Satellite Orbits

Keplerian motion
Perturbed motion
GPS satellites
GPS orbits

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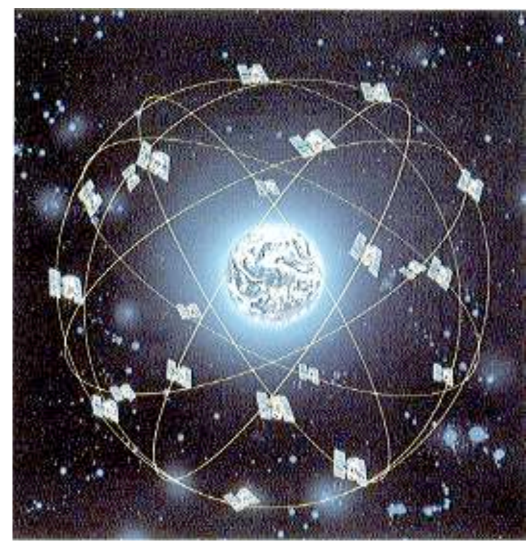
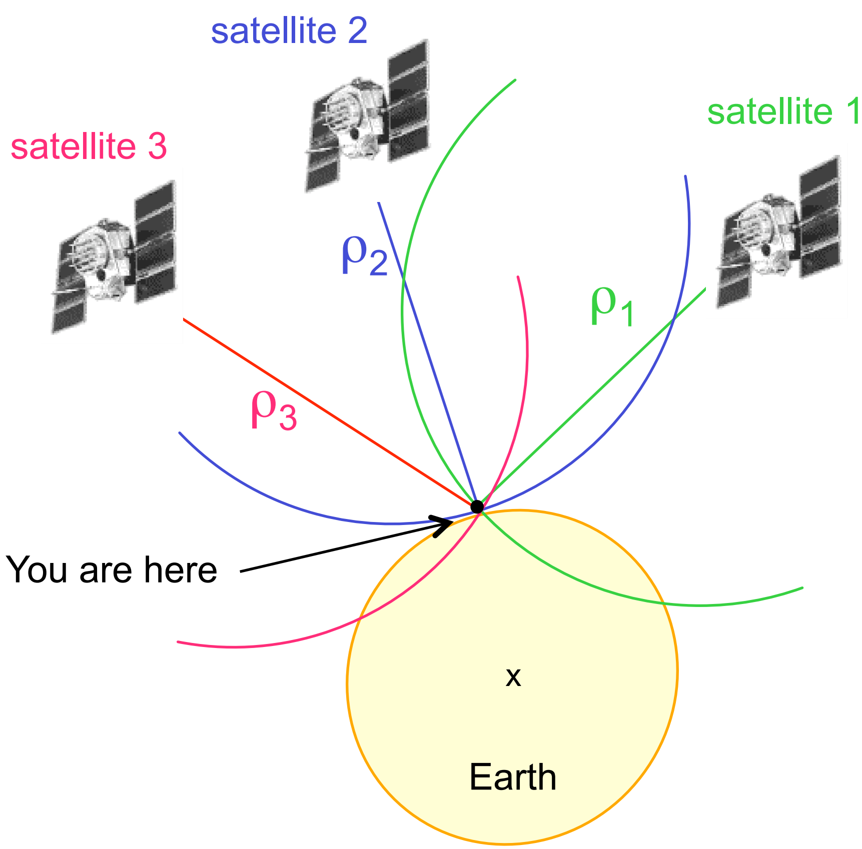


Orbits?


JOHANNES KEPLER'S UPHILL BATTLE
...SO, YOU SEE THE ORBIT OF A PLANET IS ELLIPTICAL.
WHAT'S AN ORBIT?
WHAT'S A PLANET?
WHAT'S 'ELLIPTICAL'?
S. Harris

Satellite orbits: What for?



- Principle of GPS positioning:
  - Satellite 1 sends a signal at time t_{e1}
  - Ground receiver receives it signal at time t_r
  - The range measurement ρ_1 to satellite 1 is:
    - $\rho_1 = (t_r - t_{e1}) \times$ speed of light
    - We are therefore located on a sphere centered on satellite 1, with radius ρ1
  - 3 satellites => intersection of 3 spheres

- The mathematical model is:

$$\rho_r^s = \sqrt{(X_s - X_r)^2 + (Y_s - Y_r)^2 + (Z_s - Z_r)^2}$$

  - GPS receivers measure ρ_r^s
  - If the position of the satellites in an Earth-fixed frame *(X_s, Y_s, Z_s)* is known,
  - Then one can solve for *(X_r, Y_r, X_r)* (if at least 3 simultaneous range measurements)

Dynamics of satellite orbits

- Basic dynamics in an inertial frame described by: $\sum \vec{F} = m\vec{a}$
- Case of two-body problem (two point masses), forces are:
  - Gravitational forces
  - Solar radiation pressure (drag is negligible for GPS)
  - Thruster firings (not directly modeled).
- Neglecting radiation pressure, one can write:

$$\vec{F} = \frac{Gm_s m_E}{r^2}\vec{r} \qquad and \qquad \vec{F} = m_s\vec{a}$$

$$\Rightarrow \vec{a} - \frac{Gm_E}{r^2}\vec{r} = \vec{0} \Leftrightarrow \boxed{\frac{d^2\vec{r}}{dt^2} = \frac{Gm_E}{r^2}\vec{r}}$$

- r = geocentric position vector
- $a = d^2r/dt^2$, relative acceleration vector
- G = universal gravitational constant
- m_E = Earth's mass
- m_S = satellite's mass

$$m_s\ddot{\vec{r}}_s = -G\frac{m_s m_E}{r^2}\vec{r}$$

$$m_E\ddot{\vec{r}}_E = G\frac{m_s m_E}{r^2}\vec{r}$$

$$r = \left\|\vec{r}_s - \vec{r}_E\right\|$$

$$\vec{r} = \frac{\vec{r}_s - \vec{r}_E}{r}$$

$$\ddot{\vec{r}}_s - \ddot{\vec{r}}_E = -G\frac{m_E}{r^2}\vec{r} - G\frac{m_s}{r^2}\vec{r}$$

