Earth Science Applications of Space Based Geodesy

DES-7355

Tu-Th 9:40-11:05

Seminar Room in 3892 Central Ave. (Long building)

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http://www.ceri.memphis.edu/people/smalley/ESCI7355/ESCI_7355_Applications_of_Space_Based_Geodesy.html

Class 10

Double difference observation equations

Start with

$$\nabla\Delta L_{AB}^{jk} = \nabla\Delta\rho_{AB}^{jk} + \nabla\Delta Z_{AB}^{jk} - \nabla\Delta I_{AB}^{jk} - \nabla\Delta N_{AB}^{jk}$$

Simplify to

$$L_{AB}^{jk} = \rho_{AB}^{jk} - \lambda_0 N_{AB}^{jk}$$

By dropping the $\nabla\Delta$

And assuming $\nabla\Delta Z_{AB}^{jk}$ & $\nabla\Delta I_{AB}^{jk}$ are negligible

So we have to

Write down the equations

Linearize

Solve

Let the "reference" (also KNOWN) station be A

We want to estimate (x_B, y_B, z_B)

Using observations of satellites 1, 2, 3, and 4
(common observations at all epochs)

We also need to pick a "reference" satellite
(position of all satellites known)
Pick satellite 2.

(we have to pick the reference station and satellite to properly form a linearly independent set of double differences)

For each epoch *i*

We have the following 3 linearly independent sets of double difference observations

$$\Lambda_A^2(i) = \left\{ L_{AB}^{ab}(i) \middle| a = 2; b \neq 2 \right\}$$

$$\Lambda_A^2(i) = \left\{ L_{AB}^{21}(i), L_{AB}^{23}(i), L_{AB}^{24}(i) \right\}$$

To estimate the parameter set

$$\left\{ x_B, y_B, z_B, N_{AB}^{21}, N_{AB}^{23}, N_{AB}^{24} \right\}$$

(if there were no cycle slips, else we would have to estimate additional $N_{AB}^{ij}(k)$ term for each cycle slip, *k*.

As before, the linearized observation equations can be written in terms of the "usual suspects"

$$b = Ax + v$$

Residuals – $d \times 1$

"design" matrix - $d \times p$

Parameter corrections – $p \times 1$

Observation errors – $d \times 1$

d – number linearly independent observables

p – number of parameters to estimate

Blewitt, Basics of GPS in "Geodetic Applications of GPS"

In comparison to the pseudo range data,

where we assumed the errors in the observables were independent,

the errors in double differenced data

are not – the errors are correlated.

This means that we should use

Weighted Least Squares

The WLS solution to the normal equations is

$$\hat{x} = \left(A^T W A\right)^{-1} A^T W \vec{b}$$

Where W is (an appropriately formed) data weight matrix.

The covariance matrix is now given by (does this look familiar?)

$$C_x = \left(A^T W A\right)^{-1}$$

The covariance matrix now has information about both the geometry (as before)

And new (information or effects due to) correlations between the observables.

(if we assume, as for pseudo range, that the error in measurement of the phase is the same for all measurements – we can factor out a σ,

But the differencing introduces a correlation between the "independent" measurements that makes the errors "leak" from one observable to another)

Again, one can get important information from the Covariance matrix

If it is not invertable mathematically
(linearly dependent)

If it is not invertable practically/numerically
(almost linearly dependent, large condition number)

Practically, can tell if all the integer ambiguities can be fixed.
If so, get statistically better estimations.

Blewitt, Basics of GPS in "Geodetic Applications of GPS"

Coefficients of the design matrix

Look at one row.

$$\Lambda_{AB}^{24}(i) = \left\{ \frac{\partial L_{AB}^{24}(i)}{\partial x_B}, \frac{\partial L_{AB}^{24}(i)}{\partial y_B}, \frac{\partial L_{AB}^{24}(i)}{\partial z_B}, \frac{\partial L_{AB}^{24}(i)}{\partial N_{AB}^{21}}, \frac{\partial L_{AB}^{24}(i)}{\partial N_{AB}^{23}}, \frac{\partial L_{AB}^{24}(i)}{\partial N_{AB}^{24}} \right\}$$

$$L_{AB}^{jk} = \rho_{AB}^{jk} - \lambda_0 N_{AB}^{jk}$$

$$\Lambda_{AB}^{24}(i) = \left\{ \frac{\partial \rho_{AB}^{24}(i)}{\partial x_B}, \frac{\partial \rho_{AB}^{24}(i)}{\partial y_B}, \frac{\partial \rho_{AB}^{24}(i)}{\partial z_B}, 0, 0, -\lambda_0 \right\}$$

