UNIT 1 - introduction to GPS

1. GPS SIGNAL

Each GPS satellite transmit two signal for positioning purposes:

- ✓ **L1 signal** (carrier frequency of 1,575.42 MHz). Modulated onto the L1 carrier are two pseudo-random noise (PRN) ranging codes and the navigation (*broadcast*) message. The codes (used to determine the pseudo-ranges) are (a) the 1 millisecond-long C/A-code (chipping rate about 1 MHz); (b) the weeklong segment of the P-code (chipping rate about 10 MHz). The navigation (*broadcast*) message includes orbital information (*ephemeris*), the offset *dt* of the satellite clock from the GPS system time, information on the health of the satellite and the expected accuracy of the range measurements (UERE). The message contains also *almanac* data for other satellites (used by the receiver to determine the location of each satellite). For receivers that track the weeklong P-code, the broadcast message includes a special *hand-over word* (HOW), that tells the receiver where in the P-code to start searching.
- ✓ **L2 signal** (carrier frequency of 1,227.60 MHz) is modulated by the P-code and the navigation message – the C/A code is not present.

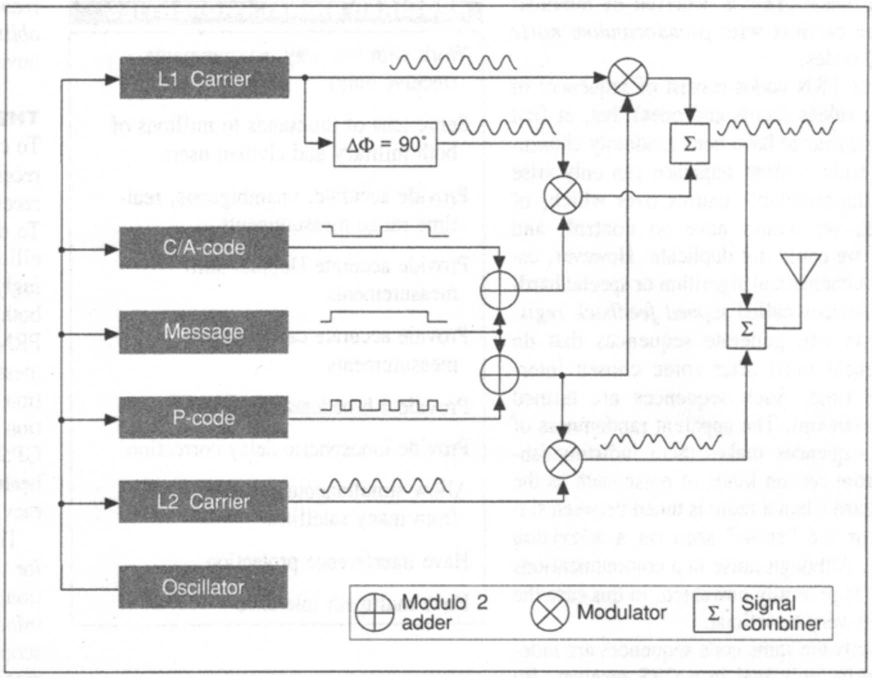


Figure 1. How the different components of the GPS signal are combined. After Langley (1990)

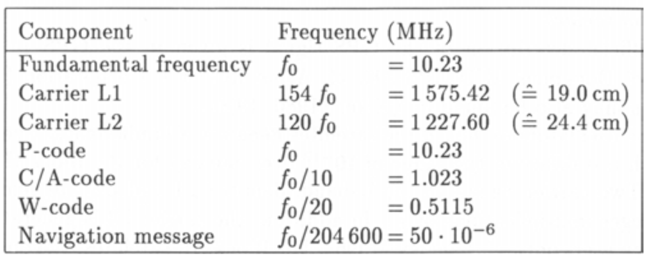
The PRN codes are unique for each satellite and the correlation between any pair of codes is very low. This allows all satellites to share the same carrier frequency.

