi_g)\sin(\lambda_g)$$

$$Z = [(1 - e^2)N + h_g]\sin(\phi_g)$$

where $e^2 = 2f - f^2$ and N (North - South radius of curvature) is

$$N^2 = a^2 / [1 - e^2 \sin^2(\phi_g)]$$

Going from XYZ to geodetic latitude is more complex (mainly because to compute the radius of curvature, you need to know the latitude).

A common scheme is iterative:

$$\mapsto N' = a/\sqrt{1 - e^2 \sin\phi'}$$

$$r' = \sqrt{X^2 + Y^2}[1 - e^2 N'/(N' + h')]$$

$$\phi' = \tan^{-1}(Z/r')$$

$$h' = \sqrt{X^2 + Y^2}/\cos\phi' - N' \text{ or } h' = Z/\sin\phi' - (1 - e^2)N'$$

iterate to $\mapsto$ until h' change is small

Coordinate Conversion: Cartesian (ECEF X, Y, Z) and Geodetic (Latitude, Longitude, and Height)

Direct Solution for Latitude, Longitude, and Height from X, Y, Z

This conversion is not exact and provides centimeter accuracy for heights < 1,000 km
(See Bowring, B. 1976. Transformation from spatial to geographical coordinates. Survey Review, XXIII: pg. 323-327)

$$\phi = \operatorname{atan}\left(\frac{Z + e'^2 b \sin^3\theta}{p - e^2 a \cos^3\theta}\right)$$

$$\lambda = \operatorname{atan2}(Y, X)$$

$$h = \frac{p}{\cos(\phi)} - N(\phi)$$

where:

ϕ, λ, h = geodetic latitude, longitude, and height above ellipsoid

X, Y, Z = Earth Centered Earth Fixed Cartesian coordinates

and:

$$p = \sqrt{X^2 + Y^2} \qquad \theta = \operatorname{atan}\left(\frac{Za}{pb}\right) \qquad e'^2 = \frac{a^2 - b^2}{b^2}$$

$N(\phi) = a / \sqrt{1 - e^2 \sin^2\phi}$ = radius of curvature in prime vertical

a = semi-major earth axis (ellipsoid equatorial radius)

b = semi-minor earth axis (ellipsoid polar radius)

$f = \frac{a - b}{a}$ = flattening

$e^2 = 2f - f^2$ = eccentricity squared

Closed form expression for small heights

From http://www.colorado.edu/geography/gcraft/notes/datum/gif/xyzllh.gif

Astronomical latitude and longitude

- There is no direct relationship between XYZ and astronomical latitude and longitude because of the complex shape of the Earth's equipotential surface.
- In theory, multiple places could have the same astronomical latitude and longitude.

Coordinate axes directions

- Origin of XYZ system these days is near center of mass of Earth (deduced from gravity field determined from orbits of geodetic satellites).

- Direction of Z-axis by convention is near mean location of rotation axis between 1900-1905.

At the time, it was approximately aligned with the maximum moments of inertia of the Earth.

review:

http://dept.physics.upenn.edu/courses/gladney/mathphys/java/sect4/subsubsection4_1_4_2.html

Motion of rotation axis

- rotation axis has moved ~10 m since 1900 (thought to be due to post-glacial rebound).
- It also moves in circle with a 10 m diameter with two strong periods: Annual due to atmospheric mass movements and 433-days which is a natural resonance frequency of an elastic rotating ellipsoid with a fluid core like the Earth.

Problems with ellipsoid and ellipsoidal heights are:

- They are new:
  - Geometric latitude and longitude have been around since Snell (optical refraction) developed triangulation in the 1500's.
  - Ellipsoidal heights could only be easily determined when GPS developed (1980's)

- Fluids flow based on the shape of the equipotential surfaces. If you want water to flow down hill, you need to use potential based heights.

Geoid height

- Difference between ellipsoidal and orthometric height allows geoid height to be determined, but can only do since GPS available.
  - Determining the geoid has been historically done using surface gravity measurements and satellite orbits.

(Satellite orbit perturbations reveal the forces acting on the satellite, which if gravity is the only effect is the first derivative of the potential [atmospheric drag and other forces can greatly effect this assumption])

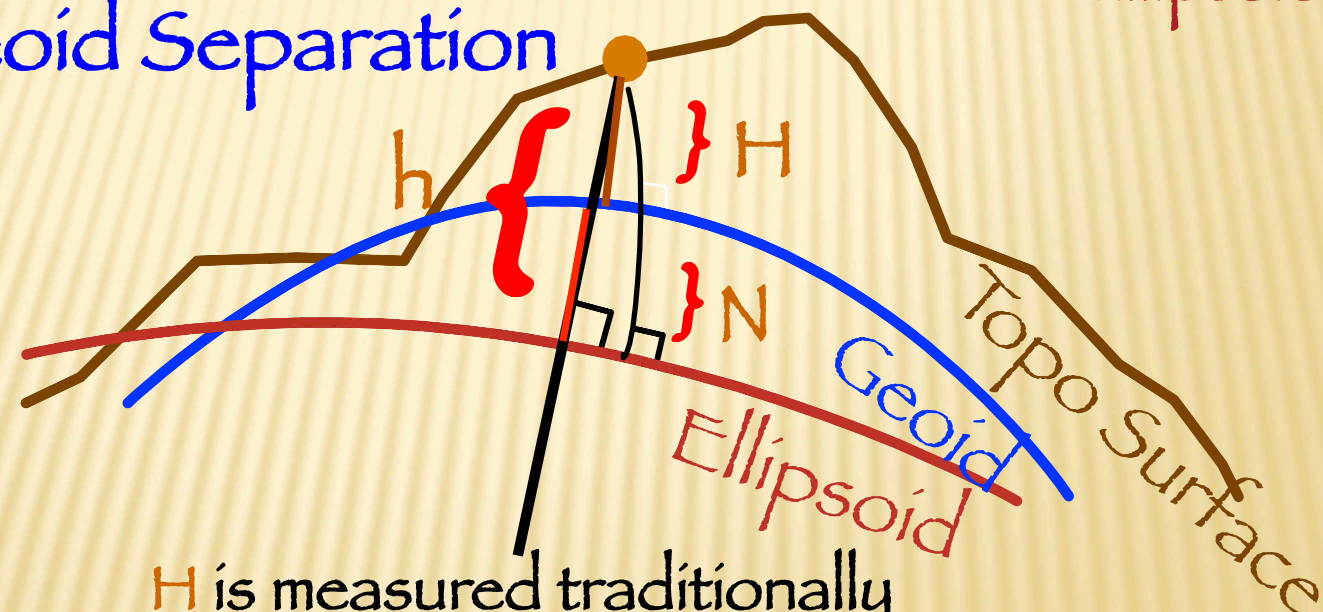
VERTICAL DATUMS

Defining the Vertical Position

H – Orthometric Height
(Height above Mean Sea Level)

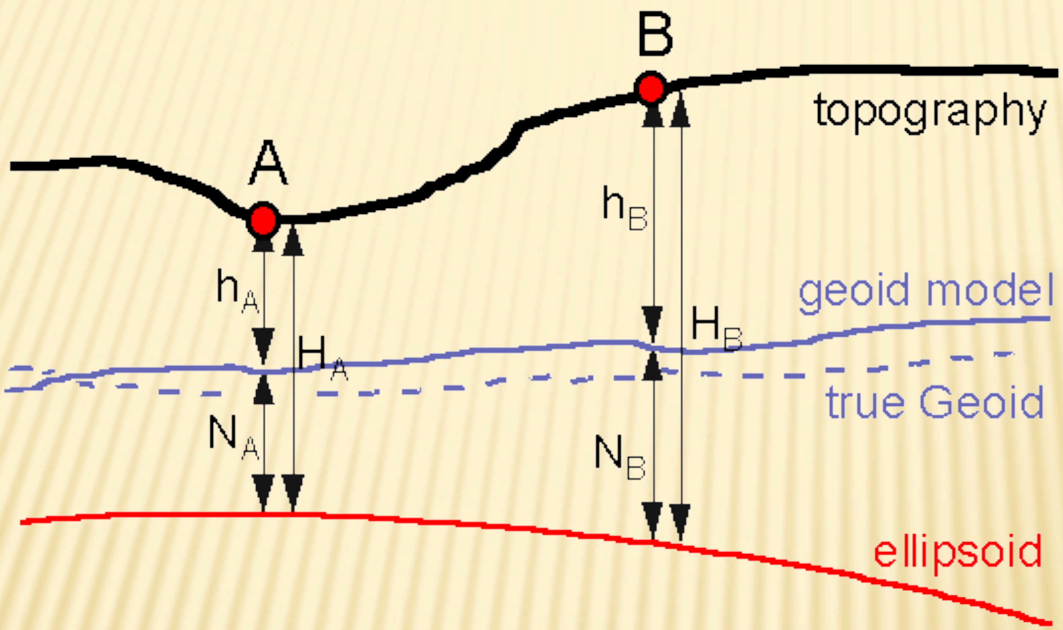
h – Geodetic Height
(Height above Ellipsoid)