$$\Rightarrow \ddot{\vec{r}} = -G(m_s + m_E)\frac{\vec{r}}{r^2}$$

$$\ddot{\vec{r}} = -\mu\frac{\vec{r}}{r^2}$$

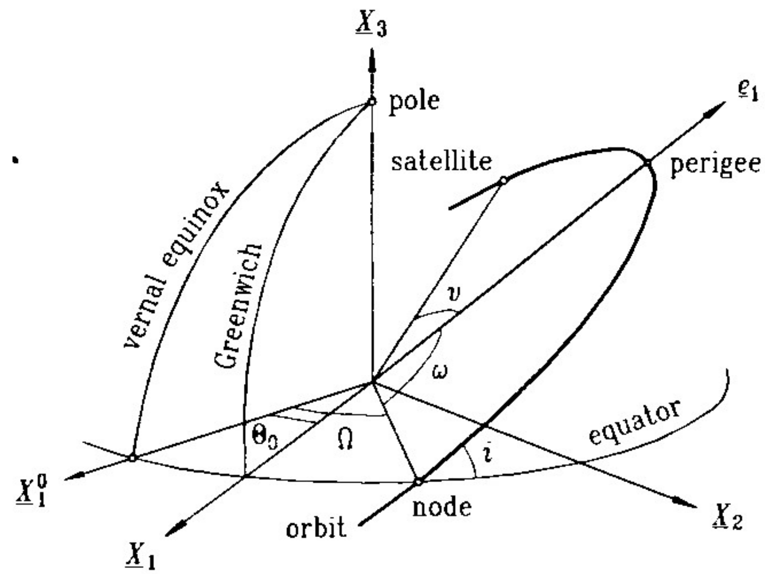
$$\mu = G(m_s + m_E)$$

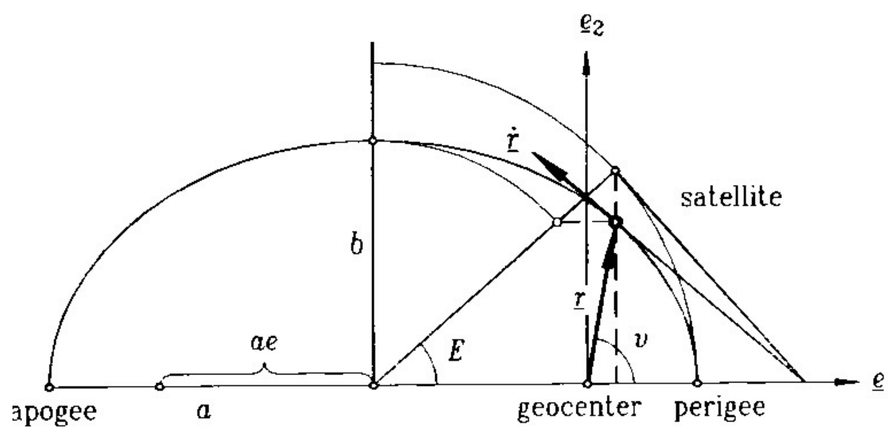
Keplerian motion

Analytical solution to this force model = Keplerian (= osculating) orbit:

- Six integration constants => 6 orbital parameters or Keplerian (= osculating) elements
- In an **inertial** reference frame, orbits can be described by an ellipse
- The orbit plane stays fixed in space
- One of the foci of the ellipse is the center of mass of the body
- These orbits are described by Keplerian elements:

a	Semi-major axis	Size and shape of orbit
e	Eccentricity	
Ω	Right ascension of ascending node	Orientation of the orbital plane in the inertial system
ω	Argument of perigee	
i	Inclination	
T_o	Epoch of perigee	Position of the satellite in the orbital plane





Keplerian motion

- Third Kepler's law (period^2/a^3=const.) relate mean angular velocity n and revolution period P:

$$n = \frac{2\pi}{P} = \sqrt{\frac{GM_E}{a^3}}$$

  – M_E = Earth's mass, GM_E = 3,986,005x10^8 m^{3}s^{-2}

- For GPS satellites:
  – Nominal semi-major axis a = 26 560,000 m
  – Orbital period = 12 sidereal hours (= 11h28mn UT), v=3.87 km/s
  – Therefore, positions of GPS satellites w.r.t. Earth's surface repeat every sidereal day

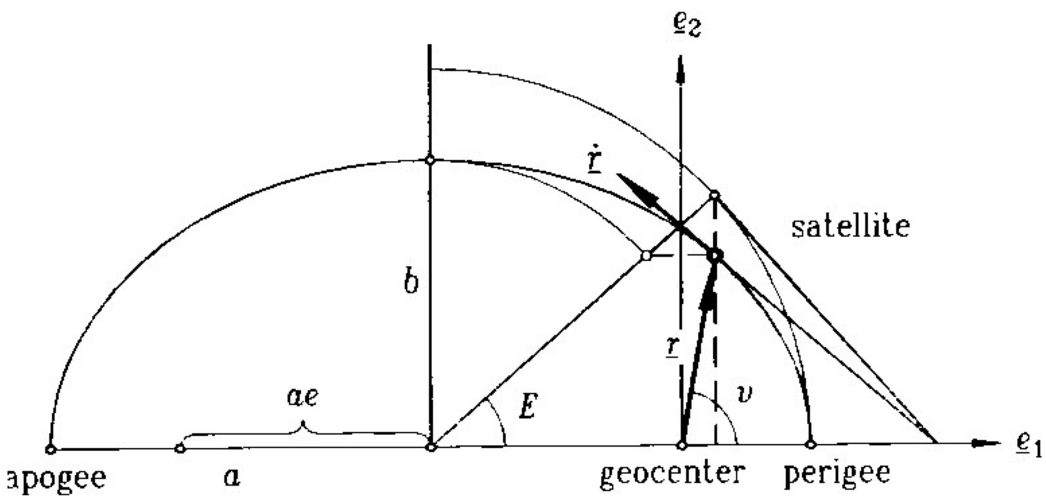
Keplerian motion

- Instantaneous position of a satellite on its orbit is defined by angular quantities called *"anomalies"*:
  - Mean anomaly: $M(t) = n \times (t-To)$
    with n = mean motion = number of orbits in 24 hours, To = time of perigee
  - Eccentric anomaly: $E(t) = M(t) + e \sin E(t)$
  - True anomaly: $v(t) = 2 \operatorname{atan} [((1+e)/(1-e))^{1/2} \tan(E(t)/2)]$

- In the coordinate system defined by the **orbital plane**:
  - $e1 = r \cos v = a \cos E - ae = a (\cos E - e)$
  - $e2 = r \sin v = (b/a) a \sin E = b \sin E = a (1-e^2)^{1/2} \sin E$
  - $e3 = 0$

$$\vec{r} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} a(\cos E - e) \\ a\sqrt{1-e^2}\sin E \\ 0 \end{bmatrix}$$

$$\|\vec{r}\| = a(1 - e\cos E)$$



Keplerian motion

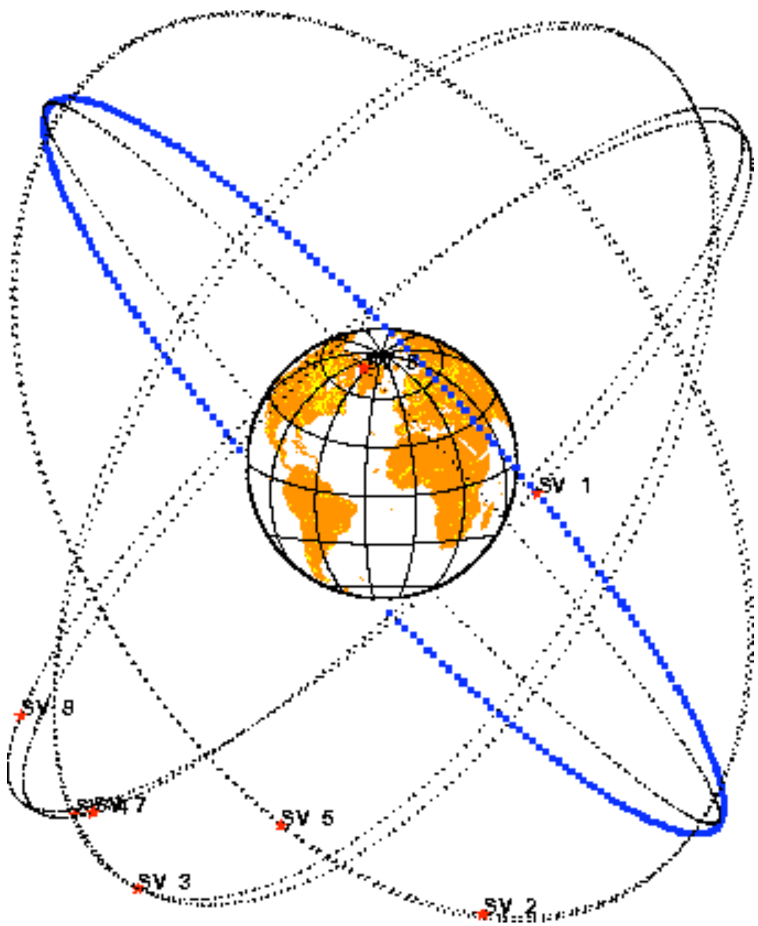


Fig. T. Herring (MIT)