Coefficients of the design matrix

Look at one derivative.

$$\frac{\partial \rho_{AB}^{24}(i)}{\partial x_B} = \frac{\partial}{\partial x_B}\left(\rho_A^2(i) - \rho_B^2(i) - \rho_A^4(i) + \rho_B^4(i)\right)$$

$$\frac{\partial \rho_{AB}^{24}(i)}{\partial x_B} = \frac{\partial \rho_A^2(i)}{\partial x_B} - \frac{\partial \rho_B^2(i)}{\partial x_B} - \frac{\partial \rho_A^4(i)}{\partial x_B} + \frac{\partial \rho_B^4(i)}{\partial x_B}$$

Independent of x_B

$$\frac{\partial \rho_{AB}^{24}(i)}{\partial x_B} = \frac{\partial \rho_B^4(i)}{\partial x_B} - \frac{\partial \rho_B^2(i)}{\partial x_B}$$

$$\frac{\partial \rho_{AB}^{24}(i)}{\partial x_B} = \frac{x_{B0} - x^4(i)}{\rho_B^4(i)} - \frac{x_{B0} - x^2(i)}{\rho_B^2(i)}$$

Coefficients of the design matrix

Finally one can use the relationship between

Range and Time

and

Time and Phase (what we measured).

$$\rho_A^j(i) = c\left(T_A(i) - T^j(i)\right)$$

$$\phi(T) = f_0 T + \phi_0$$

To write everything in terms of the observables.

Final detail

Minimum data requirements

Necessary (but not sufficient condition) that

Number of data

Exceed

Number of parameters to estimate.

So we have

$d \geq p$

(allowing perfect solution $d = p$)

If all receivers track the same satellites there are

$d = q(r-1)(s-1)$

Linearly independent double differences

Where

q is the number of epochs

r the number of receivers

s the number of satellites

Assuming no cycle slips

$$p=3+(r-1)(s-1)$$

So

$$d=q(r-1)(s-1)\geq 3+(r-1)(s-1)$$

$$(q-1)(r-1)(s-1)\geq 3$$

So for $r=2$, $s=2$

$$q\geq 4$$

(gives one double difference per epoch)

Common-mode Cancellations

Observation	Effects eliminated	Effects reduced	Option
Single differences.	Satellite or station clock (first order).	Orbit errors. GDOP. ionosphere	Constrain ambiguity.
Double differences.	Satellite and station clock (first order).	Orbit errors. GDOP. Ionosphere.	Constrain ambiguity.
Triple differences.	Satellite and station clock (first order).		Ambiguity eliminated. Find-fix cycle slips

After Glen Mattioli

RINEX files

Receiver Independent Exchange files

(standard GPS, now GNSS, observables – data – file)

ASCII files

(text – you can read them)

New competitor – may replace RINEX –

BINEX

Binary Exchange files

(binary – can't read files without program, much more general == complicated)

RINEX Files have two basic parts

Header

Data

(observables)

RINEX Header

TABLE A1
GPS OBSERVATION DATA FILE - HEADER SECTION DESCRIPTION

	HEADER LABEL (Columns 61-80)	DESCRIPTION	FORMAT	
	RINEX VERSION / TYPE	- Format version (2.10) - File type ('O' for Observation Data) - Satellite System: blank or 'G': GPS 'R': GLONASS 'S': Geostationary signal payload 'T': NNSS Transit 'M': Mixed	F9.2,11X, A1,19X, A1,19X	
	PGM / RUN BY / DATE	- Name of program creating current file - Name of agency creating current file - Date of file creation	A20, A20, A20	
*	COMMENT	Comment line(s)	A60	*
	MARKER NAME	Name of antenna marker	A60	
*	MARKER NUMBER	Number of antenna marker	A20	*
	OBSERVER / AGENCY	Name of observer / agency	A20,A40	

RINEX Header

REC # / TYPE / VERS	Receiver number, type, and version (Version: e.g. Internal Software Version)	3A20
ANT # / TYPE	Antenna number and type	2A20
APPROX POSITION XYZ	Approximate marker position (WGS84)	3F14.4
ANTENNA: DELTA H/E/N	- Antenna height: Height of bottom surface of antenna above marker - Eccentricities of antenna center relative to marker to the east and north (all units in meters)	3F14.4
WAVELENGTH FACT L1/2	- Default wavelength factors for L1 and L2 1: Full cycle ambiguities 2: Half cycle ambiguities (squaring) 0 (in L2): Single frequency instrument - zero or blank The default wavelength factor line is required and must preceed satellite-specific lines.	2I6, I6