N – Geoid Separation



H is measured traditionally
h is approximately ≈ N+H
N is modeled using Earth Geoid Model 96 or 180

Datums and Grids -- https://www.navigator.navy.mil/navigator/wgs84_0.ppt



Sam Wormley
http://www.edu-observatory.org/gps/height.html


JOHANNES KEPLER'S UPHILL BATTLE
...SO, YOU SEE THE ORBIT OF A PLANET IS ELLIPTICAL.
WHAT'S AN ORBIT?
WHAT'S A PLANET?
WHAT'S 'ELLIPTICAL'?
S. Harris

THE GLOBAL POSITIONING SYSTEM (GPS)

A. Ganse, U. Washington , http://staff.washington.edu/aganse/

1. Basic Concepts of GPS.

Space/Control/User segment
GPS measurement characteristics
selective availability (SA), antispoofing (AS)
Satellite Orbits
GPS force and measurement models for orbit determination
tracking networks
GPS broadcast ephemeris, precise GPS ephemeris.
Reference Systems
Transformation between Keplerian Elements & Cartesian Coordinates
time system and time transfer using GPS

2. GPS Observable.

Measurement types (C/A code, P-Code, L1 and L2 frequencies, pseudoranges)

atmospheric delays (tropospheric and ionospheric)

data combination (narrow/wide lane combinations, ionosphere-free combinations, single-, double-, triple-differences)

integer biases
cycle slips
clock error.

3. Processing Techniques.

Pseudorange and carrier phase processing
ambiguity removal

least squares method for state parameter determination
relative positioning

4. Earth Science GPS Applications.

Surveying
Geophysics
Geodesy
Active tectonics
Tectonic modeling
meteorological and climate research
Geoid

- Coordinate and time systems:
  - When working at the millimeter level globally, how do you define a coordinate system
  - What does latitude, longitude, and height really mean at this accuracy
  - Light propagates 30 cm in 1 nano-second, how is time defined

(Herring)

- Satellite motions
  - How are satellite orbits described and how do the satellites move
  - What forces effect the motions of satellites

  (i.e What do GPS satellite motions look like)

  - Where do you obtain GPS satellite orbits

(Herring)

- GPS observables
  - GPS signal structure and its uniqueness
  - Pseudo-range measurements
  - Carrier phase measurements
  - Initial phase ambiguities
  - Effects of GPS security: Selective availability (SA) and antispoofing (AS)
  - Data formats (RINEX)

(Herring)

- Estimation procedures
  - Simple weighted-least-squares estimation
  - Stochastic descriptions of random variables and parameters
  - Kalman filtering
  - Statistics in estimation procedures
  - Propagation of variance-covariance information

(Herring)

- Propagation medium
  - Neutral (electrically) atmosphere delay
  - Hydrostatic and water vapor contributions
  - Ionospheric delay (dispersive)
  - Multipath

(Herring)

- Mathematatic models in GPS
  - Basic theory of contributions that need be to included for millimeter level global positioning
  - Use of differenced data
  - Combinations of observables for different purposes

(Herring)

- Methods of processing GPS data
  - Available software
  - Available data (International GPS service, IGS; University NAVSTAR Consortium (UNAVCO) Facility.

  - Cycle slip detection and repair
  - Relationship between satellite based and conventional geodetic systems (revisit since this is an important topic)

(Herring)

- Applications and examples from GPS
  - Tectonic motions and continuous time series.
  - Earth rotation variations; measurement and origin.
  - Response of earth to loading.
  - Kinematic GPS; aircraft and moving vehicles.
  - Atmospheric delay studies.

(Herring)

The Global Positioning System (GPS)

What is it?

Conceived as a positioning, navigation and time transfer system for the US military

The Global Positioning System (GPS)

GPS is one of the most fantastic utilities ever devised by man.

GPS will figure in history alongside the development of the sea-going chronometer.

This device enabled seafarers to plot their course to an accuracy that greatly encouraged maritime activity, and led to the migration explosion of the nineteenth century.

http://www.ja-gps.com.au/whatisgps.html

The Global Positioning System (GPS)

GPS will effect mankind in the same way.

There are myriad applications, that will benefit us individually and collectively.

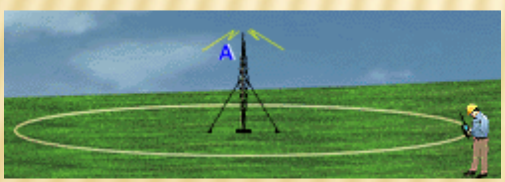
http://www.ja-gps.com.au/whatisgps.html

Trimble calls GPS the "next utility"

Brief History of Navigation

Stone age

Star age



Radio age

Satellite age

http://www.javad.com/index.html?/jns/gpstutorial/

BRIEF HISTORY OF NAVIGATION

- PreHistory - Present: Celestial Navigation
  - Ok for latitude, poor for longitude until accurate clock invented ~1760
- 13th Century: Magnetic Compass
- 1907: Gyrocompass
- 1912: Radio Direction Finding
- 1930's: Radar and Inertial Navigation
- 1940's: Loran-A
- 1960's: Omega and Navy Transit Doppler (SatNav)
- 1970's: Loran-C
- 1980's: GPS

Radio Navigation

Radio (AN) Ranges

NDB

VOR plus TACAN-DME, Localizer and ILS.