There are basically two methods to deny civilians full use of the GPS system:

- ✓ Selective Availability (SA) - adding noise to the clock and ephemeris in the navigation message (SA has been turned off in 2000)
- ✓ Anti-spoofing (AS)- the P-code is encrypted (Y-code) and available to military users only.

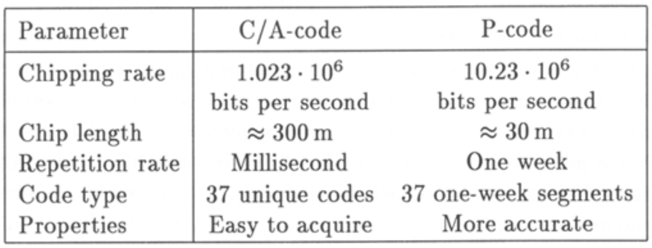
Ref. – Langley R. (1990), Why is the GPS signal so complex?, GPS World, May/June, p. 56.

Table 5.2. Components of the satellite signal



Component	Frequency (MHz)		
Fundamental frequency	f_0	$= 10.23$	
Carrier L1	$154\,f_0$	$= 1\,575.42$	($\hat{=}$ 19.0 cm)
Carrier L2	$120\,f_0$	$= 1\,227.60$	($\hat{=}$ 24.4 cm)
P-code	f_0	$= 10.23$	
C/A-code	$f_0/10$	$= 1.023$	
W-code	$f_0/20$	$= 0.5115$	
Navigation message	$f_0/204\,600$	$= 50 \cdot 10^{-6}$	

Table 5.3. PRN codes characteristics



Parameter	C/A-code	P-code
Chipping rate	$1.023 \cdot 10^6$ bits per second	$10.23 \cdot 10^6$ bits per second
Chip length	≈ 300 m	≈ 30 m
Repetition rate	Millisecond	One week
Code type	37 unique codes	37 one-week segments
Properties	Easy to acquire	More accurate

(After Hoffmann-Wellenhof et al. (1997), *GPS: Theory and Practice*, 4th Ed., Springer.)

2. GPS OBSERVABLES

The pseudorange. The GPS receiver measures the distance (pseudorange) between the satellite and the antenna by measuring the time the signal takes to propagate from the satellite to the receiver. The pseudorange is this time offset multiplied by the speed of light.

The pseudorange is biased by the lack of time synchronization between the clock in the GPS satellite and the clock in the GPS receiver. Other bias effects include the ionosphere and troposphere delay, multipath and receiver noise. The equation for the pseudorange observable is

$$p = \rho + c \times (dt - dT) + d_{ion} + d_{trop} + \varepsilon_p$$

where p is the pseudorange, ρ is the geometric range to the satellite, c is the speed of light, dt and dT are the offsets of the satellite and receiver clock from the GPS time, d_{ion} and d_{trop} the delays imparted by the ionosphere and troposphere and ε_p represents the effect of multipath and receiver noise. The receiver coordinates are hidden in the geometric range ρ.