RINEX Header

```
 +--------------------+------------------------------------------+------------+
*|WAVELENGTH FACT L1/2| - Wavelength factors for L1 and L2       |    2I6,    |*
 |                    |   1:  Full cycle ambiguities             |            |
 |                    |   2:  Half cycle ambiguities (squaring)  |            |
 |                    |   0 (in L2): Single frequency instrument |            |
 |                    | - Number of satellites to follow in list |     I6,    |
 |                    |   for which these factors are valid.     |            |
 |                    | - List of PRNs (satellite numbers with   | 7(3X,A1,I2)|
 |                    |   system identifier)                     |            |
 |                    |                                          |            |
 |                    | These opional satellite specific lines   |            |
 |                    | may follow, if they identify a state     |            |
 |                    | different from the default values.       |            |
 |                    |                                          |            |
 |                    | Repeat record if necessary.              |            |
 +--------------------+------------------------------------------+------------+
```

RINEX Header

```
+--------------------+------------------------------------------+------------+
|# / TYPES OF OBSERV | - Number of different observation types  |     I6,    |
|                    |   stored in the file                     |            |
|                    | - Observation types                      |  9(4X,A2)  |
|                    |                                          |            |
|                    |   If more than 9 observation types:      |            |
|                    |     Use continuation line(s)             |6X,9(4X,A2) |
|                    |                                          |            |
|                    | The following observation types are      |            |
|                    | defined in RINEX Version 2.10:           |            |
|                    |                                          |            |
|                    | L1, L2: Phase measurements on L1 and L2  |            |
|                    | C1    : Pseudorange using C/A-Code on L1 |            |
|                    | P1, P2: Pseudorange using P-Code on L1,L2|            |
|                    | D1, D2: Doppler frequency on L1 and L2   |            |
|                    | T1, T2: Transit Integrated Doppler on    |            |
|                    |         150 (T1) and 400 MHz (T2)        |            |
|                    | S1, S2: Raw signal strengths or SNR      |            |
|                    |         values as given by the receiver  |            |
|                    |         for the L1,L2 phase observations |            |
|                    |                                          |            |
|                    | Observations collected under Antispoofing|            |
|                    | are converted to "L2" or "P2" and flagged|            |
|                    | with bit 2 of loss of lock indicator     |            |
|                    | (see Table A2).                          |            |
|                    |                                          |            |
```

RINEX Header

```
|                    | Units : Phase       : full cycles       |            |
|                    |         Pseudorange : meters            |            |
|                    |         Doppler     : Hz                |            |
|                    |         Transit     : cycles            |            |
|                    |         SNR etc     : receiver-dependent |            |
|                    |                                          |            |
|                    | The sequence of the types in this record |            |
|                    | has to correspond to the sequence of the |            |
|                    | observations in the observation records  |            |
+--------------------+------------------------------------------+------------+
```

RINEX Header

*	INTERVAL	Observation interval in seconds	F10.3	*
	TIME OF FIRST OBS	- Time of first observation record (4-digit-year, month,day,hour,min,sec) - Time system: GPS (=GPS time system) GLO (=UTC time system) Compulsory in mixed GPS/GLONASS files Defaults: GPS for pure GPS files GLO for pure GLONASS files	5I6,F13.7, 5X,A3	
*	TIME OF LAST OBS	- Time of last observation record (4-digit-year, month,day,hour,min,sec) - Time system: Same value as in TIME OF FIRST OBS record	5I6,F13.7, 5X,A3	*
*	RCV CLOCK OFFS APPL	Epoch, code, and phase are corrected by applying the realtime-derived receiver clock offset: 1=yes, 0=no; default: 0=no Record required if clock offsets are reported in the EPOCH/SAT records	I6	*
*	LEAP SECONDS	Number of leap seconds since 6-Jan-1980 Recommended for mixed GPS/GLONASS files	I6	*
*	# OF SATELLITES	Number of satellites, for which observations are stored in the file	I6	*

RINEX Header

*	PRN / # OF OBS	PRN (sat.number), number of observations for each observation type indicated in the "# / TYPES OF OBSERV" - record. If more than 9 observation types: Use continuation line(s) This record is (these records are) repeated for each satellite present in the data file	3X,A1,I2,9I6 * 6X,9I6
	END OF HEADER	Last record in the header section.	60X