OMEGA, LORAN

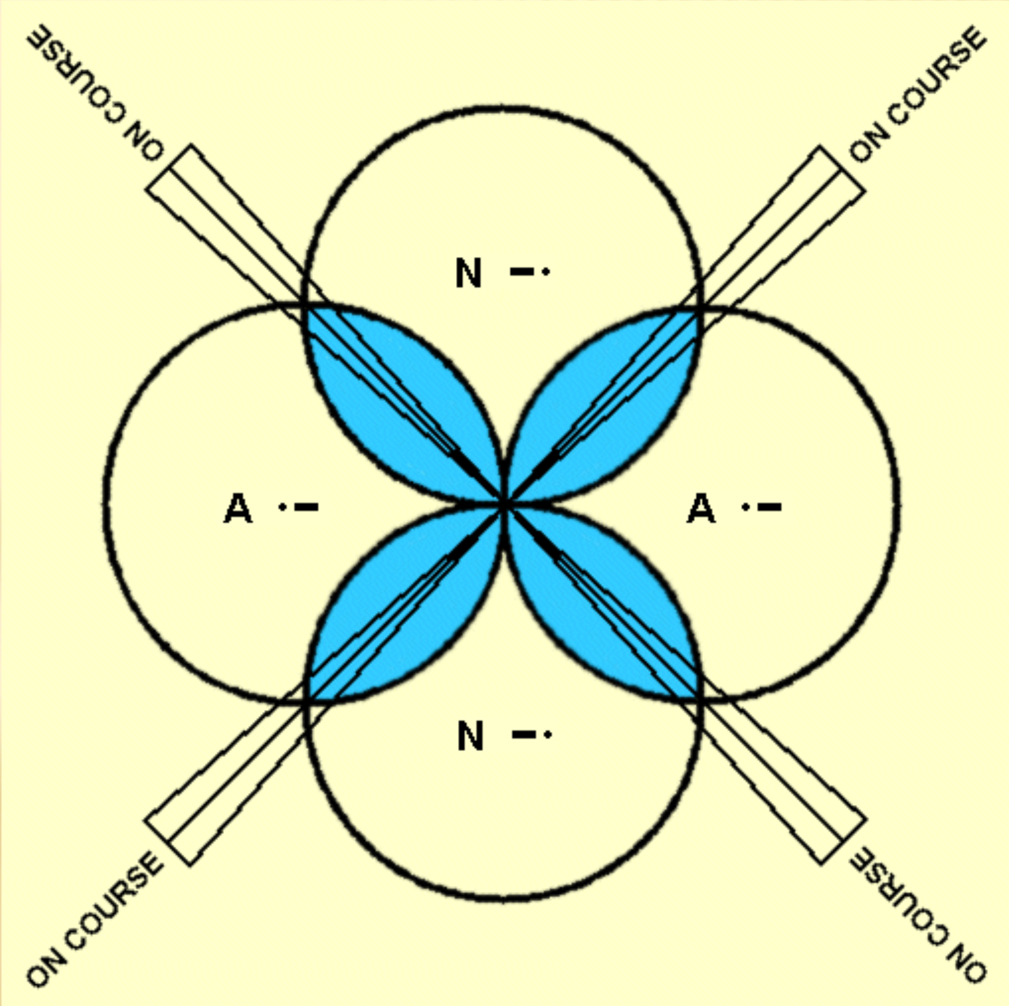
Doppler

Radio Navigation

Radio (AN) Ranges

Build a network of these all over

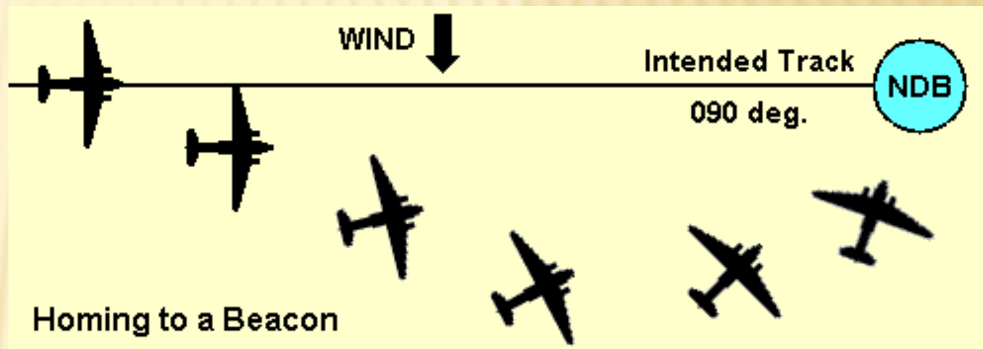
2-D only



Radio Navigation

NDB

Build a network of these all over



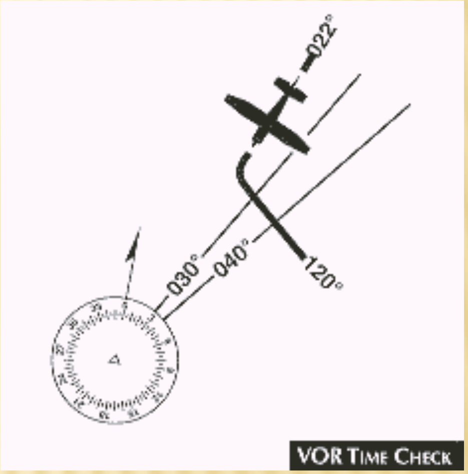
2-D only

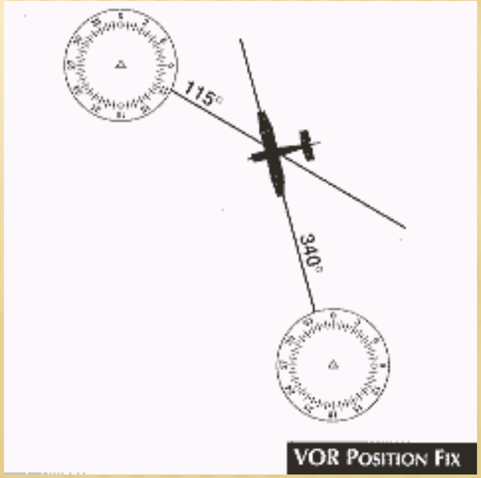
Radio Navigation

VOR plus TACAN-DME, Localizer and ILS.

Build a network of these all over

2-D only



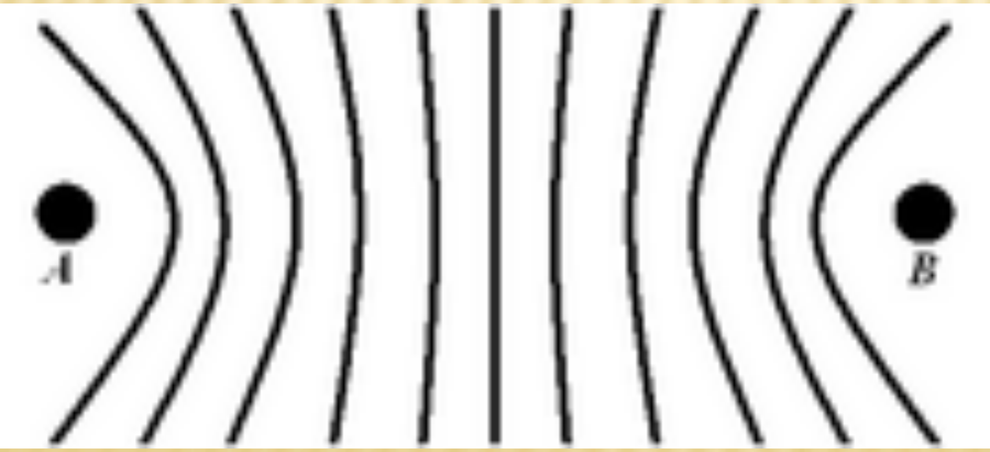


Radio Navigation

LORAN

LORAN (LOng RAnge Navigation)
5% surface, not global
250 m
Transmitters on surface – gives 2D, not 3D location