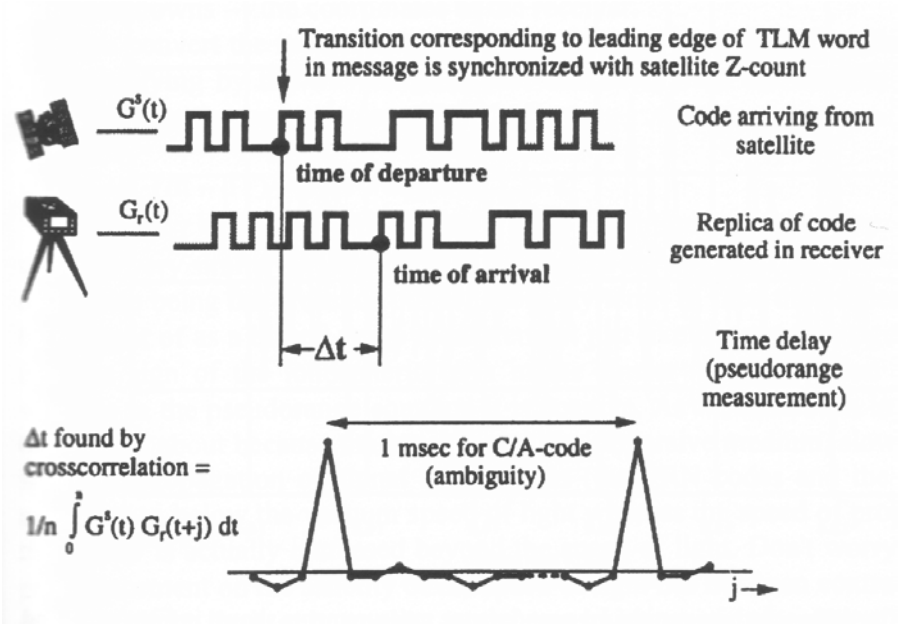


Figure 2. How the pseudorange is measured, after Langley (1998), in *GPS for Geodesy*, p. 151.

Carrier phase. A more precise observable than the pseudo-range is the phase of the received carrier with respect to the phase generated by an oscillator in the GPS receiver. The difference between the received carrier and the receiver generated one is called the carrier beat phase. The problem is that the GPS receiver cannot distinguish one cycle of a carrier from another. The receiver measures the fractional phase, and keeps track of changes to the phase. The initial phase is undetermined, or ambiguous, by an integer number of cycles *N*.

If we convert the carrier beat phase into an equivalent distance by multiplying by the carrier wavelength λ, we get

$$\Phi = \rho + c \times (dt - dT) + \lambda \times N + d_{ion} + d_{trop} + \varepsilon_p$$

which is very similar to the pseudorange expression, the major difference being the presence of the ambiguity term $\lambda \times N$.

Linear combinations. We can form what are known as between-receivers (or between- satellites) differences to obtain new observable with significantly reduced errors.

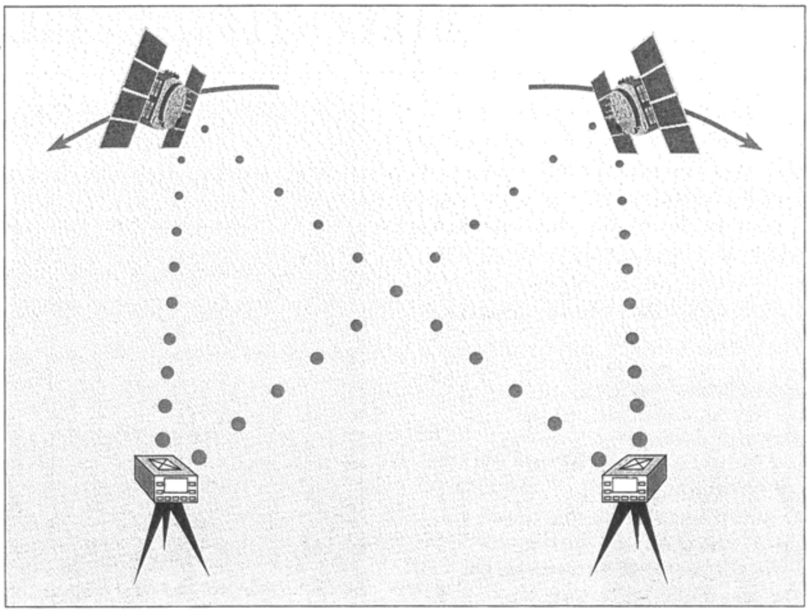


Figure 3. Linear combinations. After Langley (1993).