Records marked with * are optional

Header example

```
     2.10           OBSERVATION DATA    M (MIXED)           RINEX VERSION / TYPE
BLANK OR G = GPS,  R = GLONASS,  T = TRANSIT,  M = MIXED    COMMENT
XXRINEXO V9.9       AIUB                24-MAR-01 14:43     PGM / RUN BY / DATE
EXAMPLE OF A MIXED RINEX FILE                               COMMENT
A 9080                                                      MARKER NAME
9080.1.34                                                   MARKER NUMBER
BILL SMITH          ABC INSTITUTE                           OBSERVER / AGENCY
X1234A123           XX                  ZZZ                 REC # / TYPE / VERS
234                 YY                                      ANT # / TYPE
  4375274.       587466.      4589095.                      APPROX POSITION XYZ
         .9030         .0000         .0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     1     2     6   G14   G15   G16   G17   G18   G19      WAVELENGTH FACT L1/2
     0                                                      RCV CLOCK OFFS APPL
     4    P1    L1    L2    P2                              # / TYPES OF OBSERV
    18.000                                                  INTERVAL
  2001     3    24    13    10   36.0000000                 TIME OF FIRST OBS
                                                            END OF HEADER
```

(I've not seen many headers with the "time of last observation" line)

Another header example

```
     2.10           OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
teqc  2005Feb10      You don't know?     20050411 15:07:57UTCPGM / RUN BY / DATE
      Linux 2.0.36|Pentium II|gcc|Linux|486/DX+                   COMMENT
      BIT 2 OF LLIGFLAGS DATA COLLECTED UNDER A/S CONDITION       COMMENT
    CJTR                                                        MARKER NAME
 -Unknown-           -Unknown-                               OBSERVER / A ENCY
664                  ASHTECH Z-12        CD00                REC # / TYPE / VERS
    943                  -Unknown-                             ANT # / TYPE
       0.0000        0.0000        0.0000                    APPROX POSITION XYZ
       0.0000        0.0000        0.0000                    ANTENNA: DELTA H/E/N
     1     1                                                 WAVELENGTH FACT L1/2
     5     L1    L2    C1    P1    P2                        # / TYPES OF OBSERV
       SNR is mapped to RINEX snr flag  value [0-9]               COMMENT
        L1 & L2:  2-19 dBHz = 1, 20-27 dBHz = 2, 28-31 dBHz = 3   COMMENT
                 32-35 dBHz = 4, 36-38 dBHz = 5, 39-41 dBHz = 6   COMMENT
                 42-44 dBHz = 7, 45-48 dBHz = 8, >= 49 dBHz = 9   COMMENT
      pseudorange smoothing corrections not applied               COMMENT
   2004    12    26     0     0   30.0000000                  TIME OF FIRST OBS
                                                                END OF HEADER
```

Not having an X0 estimate makes processing more difficult

RINEX Observations (data)

TABLE A2

GPS OBSERVATION DATA FILE - DATA RECORD DESCRIPTION

OBS. RECORD	DESCRIPTION	FORMAT
EPOCH/SAT	- Epoch :	
or	- year (2 digits, padded with 0 if necessary)	1X,I2.2,
EVENT FLAG	- month,day,hour,min,	4(1X,I2),
	- sec	F11.7,
	- Epoch flag 0: OK	2X,I1,
	1: power failure between previous and current epoch	
	>1: Event flag	
	- Number of satellites in current epoch	I3,
	- List of PRNs (sat.numbers with system identifier, see 5.1) in current epoch	12(A1,I2),
	- receiver clock offset (seconds, optional)	F12.9
	If more than 12 satellites: Use continuation line(s)	32X, 12(A1,I2)
	If epoch flag 2-5:	

```
01  3 24 13 10 36.0000000  0  3G12G 9G 6                                -.123456789
 23629347.915           .300 8        -.353    23629364.158
 20891534.648          -.120 9        -.358    20891541.292
 20607600.189          -.430 9         .394    20607605.848

01  3 24 13 10 54.0000000  0  5G12G 9G 6R21R22                          -.123456789
 23619095.450      -53875.632 8    -41981.375    23619112.008
 20886075.667      -28688.027 9    -22354.535    20886082.101
 20611072.689       18247.789 9     14219.770    20611078.410
 21345678.576       12345.567 5
 22123456.789       23456.789 5
```