Uses difference of arrival time from Master and several slave transmitters

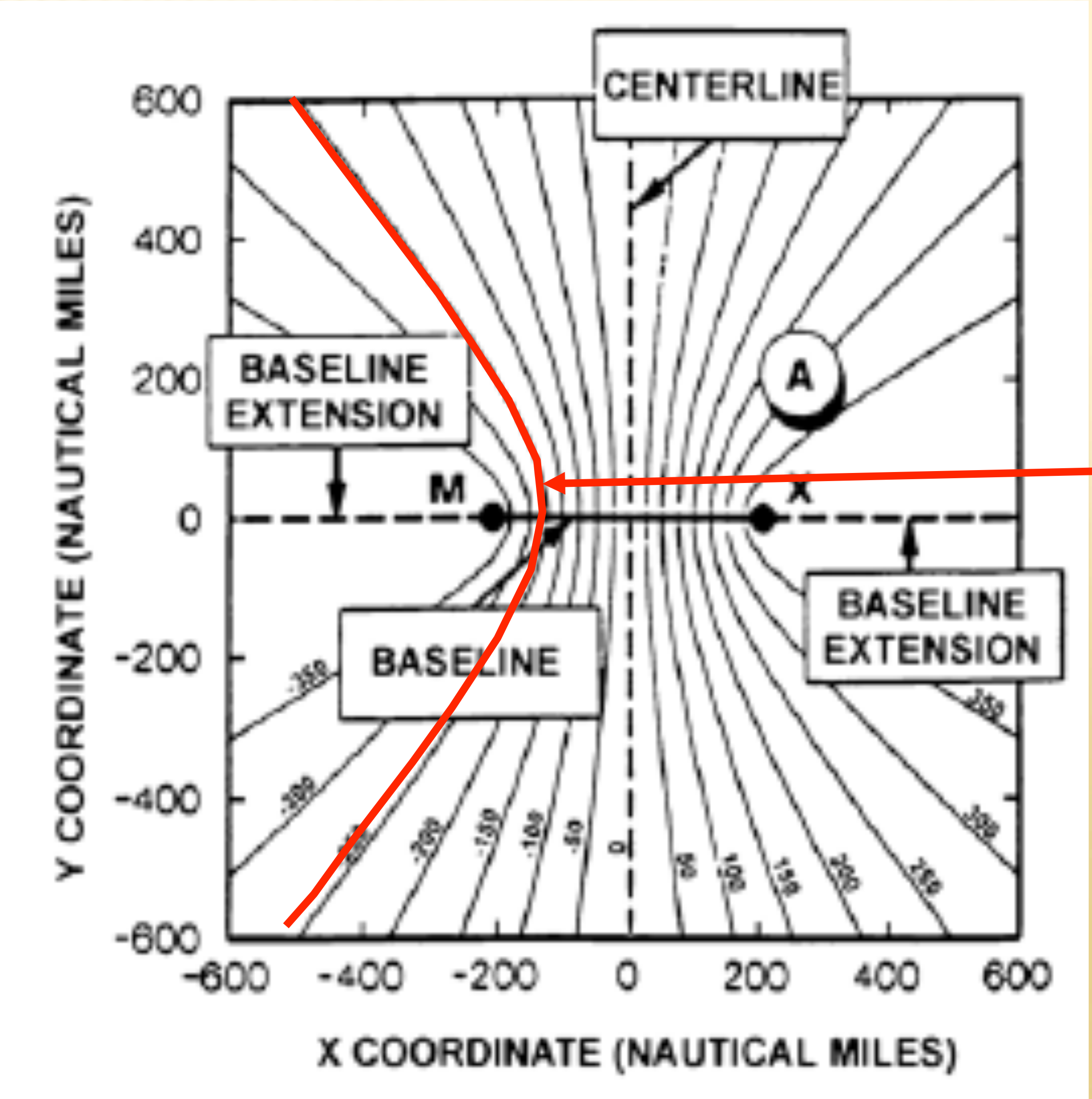


(like using s-p times to locate earthquakes)

Build a network of these all over

Radio Navigation

LORAN



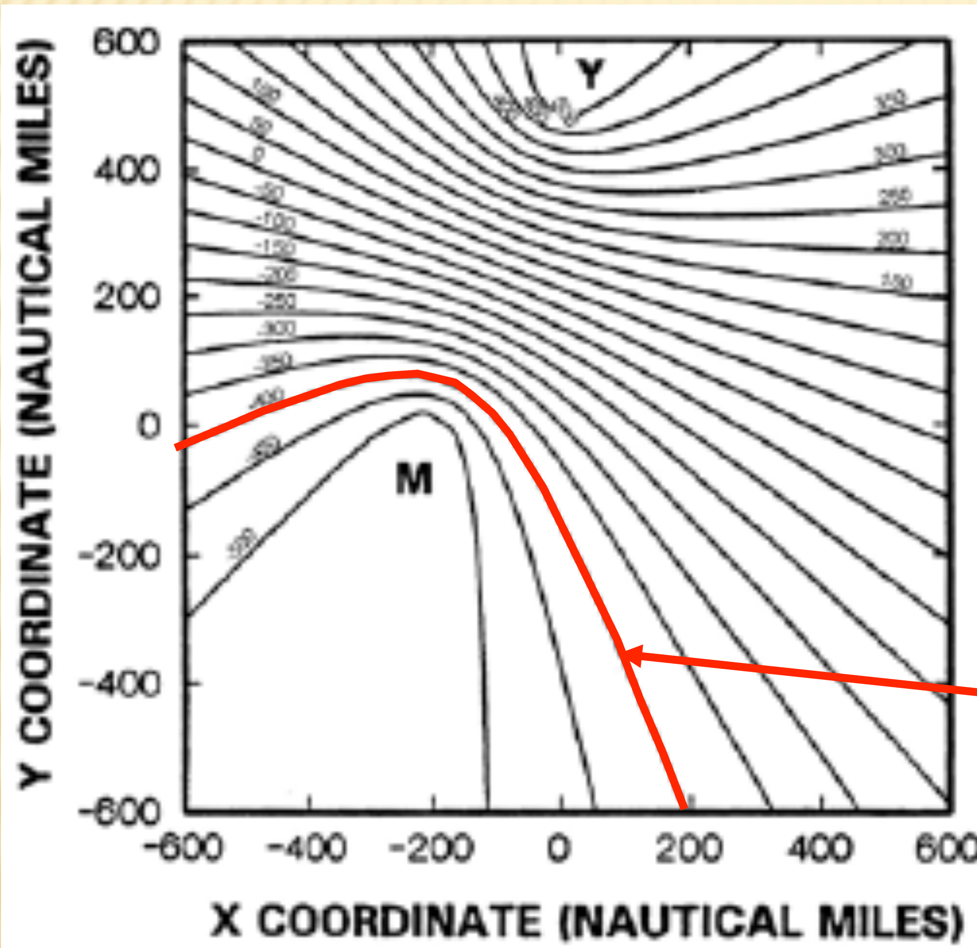
Time differences of signal from Master (M) and Slave (X) give hyperbolas

For given time difference (TD) you are on one of them

Called Line of Position (LOP)

Radio Navigation

LORAN



Locating yourself with LORAN

M – Master (same one)

Y – another secondary

TD puts you somewhere on LOP between these two stations

Radio Navigation

LORAN

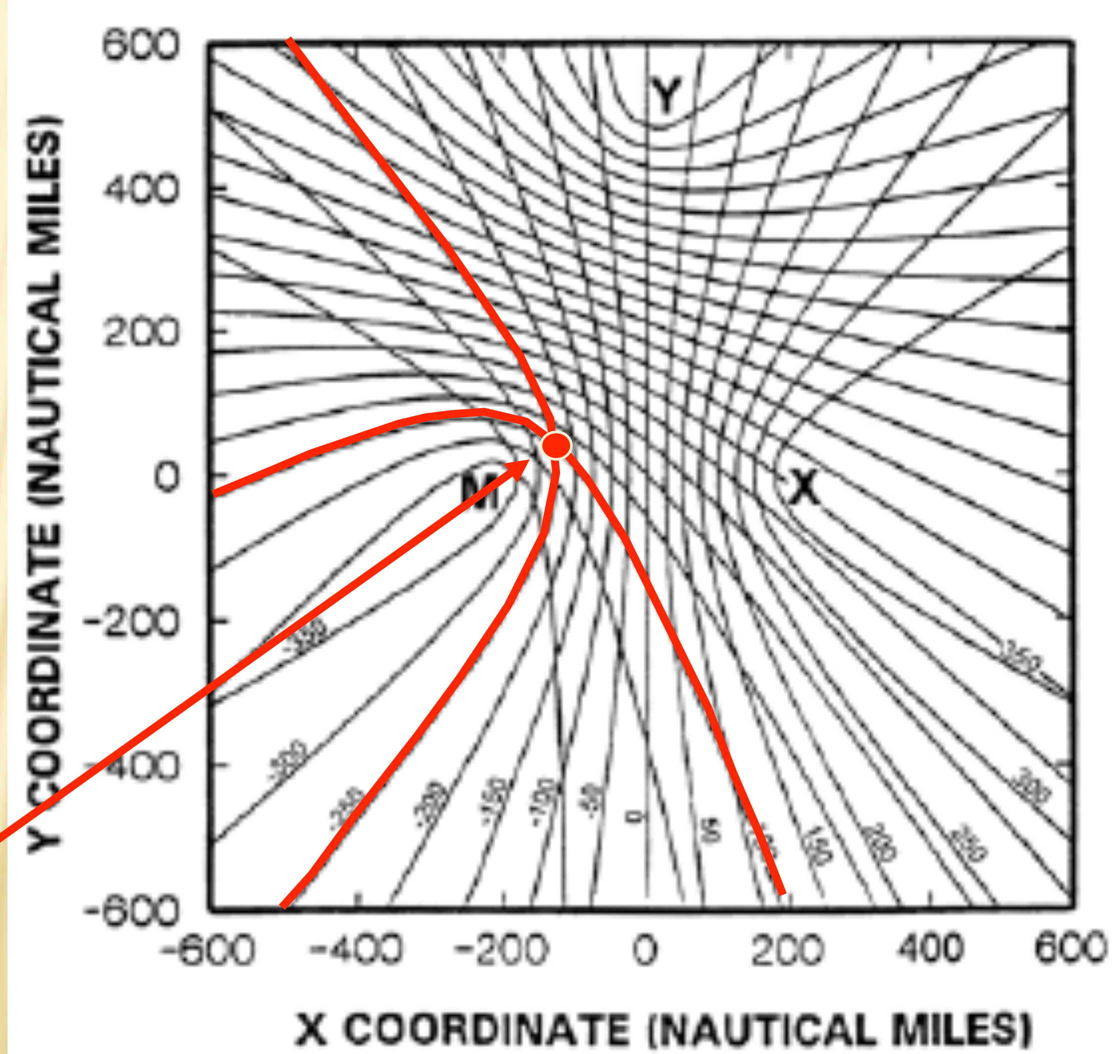
Locating yourself with LORAN

Combine

M – Master (same one)