- ✓ The <u>between-receivers single difference</u> (two different receivers tracking the same satellite) - eliminates the satellite clock offset

$$\Delta\Phi = \Delta\rho - c \times \Delta dT + \lambda \times \Delta N + \Delta d_{ion} + \Delta d_{trop} + \Delta\varepsilon_p$$

- ✓ The <u>between-satellites single difference</u> (one receiver tracking two satellites) - eliminates the receiver clock offset

$$\nabla\Phi = \nabla\rho - c \times \nabla dt + \lambda \times \nabla N + \nabla d_{ion} + \nabla d_{trop} + \nabla\varepsilon_p$$

- ✓ The <u>double difference</u> (we can difference either the between receivers or the between-satellite difference pairs) - eliminate both the receiver and satellite clock offset

$$\Delta\nabla\Phi = \Delta\nabla\rho + \lambda \times \Delta\nabla N + \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \Delta\nabla\varepsilon_p$$

- ✓ The <u>LC (ionosphere free) combination.</u> The linear combination of the L1 and L2 phase measurements reduces the effect of the ionosphere, but may amplify other sources of error

$$\Phi_{LC} = \frac{f_{L1}^2}{f_{L1}^2 - f_{L2}^2}\Phi_{L1} - \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2}\Phi_{L2}$$

- ✓ Wide-lane and narrow-lane combinations (applied for ambiguity resolution)

$$\phi_{WL} = \frac{\Phi_{L1}}{\lambda_{L1}} - \frac{\Phi_{L2}}{\lambda_{L2}} \quad \lambda_{NL} = \frac{\Phi_{L1}}{\lambda_{L1}} + \frac{\Phi_{L2}}{\lambda_{L2}} \; m$$

$$\lambda_{WL} = \frac{c}{f_{L1} - f_{L2}} \approx 0.86\ m \quad \lambda_{NL} = \frac{c}{f_{L1} + f_{L2}} \approx 0.11\ m$$

Ref. – Langley R. (1993), The GPS observables, GPS World, April, p. 52.

3. ERROR BUDGET.

Both systematic errors (biases) and random noise affect the code pseudoranges *p* and phase pseudoranges Φ. The error sources can be classified into three groups (see Table 6.1)

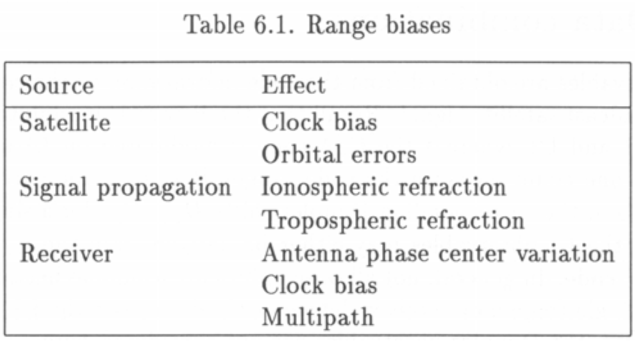


Table 6.1. Range biases

Source	Effect
Satellite	Clock bias
	Orbital errors
Signal propagation	Ionospheric refraction
	Tropospheric refraction
Receiver	Antenna phase center variation
	Clock bias
	Multipath

(After Hoffmann-Wellenhof et al. (1997), *GPS: Theory and Practice*, 4th Ed., Springer.)