Notice the order of satellites,

and which satellites are recorded is different for each epoch

RINEX Observations (data)

```
|              |    - Event flag:                                 |  [2X,I1,]  |
|              |       2: start moving antenna                    |            |
|              |       3: new site occupation (end of kinem. data)|            |
|              |          (at least MARKER NAME record follows)   |            |
|              |       4: header information follows             |            |
|              |       5: external event (epoch is significant,  |            |
|              |          same time frame as observation time tags)|           |
|              |                                                  |            |
|              |    - "Number of satellites" contains number of   |    [I3]    |
|              |      special records to follow.                  |            |
|              |      Maximum number of records: 999              |            |
|              |                                                  |            |
|              |    - For events without significant epoch the    |            |
|              |      epoch fields can be left blank              |            |
|              |                                                  |            |
|              | If epoch flag = 6:                               |            |
|              |       6: cycle slip records follow to optionally |            |
|              |          report detected and repaired cycle slips|            |
|              |          (same format as OBSERVATIONS records;   |            |
|              |           slip instead of observation; LLI and   |            |
|              |           signal strength blank or zero)         |            |
+--------------+--------------------------------------------------+------------+
```

```
|OBSERVATIONS | - Observation      | rep. within record for    |  m(F14.3, |
|             | - LLI              | each obs.type (same seq   |     I1,   |
|             | - Signal strength  | as given in header)       |     I1)   |
|             |                                                 |           |
|             | If more than 5 observation types (=80 char):   |           |
|             | continue observations in next record.           |           |
|             |                                                 |           |
|             | This record is (these records are) repeated for |           |
|             | each satellite given in EPOCH/SAT - record.     |           |
|             |                                                 |           |
|             | Observations:                                   |           |
|             |   Phase  : Units in whole cycles of carrier     |           |
|             |   Code   : Units in meters                       |           |
|             | Missing observations are written as 0.0         |           |
|             | or blanks.                                      |           |
|             |                                                 |           |
|             | Phase values overflowing the fixed format F14.3 |           |
|             | have to be clipped into the valid interval (e.g.|           |
|             | add or subtract 10**9), set LLI indicator.      |           |
|             |                                                 |           |
|             | Loss of lock indicator (LLI). Range: 0-7        |           |
|             |  0 or blank: OK or not known                    |           |
|             |  Bit 0 set : Lost lock between previous and     |           |
|             |              current observation: cycle slip    |           |
|             |              possible                           |           |
|             |  Bit 1 set : Opposite wavelength factor to the  |           |
|             |              one defined for the satellite by a |           |
|             |              previous WAVELENGTH FACT L1/2 line.|           |
|             |              Valid for the current epoch only.  |           |
```

RINEX Observations (data)

```
|             |  Bit 2 set : Observation under Antispoofing          |            |
|             |              (may suffer from increased noise)       |            |
|             |                                                      |            |
|             |  Bits 0 and 1 for phase only.                        |            |
|             |                                                      |            |
|             | Signal strength projected into interval 1-9:         |            |
|             |  1: minimum possible signal strength                 |            |
|             |  5: threshold for good S/N ratio                     |            |
|             |  9: maximum possible signal strength                 |            |
|             |  0 or blank: not known, don't care                   |            |
+-------------+------------------------------------------------------+------------+
```

Phase in cycles, Range in meters

```
04 12 26  0  0 30.0000000  0  9G 4G24G 5G17G 6G10G30G 2G29
 -7408143.20348  -5712212.12343  23722895.4574   23722895.8514   23722901.0124
-11151164.34848  -8348759.79145  23027140.6794   23027140.3024   23027147.6974
-17702667.27649 -13496720.20047  21946318.4604   21946318.0704   21946325.1504
-20607717.25049 -16031193.33649  20980332.7214   20980332.1484   20980339.2334
-10697009.82948  -8319281.13543  23671597.2204   23671597.2244   23671604.0324
-25994074.45749 -20224979.69249  20080903.8494   20080902.8804   20080910.1054
-17497598.39549 -13604851.76347  21641129.8624   21641129.7384   21641136.2574
-24900942.06749 -19353992.61648  20874424.4194   20874423.9874   20874428.6824
 -2640345.03446  -1780147.16442  24402022.2324   24402021.1924   24402029.2134
```

L1 L2 C1 P1 P2

Input format is "fortranny"
(fixed number of digits per data entry field, in fixed "card columns", can leave field blank for zero or no data)

Plus more for

Navigation

"met" (METEOROLOGICAL)

Tilt

Other?