X and Y – secondaries

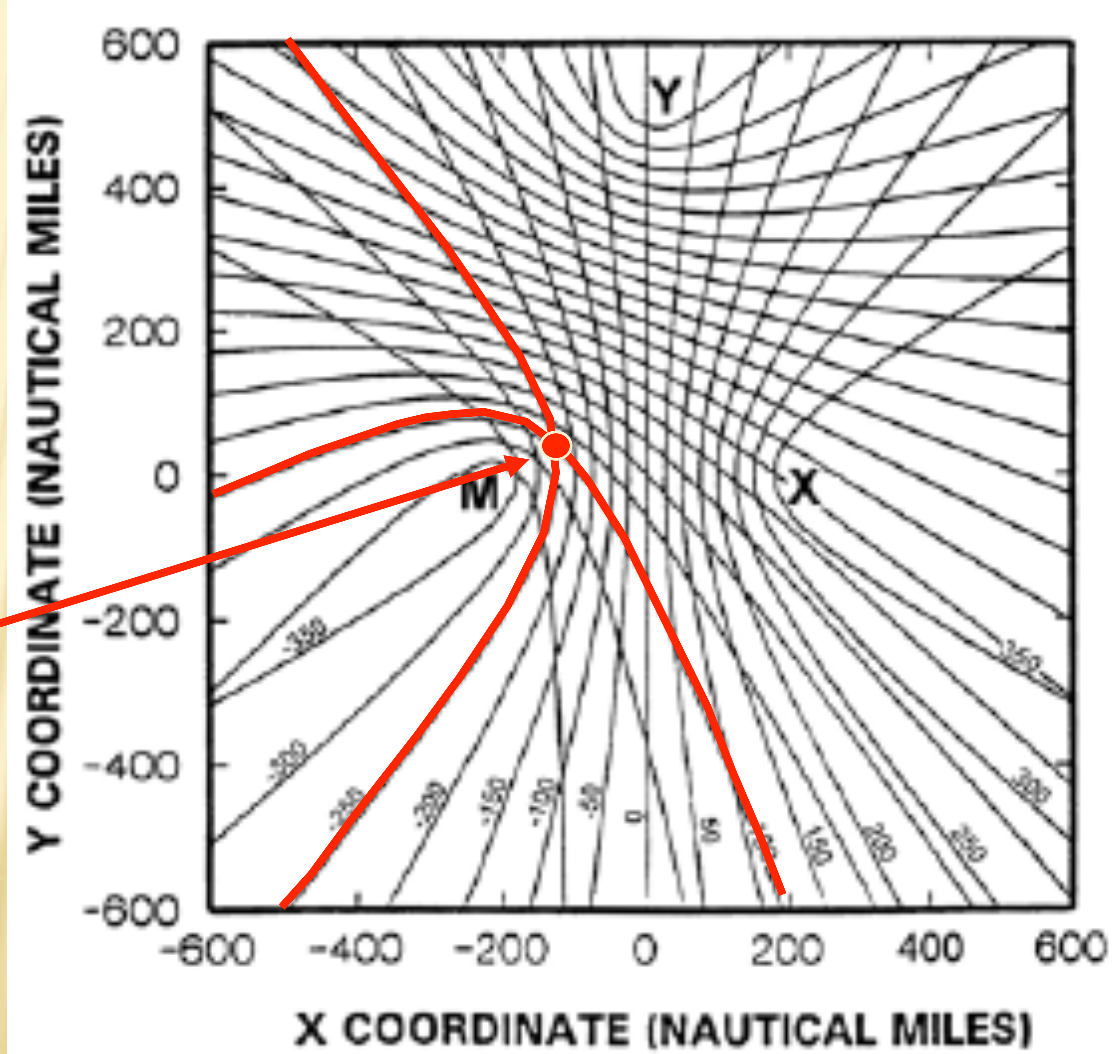
You are at intersection of LOPs



Radio Navigation

LORAN

Locating yourself with LORAN



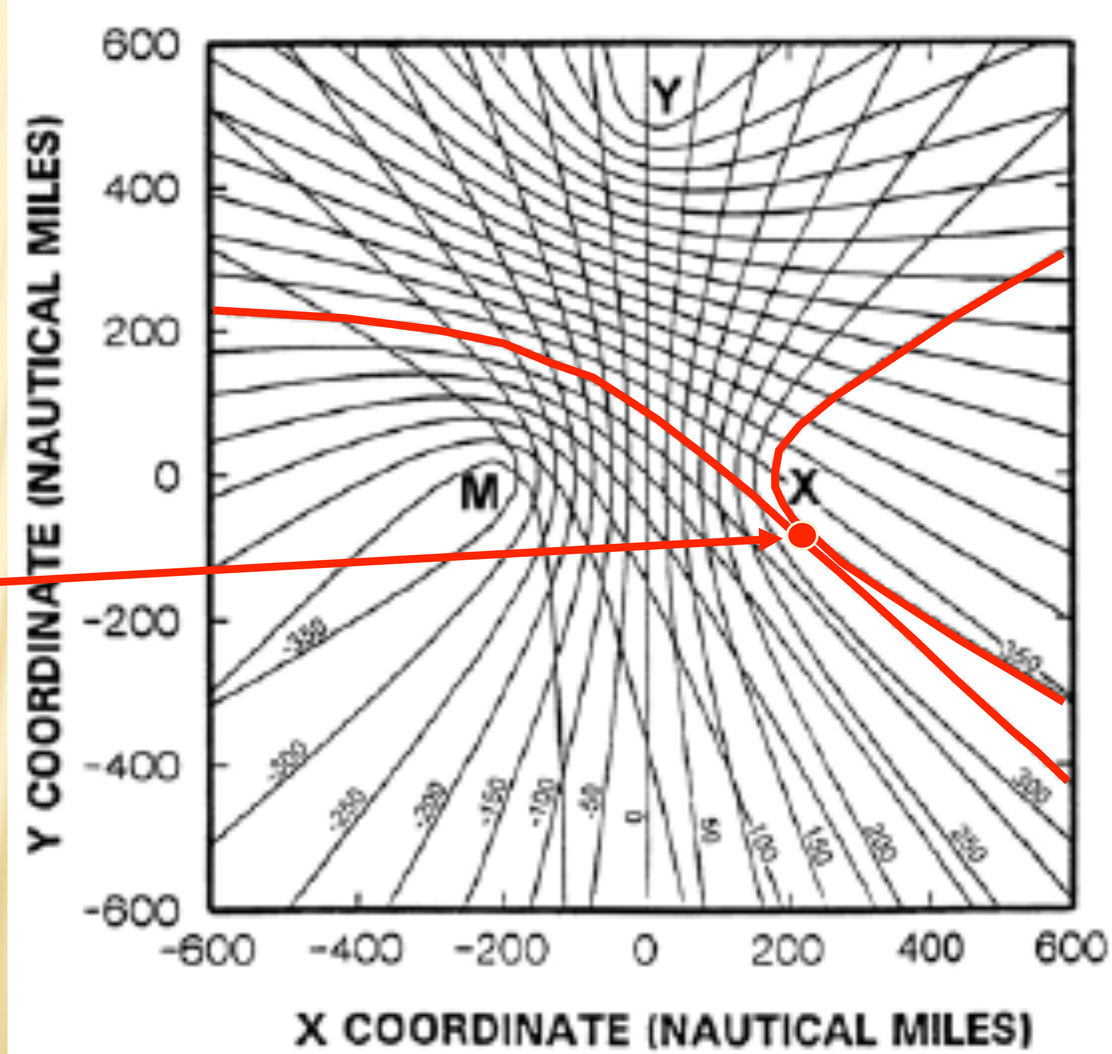
Good location when the 2 LOP are perpendicular (or close to it)