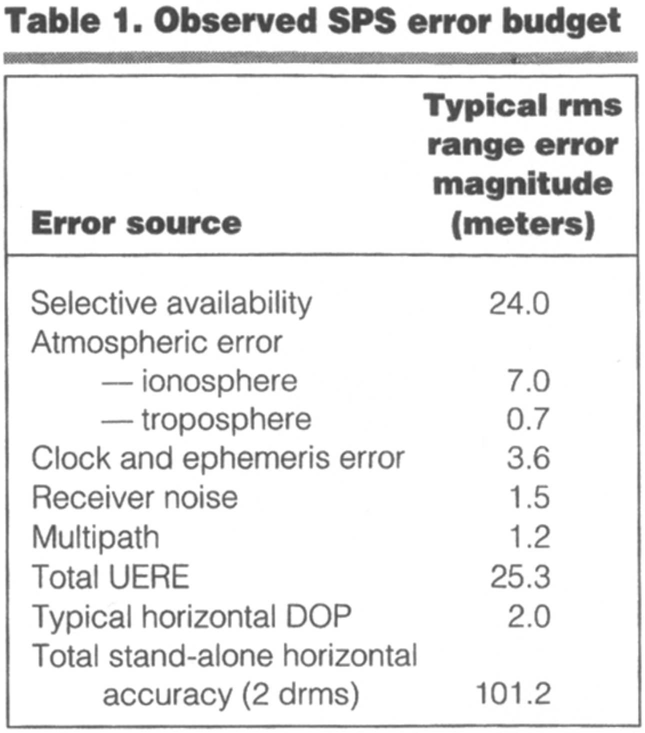


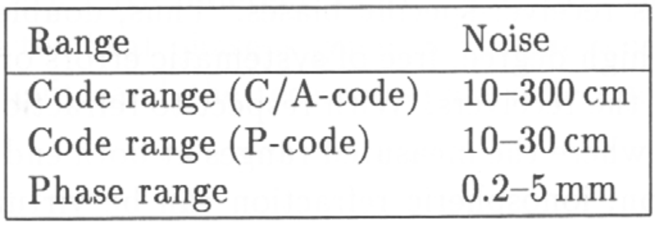
Table 1. Observed SPS error budget

Error source	**Typical rms range error magnitude (meters)**
Selective availability	24.0
Atmospheric error	
— ionosphere	7.0
— troposphere	0.7
Clock and ephemeris error	3.6
Receiver noise	1.5
Multipath	1.2
Total UERE	25.3
Typical horizontal DOP	2.0
Total stand-alone horizontal accuracy (2 drms)	101.2

UERE stands for User Equivalent Range Error. After Langley (1997)

Orbital Errors /Clock Bias/Measurement Noise: As mentioned earlier, GPS signals contain information about ephemeris (orbital position) errors, and about the rate of clock drift for the broadcasting satellite. The data concerning ephemeris errors may not exactly model the true satellite motion or the exact rate of clock drift. Distortion of the signal by measurement noise can further increase positional error. The disparity in ephemeris data can introduce 1-5 meters of positional error, clock drift disparity can introduce 0-1.5 meters of positional error and measurement noise can introduce 0-10 meters of positional error (see Table 6.2).

Table 6.2. Range noise



Range	Noise
Code range (C/A-code)	10–300 cm
Code range (P-code)	10–30 cm
Phase range	0.2–5 mm

(After Hoffmann-Wellenhof et al. (1997), *GPS: Theory and Practice*, 4th Ed., Springer.)

Signal propagation: The ionosphere and troposphere both refract the GPS signals. This causes the speed of the GPS signal in the ionosphere and troposphere to be different from the speed of the GPS signal in space. Therefore, the distance calculated from "Signal Speed x Time" will be different for the portion of the GPS signal path that passes through the ionosphere and troposphere and for the portion that passes through space.

Multipath: A GPS signal bouncing off a reflective surface prior to reaching the GPS receiver antenna is referred to as multipath. Because it is difficult to completely correct multipath error, even in high precision GPS units, multipath error is a serious concern to the GPS user.

Selective Availability (turned off in January 2000): Ephemeris errors should not be confused with Selective Availability (SA), which is the intentional alteration of the time and ephemeris signal by the Department of Defense. SA can introduce 0-70 meters of positional error. Fortunately, positional errors caused by SA can be removed by differential correction.

Dilution of Precision (DOP). The UERE is mapped into the computed position by a geometrical factor called DOP. The DOP is a mathematical function involving the relative coordinates of the receiver and the satellite and can be easily computed for a particular satellite arrangement. The more spread out the satellites are in the sky, the smaller the DOP value. A typical value for the horizontal dilution of precision (HDOP), assuming that a receiver is processing the signals of 4 satellites only, is 2.0.

Ref. - Langley, R. B. (1997), The GPS error budget. GPS World , Vol. 8, No. 3, pp. 51-56.