GPS orbit, inertial frame

- GPS orbit in an inertial frame (+Earth rotation)
- To compute site position on the Earth, we need the satellite orbit to an Earth-fixed frame…

Keplerian motion

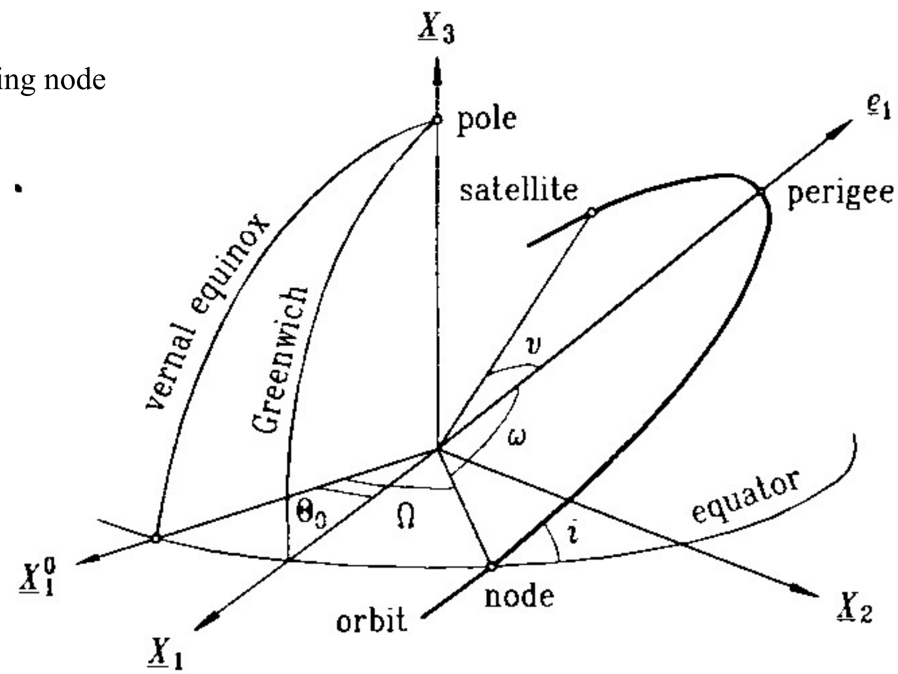
In the **Earth centered inertial system** (X_1^0, X_2^0=X_2, X^O_3=X_3), ρ relates to *r* through the combination of 3 rotations:

$$\vec{\rho} = R\vec{r}$$
$$R = R_3\{-\Omega\}R_1\{-i\}R_3\{-\omega\}$$

$$R = \begin{bmatrix} \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i & -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i & \sin\Omega\sin i \\ \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i & -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i & -\cos\Omega\sin i \\ \sin\omega\sin i & \cos\omega\cos i & \cos i \end{bmatrix}$$

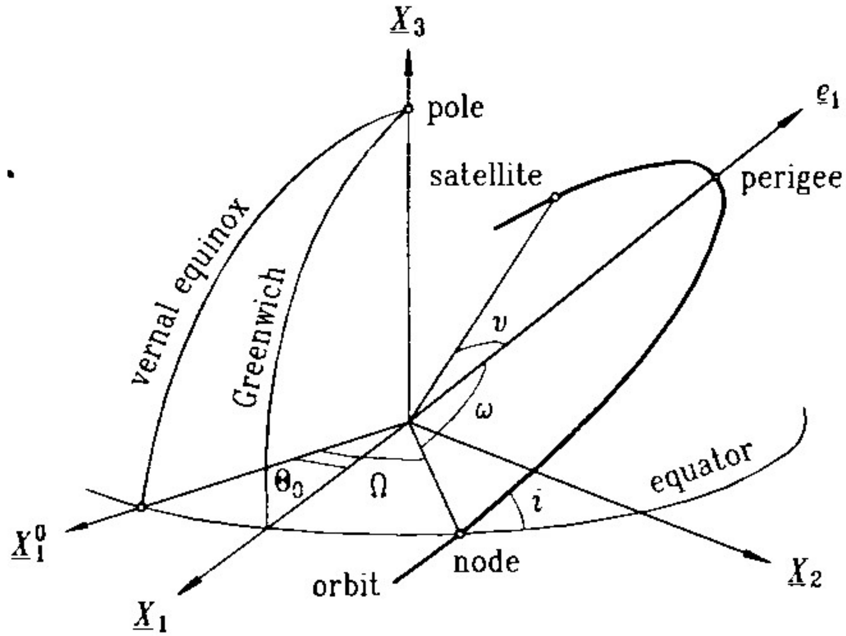
[think of what it takes for (e_1, e_2, e_3) to align with (X_1^0,X_2,X_3)]

Ω = ascension of ascending node
ω= argument of perigee
i = inclination
v = true anomaly



Keplerian motion

- With 3 rotations (Ω,i,ω), we went from orbital plane coordinates (e1,e3,e3) to Earth-centered inertial coordinates (r)
- To convert r to an **Earth fixed system** (X_1, X_2, X_3), we need an additional rotation of Θ_o (related to the Greenwich Sidereal Time) about X_3 :



$$\vec{\rho} = R'\vec{r}$$

$$R' = R_3\{\Theta_o\}R = R_3\{\Theta_o\}R_3\{-\Omega\}R_1\{-i\}R_3\{-\omega\}$$

$$\textit{defining}: \quad l = \Omega - \Theta_o$$

$$R' = \begin{bmatrix} \cos l \cos\omega - \sin l \sin\omega \cos i & -\cos l \sin\omega - \sin l \cos\omega \cos i & \sin l \sin i \\ \sin l \cos\omega + \cos l \sin\omega \cos i & -\sin l \sin\omega + \cos l \cos\omega \cos i & -\cos l \sin i \\ \sin\omega \sin i & \cos\omega \cos i & \cos i \end{bmatrix}$$