Radio Navigation

LORAN

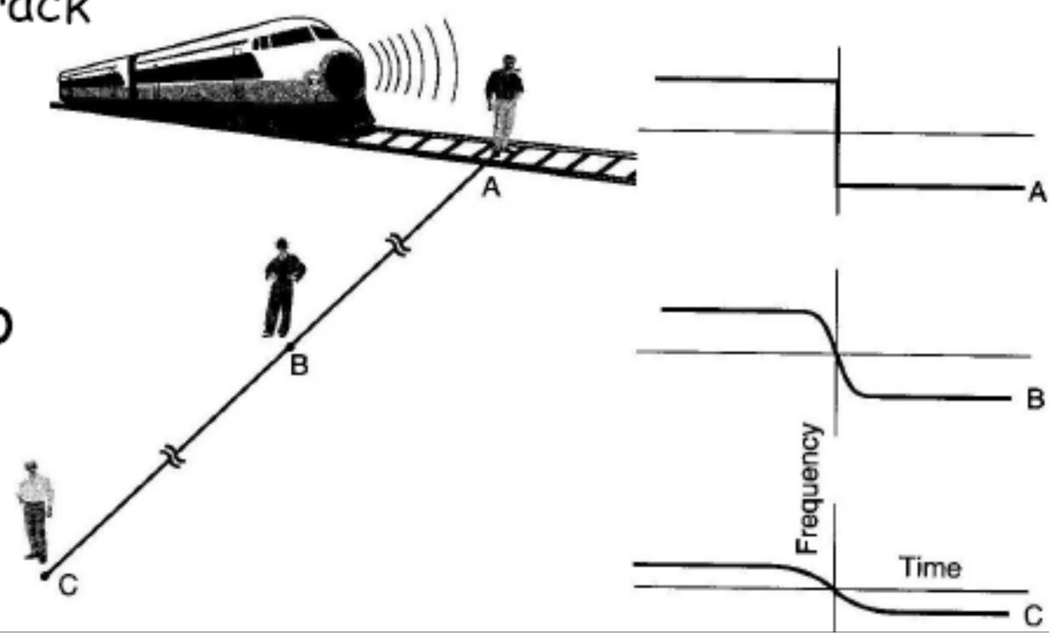
Locating yourself with LORAN

Problem when the two LOPs cross at small angle or are tangent.



Doppler Positioning

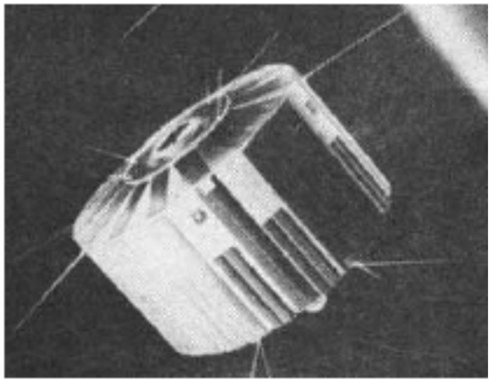
- All familiar with the change in tone of a train's whistle as it speeds past
  - More cycles received than transmitted due to the shrinking distance between train and person
    - Pitch seems higher
  - Shape of **Doppler Curve** is function of **range rate**
    - Gives distance from track



- With train schedule and watch, could also do 2D positioning

Useful Doppler Ranging

- Doppler ranging is a good way to do 2D ranging, but how often is a train around?
- DoD (USN) replaced train with a satellite
  - Satellite orbit very predictable (as if "on rails")
  - Each satellite pass gives 2D position
    - Wait for further passes to get more 2D position updates
    - Combine to get 3D estimate



- Basis of DoD **Transit System (1961)**
  - 4—7 satellites in LEO, updates every 100mins
  - Used by submarines to obtain 3D updates ~10-25m accuracy
  - But limited satellite coverage and low accuracy

From J. HOW, MIT

Disadvantages of other navigation systems

Landmarks:

Only work in local area.
Subject to movement or destruction by environmental factors.

Disadvantages of other navigation systems

Dead Reckoning:

Very complicated.
Accuracy depends on measurement tools which are usually relatively crude.
Errors accumulate quickly.

(actually is from "deduced" reckoning and should be "ded"-reckoning

.

Not from "you're dead if you don't reckon right ")

Disadvantages of other navigation systems

Celestial:

Complicated.
Only works at night in good weather.
Limited precision.

Disadvantages of other navigation systems

OMEGA:

Based on relatively few radio direction beacons.
Accuracy limited and subject to radio interference.

LORAN:

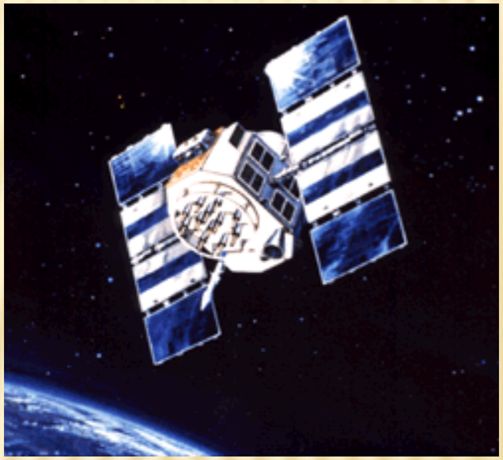
Limited coverage.
Accuracy variable, affected by geographic and weather situation.
Easy to jam or disturb.

Disadvantages of other navigation systems

SatNav (Transit doppler):

Based on low-frequency doppler measurements so it's sensitive to small movements at receiver.
Few satellites so updates are infrequent.

FINALLY GPS



BRIEF HISTORY OF GPS

- Original concept developed around 1960
  - In the wake of Sputnik & Explorer

- Preliminary system, *Transit* (doppler based), operational in 1964
  - Developed for nuke submarines
  - 5 polar-orbiting satellites, Doppler measurements only

- *Timation* satellites, 1967-69, used the first onboard precise clock for passive ranging

A. Ganse, U. Washington , http://staff.washington.edu/aganse/

BRIEF HISTORY OF GPS

- Fullscale GPS development begun in 1973
  - Renamed *Navstar*, but name never caught on
- First 4 SV's launched in 1978
- GPS IOC in December 1993 (FOC in April 1995)