GPS Orbits

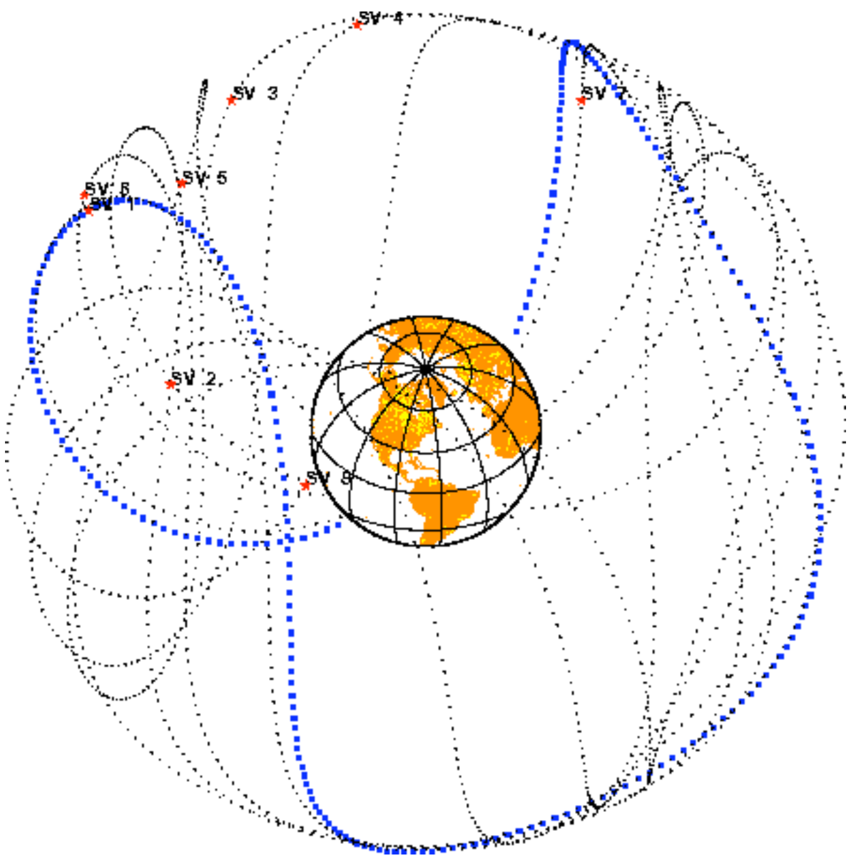


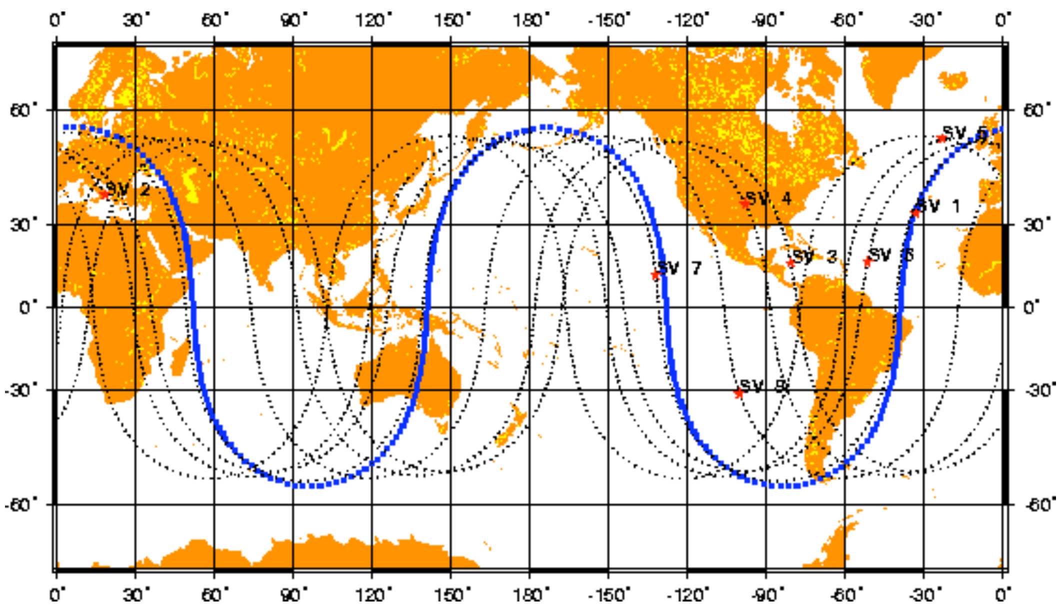
Fig. T. Herring (MIT)

GPS orbit, Earth-fixed frame

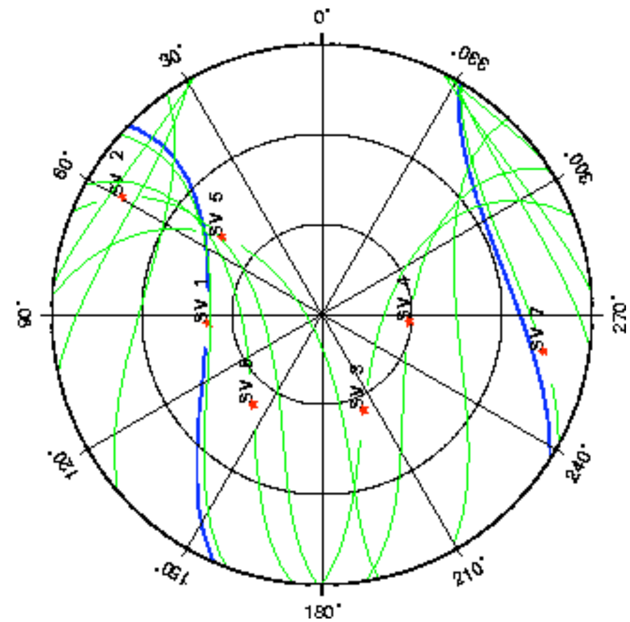
1. Broadcast ephemerides (in GPS signal) contain **orbital parameters** $M, e, a, \Omega, i, \omega, n$ (=angular velocity)
2. Orbital parameters are used to compute orbit in **inertial frame**
3. Orbit in inertial frame is then **rotated into terrestrial frame** (Earth-fixed)
4. In order to do this accurately over long time periods, one need to account for variations in Earth orientation and rotation parameters => use precise UT1, nutations, and polar motion.
5. And take into account orbit perturbations (Earth's gravity field, solar pressure, attraction from Sun and Moon): also provided in broadcast ephemerides

GPS Orbits

Other representations of GPS orbits



GPS orbit in Earth-fixed frame =
"ground track"



GPS orbit in
topocentric frame =
"sky plot"

Perturbed motion

Central gravitational force = main force acting on GPS satellites, but there are other significant perturbations:

- Gravitational forces:
  - Non sphericity of the Earth gravitational potential:
    - Major contribution from the Earth's flattening (J_2)
    - GPS orbits high (20 000 km) and attraction force attenuates rapidly with altitude => only a few terms of the Earth's gravitational potential are necessary for modeling GPS orbits
  - Third body effect: direct attraction of Moon and Sun => lunar and solar ephemerides necessary to model GPS orbits
  - [Tidal effects of Sun and Moon => deforms Earth => modifies Earth's gravitational potential: negligible for GPS satellites]
- Non-gravitational forces:
  - Solar radiation pressure:
    - Impact on the satellite surfaces of photons emitted by the Sun and reflected by the Earth surface: can be modeled, knowing the 3D geometry and the attitude of the satellite.
    - Effects on GPS satellite position: 5-10 m
    - Eclipse periods = satellite in the Earth's shadow (1-2/year, lasts about 1 hr): transition to Sun light difficult to model, usually this part of the orbit is simply edited out!
  - [Atmospheric drag = negligible for GPS satellites]
  - Satellite maneuvers

Solar radiation pressure

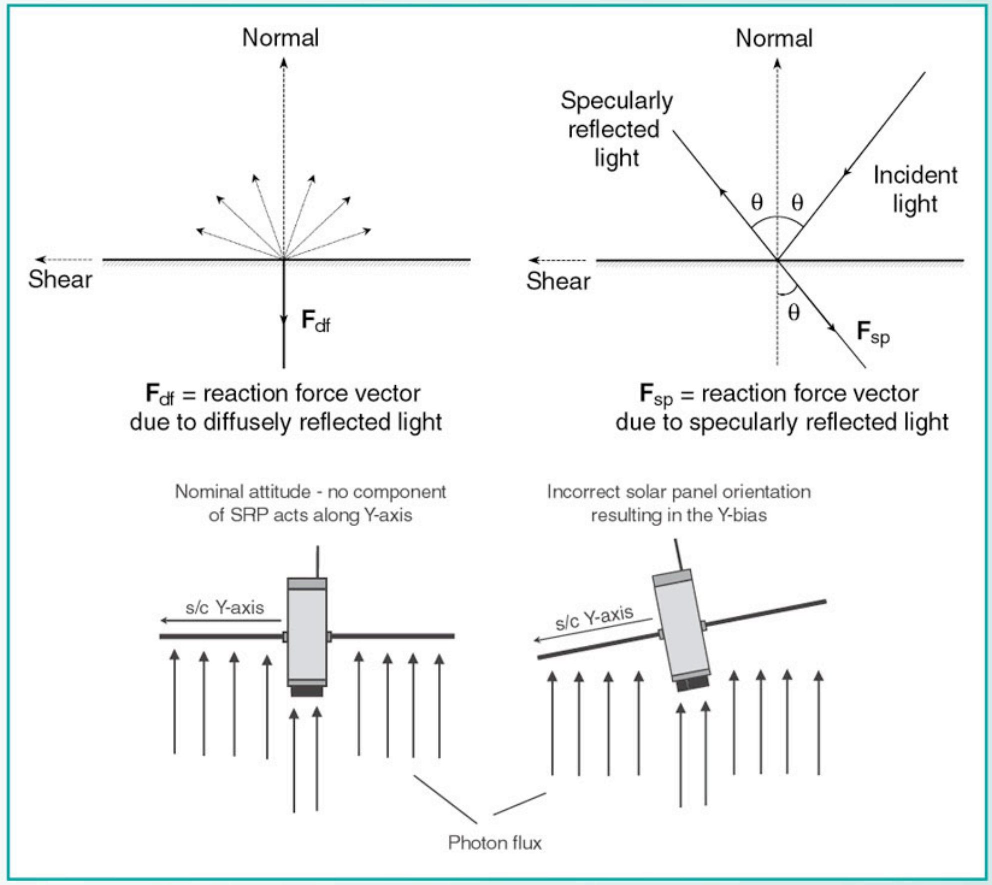
- Results from impact of Sun photons on the satellite's surface.
- Depends on:
  - Effective area (surface normal to the incident radiation)
  - Surface reflectivity
  - Luminosity of the Sun
  - Distance to the Sun
- GPS satellites oriented so that the "Y-axis" is perpendicular to the direction of the Sun (solar sensors)
- For satellites in the Earth shadow region (eclipsing), the solar radiation pressure is zero.
- For precise orbit determination, the shadow region must be carefully determined using the relative positions of the Sun, Earth, and the satellites.

Terminology

Reflectivity (ν) – the proportion of radiation incident on a surface that is reflected, the reflected radiation being separated into diffuse (scattered) and specular (beamed) components.

Specularity (μ) – the proportion of reflected radiation that is reflected specularly. Specular reflection implies that the surface behaves like a perfect mirror.

Y-bias – a force acting along the spacecraft BFS Y-axis and believed to derive from NCF effects. A likely mechanism for the Y-bias is due to non-orthogonality of the solar panels with respect to the solar photon flux, as a result of attitude bias or variations. However, another possible contribution could come from heat dissipation effects of payload components.



Reflected radiation from a spacecraft may be separated into diffuse and specular components. If a spacecraft's solar panels are not oriented precisely orthogonal to the photon flux, an anomalous bias force is generated along the spacecraft Y-axis.

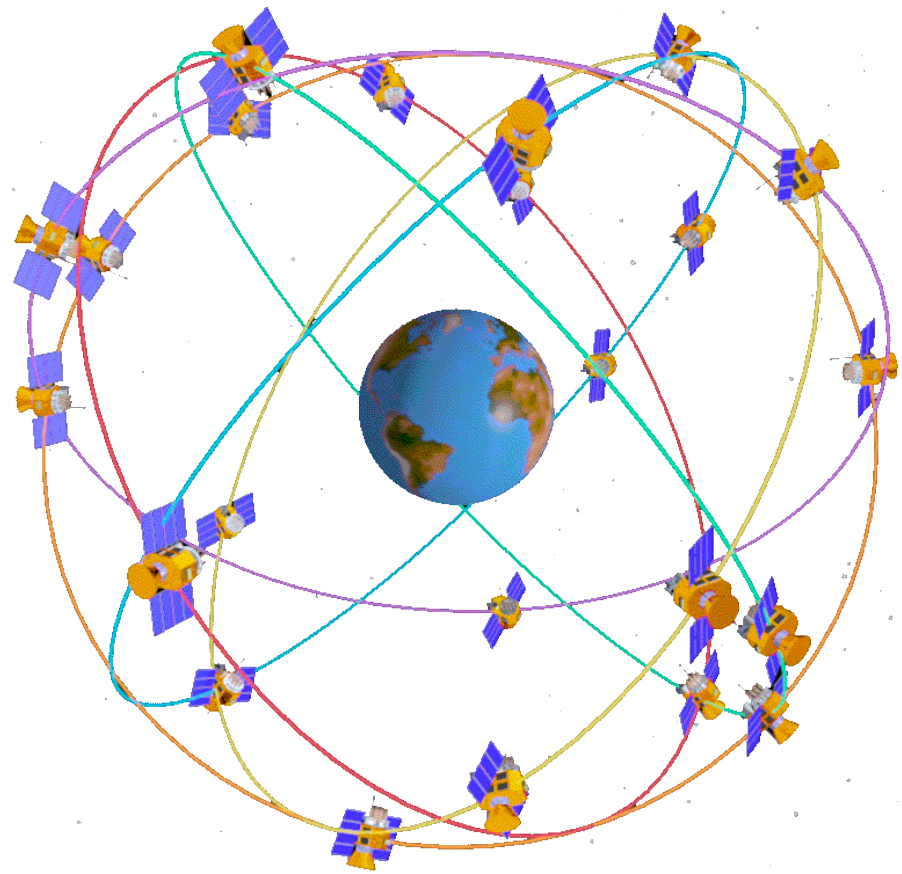
Perturbed motion

Term	Acceleration (m/sec^2)	Perturbation (after 2 days)
Central	0.6	-
J_2	$5x10^{-5}$	14 km
Other gravity harmonics	$3x10^{-7}$	0.1-1.5 km
Third body	$5x10^{-6}$	1-3 km
Earth tides	10^{-9}	0.5-1 m
Ocean tides	10^{-10}	0-2 m
Drag	~0	-
Solar radiation	10^{-7}	0.1-0.8 km
Albedo radiation	10^{-9}	1-1.5 m