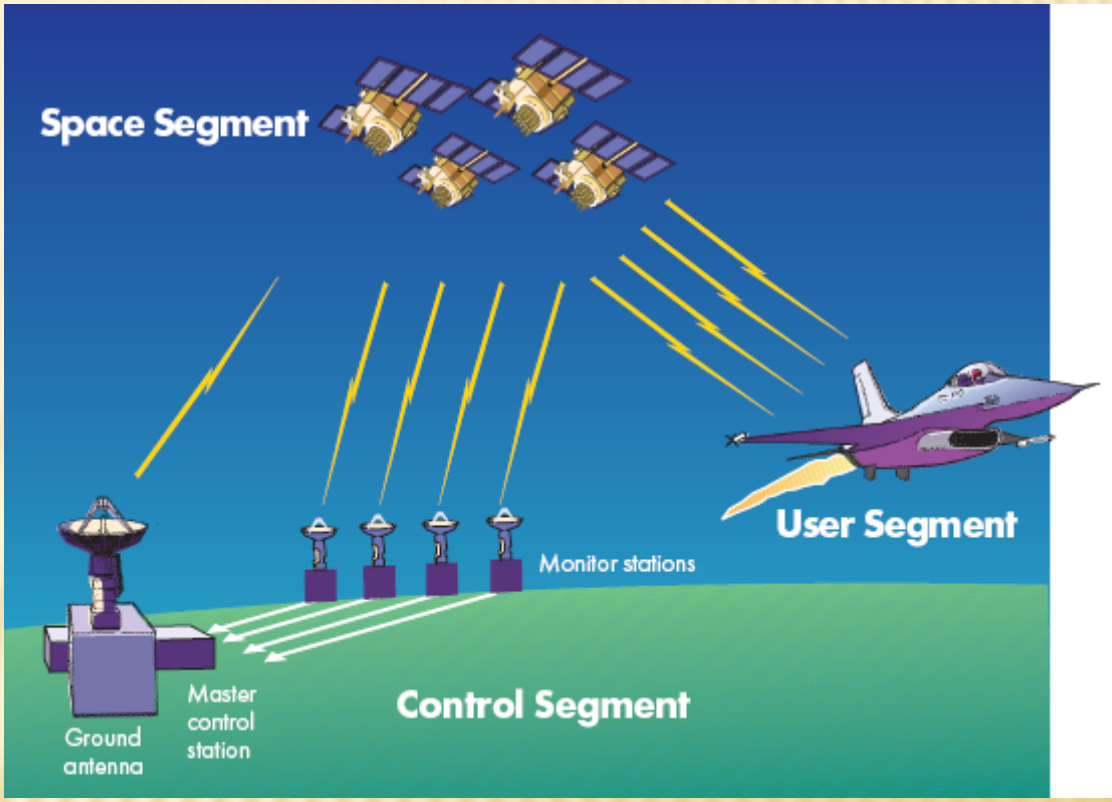
GPS

- Transit improved by increasing constellation size + improving accuracy → **GPS NAVSTAR**
  - Navigation System with Timing And Ranging
- GPS enabled by:
  - Stable platforms in predictable orbits
    - So station locations are highly predictable
  - Ultra-stable clocks onboard
    - Time-synchronization feasible → can use trilateration
  - Spread spectrum signaling
    - Use CDMA to access multiple transmitters at once
  - Low-cost integrated circuits
    - Trades cheaper receiver clock for more processing

GPS TIDBITS

- Development costs estimate ~$12 billion
- Annual operating cost ~$400 million

GPS Tidbits



- 3 Segments:
  - Space: Satellites
  - User: Receivers
  - Control: Monitor & Control stations

A. Ganse, U. Washington , http://staff.washington.edu/aganse/

GPS TIDBITS

- Prime Space Segment contractor: Rockwell International

- Coordinate Reference: WGS-84 ECEF

- Operated by US Air Force Space Command (AFSC)
  - Mission control center operations at Schriever (formerly Falcon) AFB, Colorado Springs

WHO USES IT?

- Everyone!
- Merchant, Navy, Coast Guard vessels
  - Forget about the sextant, Loran, etc.
- Commercial Airliners, Civil Pilots
- Surveyors
  - Has completely revolutionized surveying
- Commercial Truckers
- Hikers, Mountain Climbers, Backpackers
- Cars now being equipped
- Communications and Imaging Satellites
  - Space-to-Space Navigation
- Any system requiring accurate timing

WHO USES IT?

- GEOPHYSICISTS and GEODESISTS

(not even mentioned by Ganse!)

Basics of Navigation

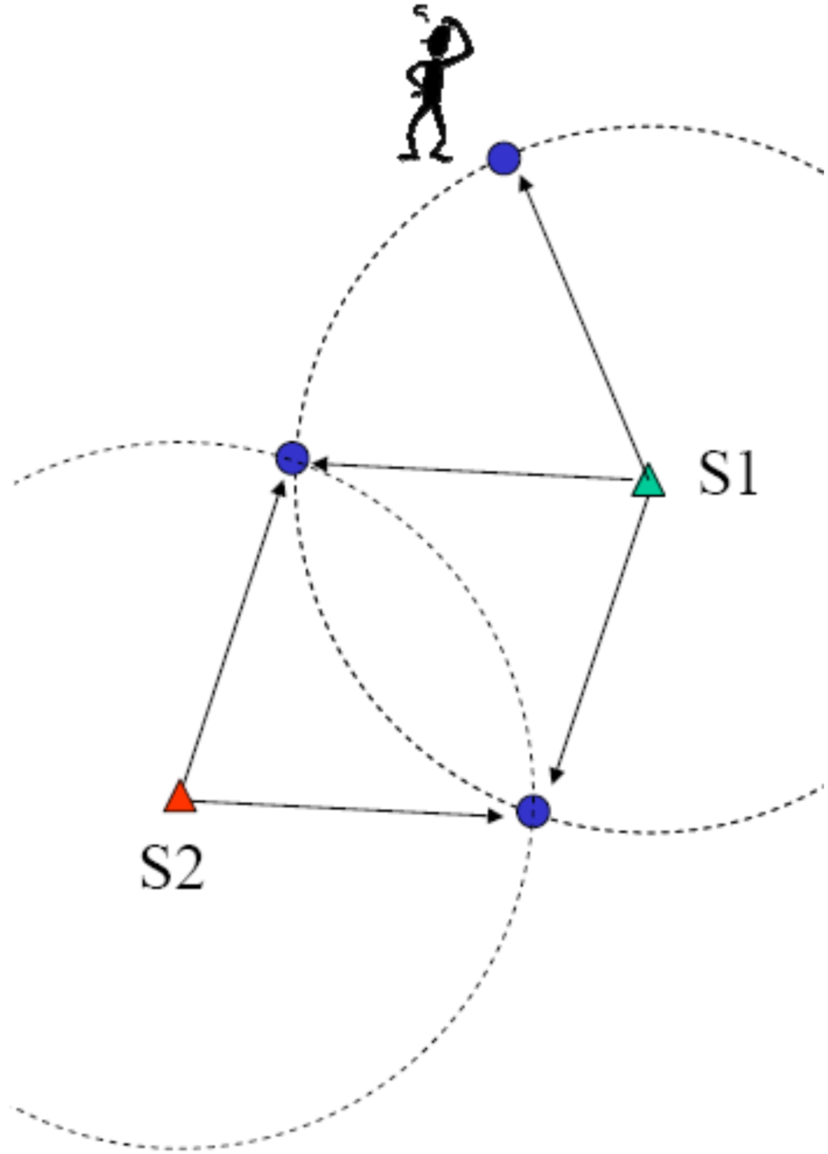
- Goal: determine our location relative to a known coordinate frame

- Basic approach:
  - Ultrasonic/laser distance meter
    - Range: 0.1 ft - 330ft (0.3m - 100m)
    - Accuracy: ±0.2in (5mm)
  - Measure **range** r_k to **known stations** (x_k, y_k) in the specified coordinate frame
  - In 2D: $r_k = \sqrt{(x_k - x)^2 + (y_k - y)^2} + v_k$

From J. HOW, MIT

2D Example Continued

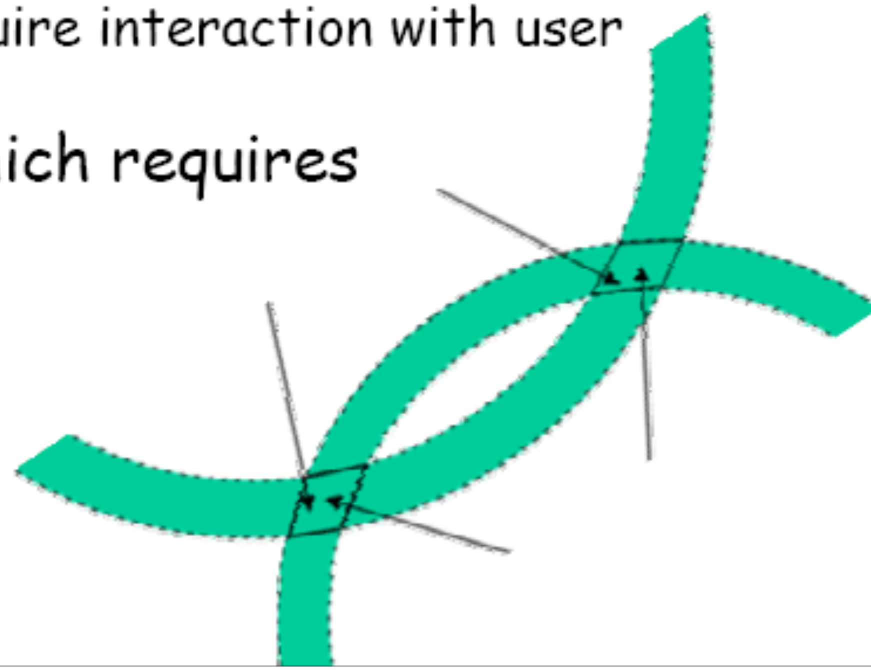
- Ranging to 1 station places us anywhere on a circle
- Ranging to 2 stations reduces uncertainty to only 2 points
- Could use a 3rd station to determine unique position estimate



From J. HOW, MIT

Basic Navigation

- Process called *trilateration* (triangulation based on circular constraints from the range)
  - Extends to 3D relatively easily, but requires stations "out of the plane" → explains why terrestrial navigation in 2D
- Describes an **active system**. Could use **passive system**
  - Transmitters at stations, user just receives a signal
  - Ideal for military, doesn't require interaction with user
- ***Time-of-Arrival*** system, which requires tight clock synchronization
  - 1μs error → 300m error
  - Better clock → higher cost



From J. HOW, MIT

GPS Timing

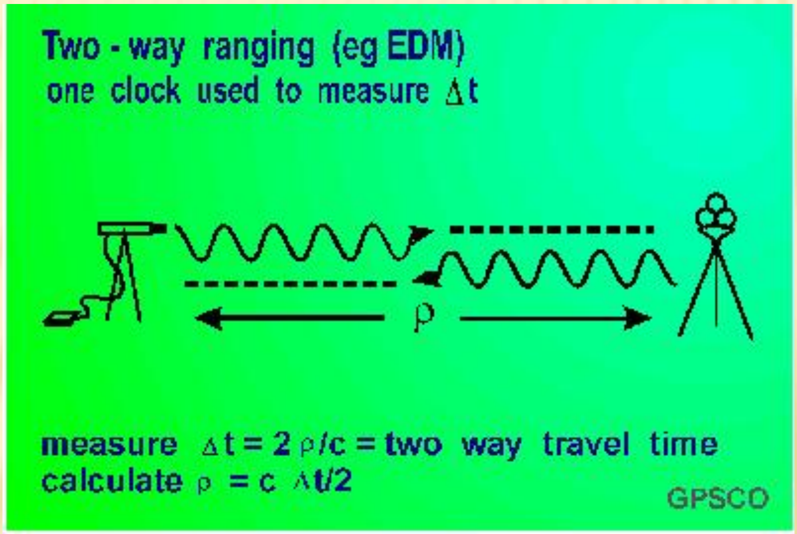
- Trilateration requires synchronized clocks:

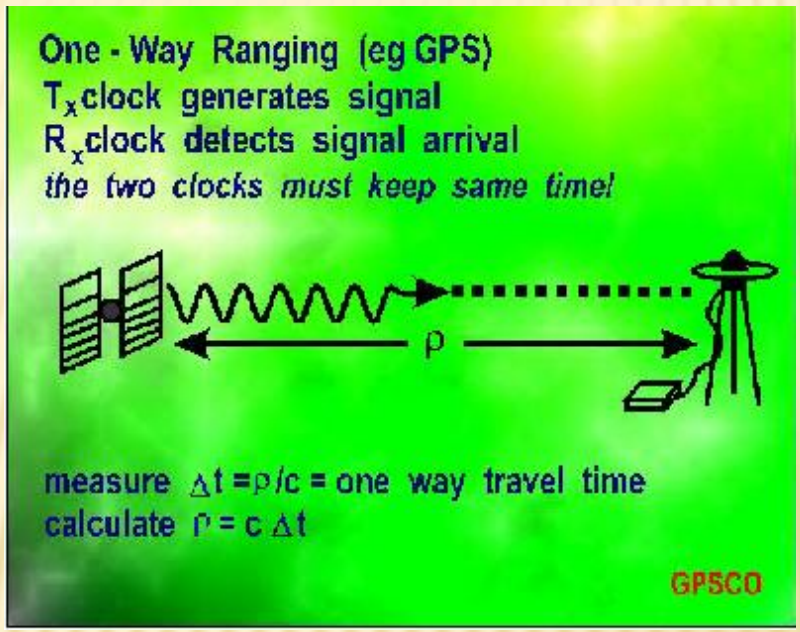
(Measured time of arrival) - (Time sent) = (Time of flight) ➔ Range

- But requiring global synchronization of user & transmitter clocks would be very **cumbersome**
  - So GPS satellites use 4 oven-controlled atomic clocks
  - And to reduce cost, receiver uses low-cost crystal oscillator
    - Adds ***user clock error*** that is common to all measurements

- GPS design:
  - Transmitters are synchronized
  - They broadcast the time that the signals were sent

➔ User clock error is a fourth variable that we must solve for in real-time (need 4 measurements)


Two - way ranging (eg EDM)
one clock used to measure Δt
ρ
measure Δt = 2 ρ/c = two way travel time
calculate ρ = c Δt/2
GPSCO


One - Way Ranging (eg GPS)
T_x clock generates signal
R_x clock detects signal arrival
the two clocks must keep same time!
ρ
measure $\Delta t = \rho / c$ = one way travel time
calculate $\rho = c \, \Delta t$
GPSCO

Advantages of One-Way Ranging

Receiver doesn't have to generate signal, which means

- We can build inexpensive portable receivers
- Receiver cannot be located (targeted)
- Receiver cannot be charged

http://www.geology.buffalo.edu/courses/gly560/Lectures/GPS

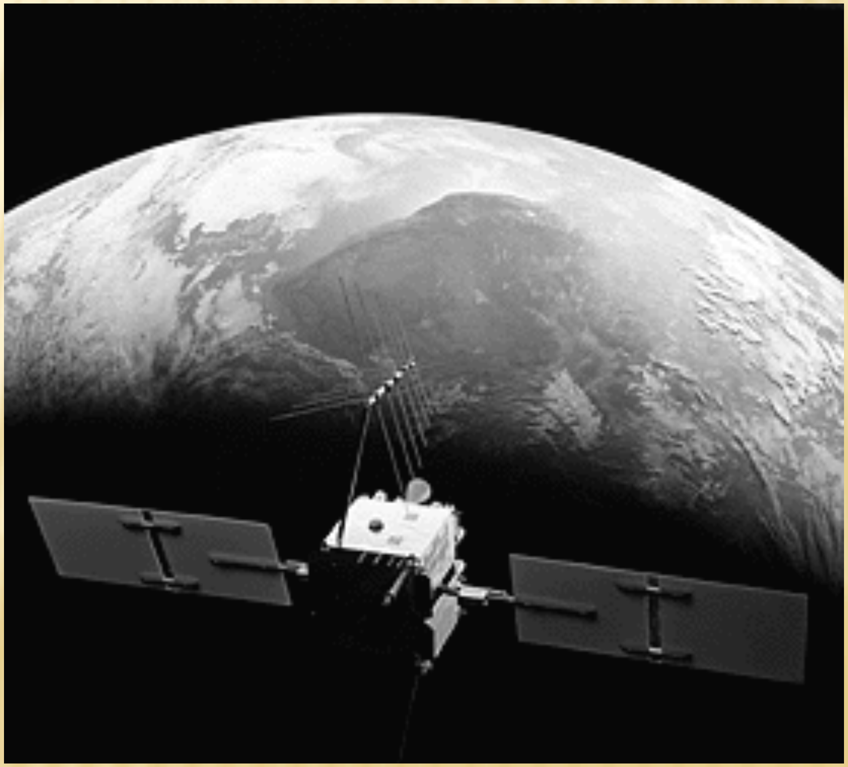
Determining Range (Distance)

- Measure time it takes for radio signal to reach receiver, use speed of light to convert to distance.

- This requires
- Very good clocks
- Precise location of the satellite
- Signal processing over background

http://www.geology.buffalo.edu/courses/gly560/Lectures/GPS we will break the process into five conceptual pieces

step 1: using satellite ranging
step 2: measuring distance from satellite
step 3: getting perfect timing
step 4: knowing where a satellite is in space
step 5: identifying errors



Mattioli-http://comp.uark.edu/~mattioli/geol_4733.html