GPS orbits

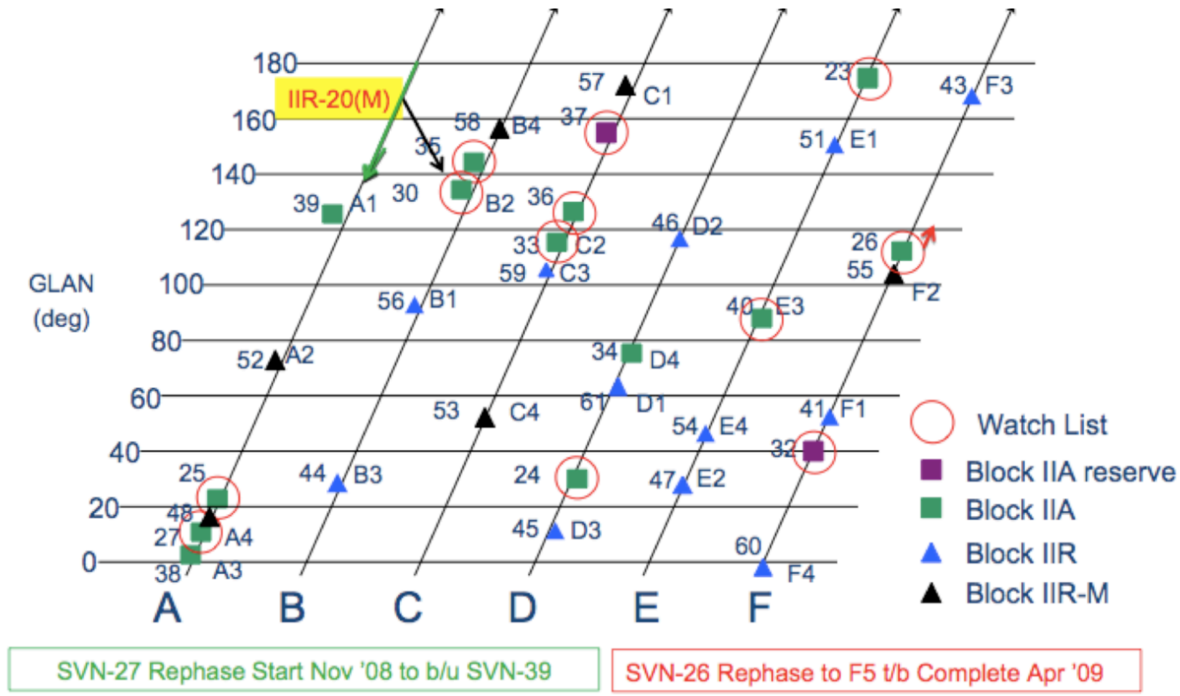
- Orbit characteristics:
  - Semi-major axis = 26,400 km
  - Period = 12 sidereal hour
  - 6 orbital planes
  - Inclination = 55.5 degrees (except old block I sats = 63 deg., but all dead)
  - Eccentricity nearly 0 (largest 0.02) => quasi-circular orbits

- Full constellation = 24 satellites, completed March 9, 1994

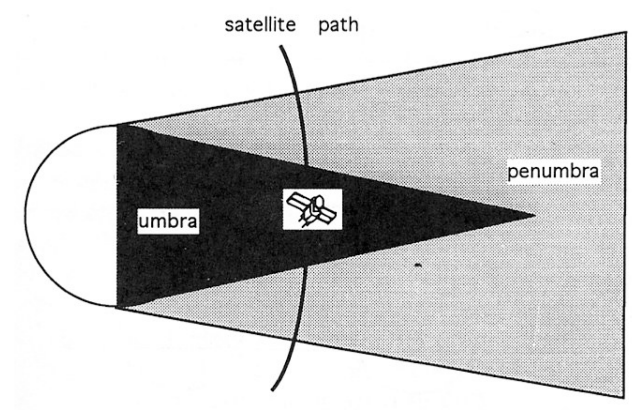


GPS Orbital Constellation

- 27 satellites (24 operational + 3 spares = nominal constellation)
- 6 evenly spaced orbital planes (A to F), inclination 55°
- 4-6 satellites per plane, spacing for optimized visibility
- Period = 12 sidereal hours (= 11h58mn “terrestrial” hours) => in a terrestrial frame, the constellation repeats every 23h56mn.
- As Earth orbits around the Sun => eclipse periods (solar radiation pressure = 0, transition to shadow difficult to model, often simply edited out)

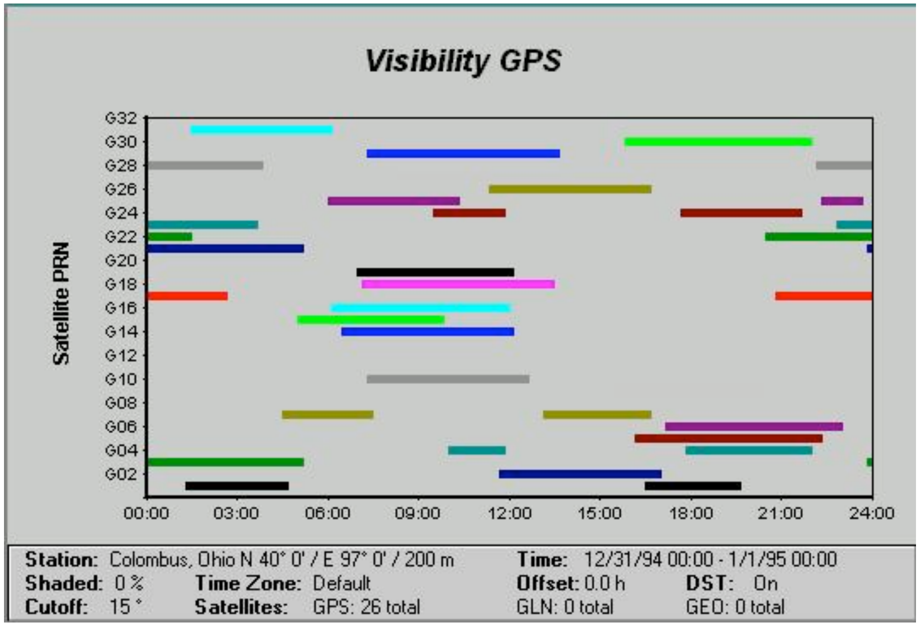


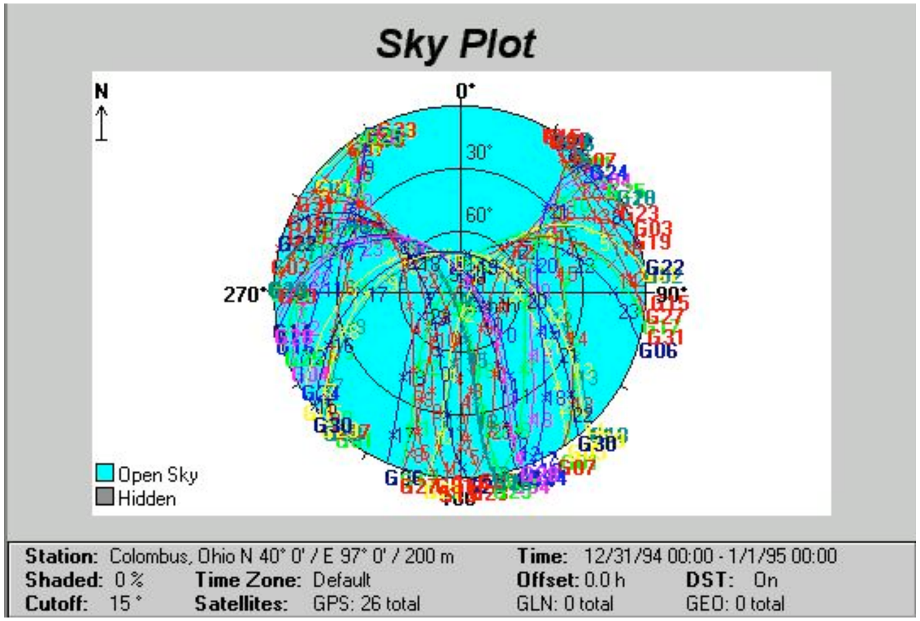
y-axis =longitude at which the satellite crosses the equator in the eastern hemisphere



GPS satellite visibility

- Satellite visible up to 6 hours in a row (from rising to setting)
- In practice, 6-12 satellites are visible simultaneously, depending on:
  - Constellation geometry
  - User location
  - Elevation cut-off angle (chosen by the user)
- Problematic environments:
  - Forest
  - “urban canyons”
  - Mission planning may become necessary: determine best time of day to make measurements
- Site selection:
  - Must account for masks (minimal)
  - A! masks can grow (trees, houses)





GPS constellation

- Current status posted daily on ftp://tycho.usno.navy.mil/pub/gps/gpstd.txt
- Notice Advisories to NAVSTAR Users (NANUs) posted on: http://tycho.usno.navy.mil/gps_datafiles.html
- Or: http://www.navcen.uscg.gov/navinfo/Gps/ActiveNanu.aspx

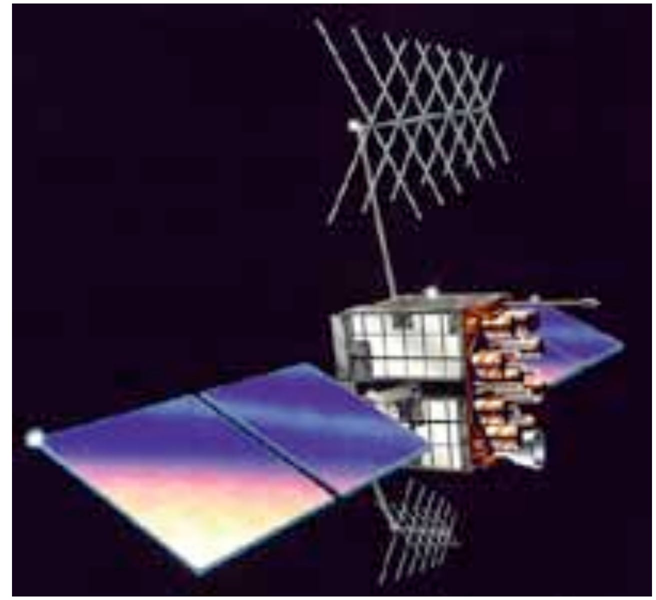
CURRENT BLOCK II/IIA/IIR/IIR-M SATELLITES

LAUNCH ORDER	PRN	SVN	LAUNCH DATE	FREQ STD	PLANE	US SPACE COMMAND **
*II-1		14	14 FEB 1989			19802
*II-2		13	10 JUN 1989			20061
*II-3		16	18 AUG 1989			20185
*II-4		19	21 OCT 1989			20302
*II-5		17	11 DEC 1989			20361
*II-6		18	24 JAN 1990			20452
*II-7		20	26 MAR 1990			20533
*II-8		21	02 AUG 1990			20724
*II-9		15	01 OCT 1990			20830
IIA-10	32	23	26 NOV 1990	Rb	E5	20959
IIA-11	24	24	04 JUL 1991	Cs	D5	21552
IIA-12	25	25	23 FEB 1992	Rb	A5	21890
*IIA-13		28	10 APR 1992			21930
IIA-14	26	26	07 JUL 1992	Rb	F5	22014
IIA-15	27	27	09 SEP 1992	Cs	A4	22108
*IIA-16		32	22 NOV 1992		F6	22231
*IIA-17		29	18 DEC 1992			22275
*IIA-18		22	03 FEB 1993			22446
*IIA-19		31	30 MAR 1993			22581
*IIA-20		37	13 MAY 1993			22657
IIA-21	09	39	26 JUN 1993	Cs	A1	22700
*IIA-22		35	30 AUG 1993	Rb	B5	22779
IIA-23	04	34	26 OCT 1993	Rb	D4	22877
IIA-24	06	36	10 MAR 1994	Rb	C5	23027
IIA-25	03	33	28 MAR 1996	Cs	C2	23833
IIA-26	10	40	16 JUL 1996	Cs	E3	23953
IIA-27	30	30	12 SEP 1996	Cs	B2	24320
IIA-28	08	38	06 NOV 1997	Cs	A3	25030
***IIR-1		42	17 JAN 1997			
IIR-2	13	43	23 JUL 1997	Rb	F3	24876
IIR-3	11	46	07 OCT 1999	Rb	D2	25933
IIR-4	20	51	11 MAY 2000	Rb	E1	26360
IIR-5	28	44	16 JUL 2000	Rb	B3	26407
IIR-6	14	41	10 NOV 2000	Rb	F1	26605
IIR-7	18	54	30 JAN 2001	Rb	E4	26690
IIR-8	16	56	29 JAN 2003	Rb	B1	27663
IIR-9	21	45	31 MAR 2003	Rb	D3	27704
IIR-10	22	47	21 DEC 2003	Rb	E2	28129
IIR-11	19	59	20 MAR 2004	Rb	C3	28190
IIR-12	23	60	23 JUN 2004	Rb	F4	28361
IIR-13	02	61	06 NOV 2004	Rb	D1	28474
IIR-14M	17	53	26 SEP 2005	Rb	C4	28874
IIR-15M	31	52	25 SEP 2006	Rb	A2	29486
IIR-16M	12	58	17 NOV 2006	Rb	B4	29601
IIR-17M	15	55	17 OCT 2007	Rb	F2	32260
IIR-18M	29	57	20 DEC 2007	Rb	C1	32384
IIR-19M	07	48	15 MAR 2008	Rb	A6	32711
IIR-20M	01	49	24 MAR 2009	Rb	B2	34661
IIR-21M	05	50	17 AUG 2009	Rb	E6	35752

* Satellite is no longer in service.
** US SPACE COMMAND, previously known as the NORAD object number; also referred to as the NASA Catalog number. Assigned at successful launch. Catalog numbers retrieved from SPACEWARN Bulletins: http://nssdc.gsfc.nasa.gov/spacewarn/
*** Unsuccessful launch.

GPS satellites

- Solar powered
- S-Band (SGLS) communications for control and telemetry + UHF cross-link between spacecraft.
- Two L-Band navigation signals at 1575.42 MHz (L1) and 1227.60 MHz (L2).
- Each spacecraft carries 2 rubidium or 2 cesium clocks.
- Several generations of GPS satellites have been built and launched (next slide):
  - Different masses and phase center
  - Different capabilities

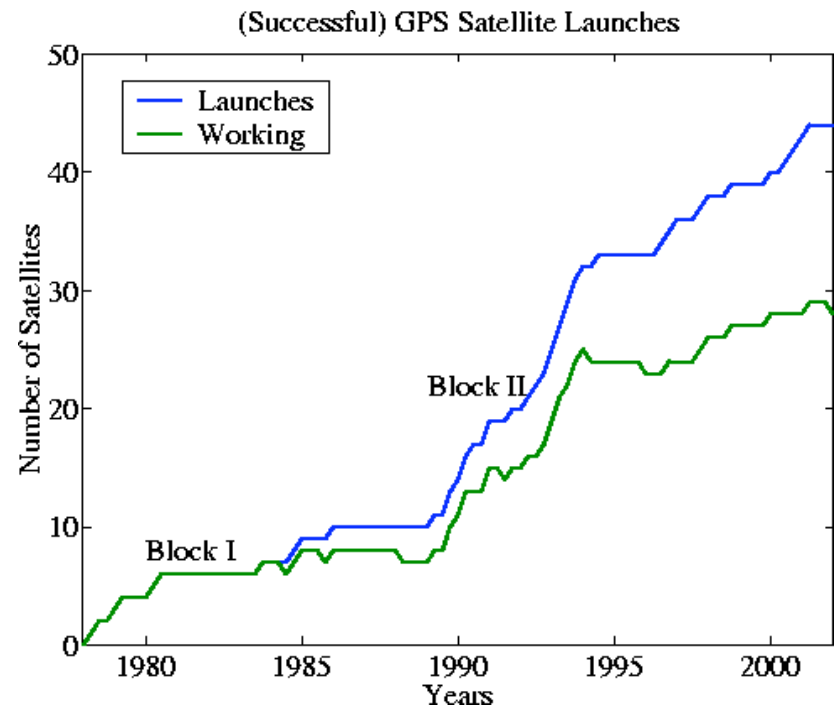


Block IIR satellite

Dec. 20, 2007, launch

GPS satellites

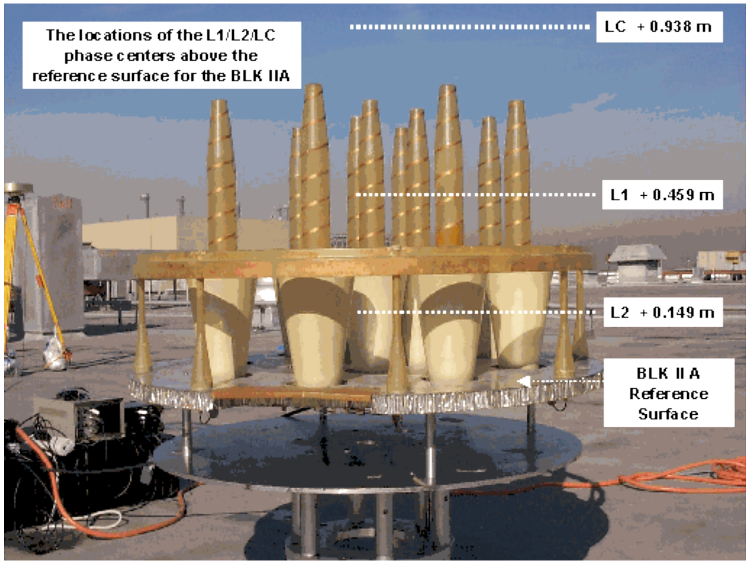
- Block I: "proof-of-concept", 1978 to 1985.
  - 11 satellites launched between 1978 and 1985 Life expectancy = 4.5 years, actual mean life = 7.1 years
  - Signal entirely accessible to civilian users
  - Last block I satellite died on Feb. 28, 1994
- Block II (II,IIA,IIR, IIR-M)
  - First launch 1989, latest launch December 20, 2007
  - Possibility to degrade the signal for civilian users (= selective availability)
- Next generation: Block III, 2010
  - Selective availability eliminated
  - Additional navigation signals
- Details on: http://www.spaceandtech.com/spacedata/constellations/navstar-gps_consum.shtml



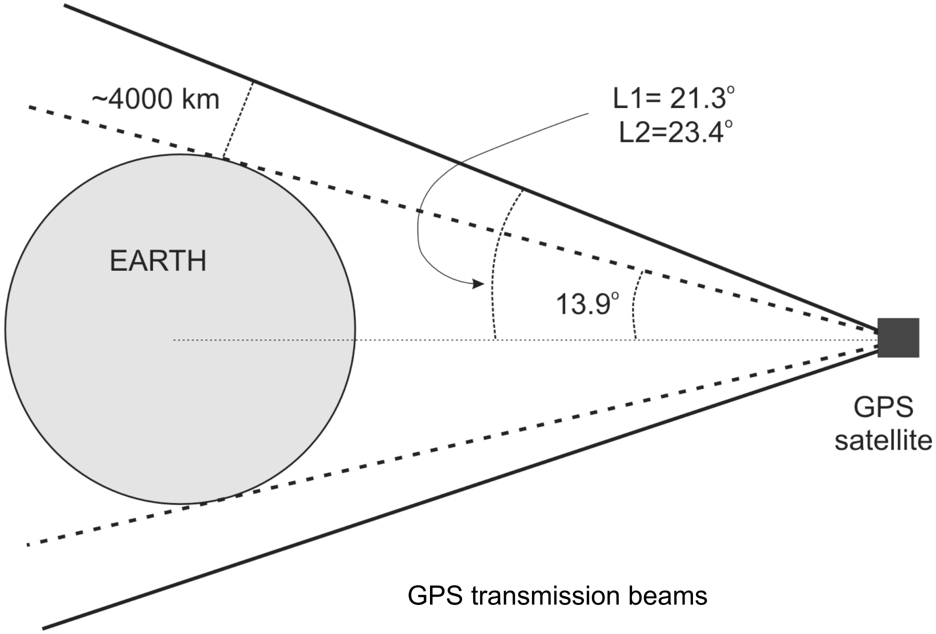
	Block I	Block II/ IIA	Block IIR	Block IIF
Number	11	28	21	12
First launch	1978	1989	1997	2005
Mass (kg)	760	1660	2000	?
Power (W)	410	710	1100	2400
Design life (yr)	5	7.5	10	15
Cost	?	43 M$	30M$	28M$
Contractor	Rockwell	Rockwell	Lockheed Martin	Rockwell

Satellite transmissions

- GPS satellites broadcast continuously on 2 frequencies in the L-band
  - 1575.42 MHz (L1)
  - 1227.60 MHz (L2)
- GPS antennas point their transmission antenna to the center of the Earth (controlled by solar sensors).
- Main beam = 21.4/23.4 (L1/L2) half width.
- Phase center of antenna does not coincide with center of gravity of satellite.



Transmission antenna of a block II-R GPS satellite



GPS transmission beams

Satellite clocks

- Frequencies broadcast by GPS satellites are derived from a **fundamental frequency of 10.23 Mhz**

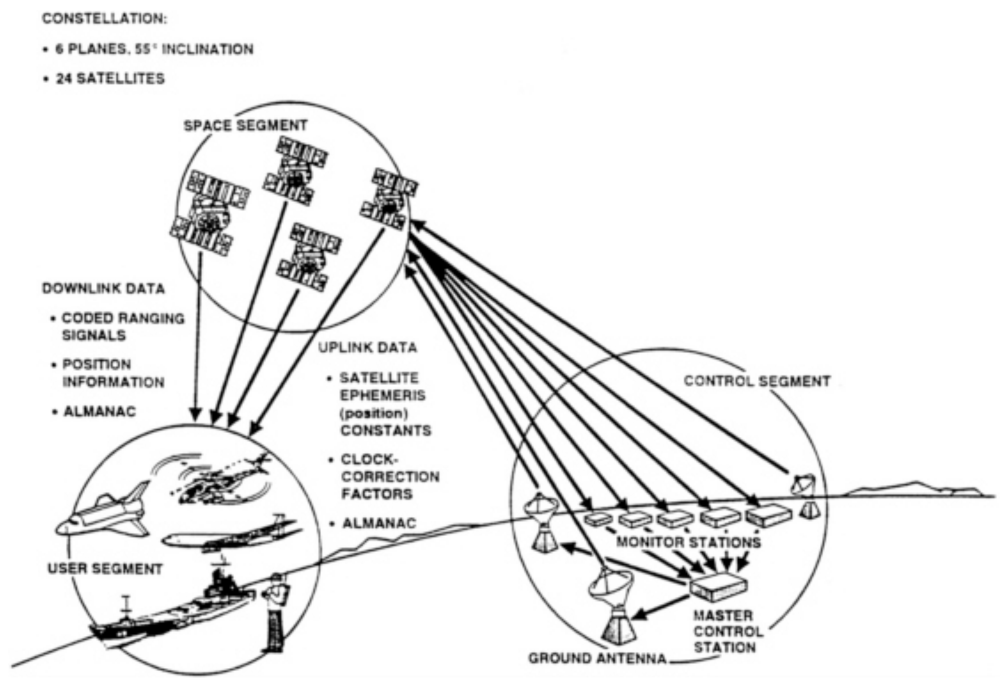
- Fundamental frequency provided by 2 or 4 atomic clocks (Ce/Rb)
  - Clocks run on GPS time = UTC not adjusted for leap seconds
  - Clock stability over 1 day = 10^{-13} (Rb) to 10^{-14} (Ce), ~ 1 ns/day
  - Clocks **synchronized between all satellites**

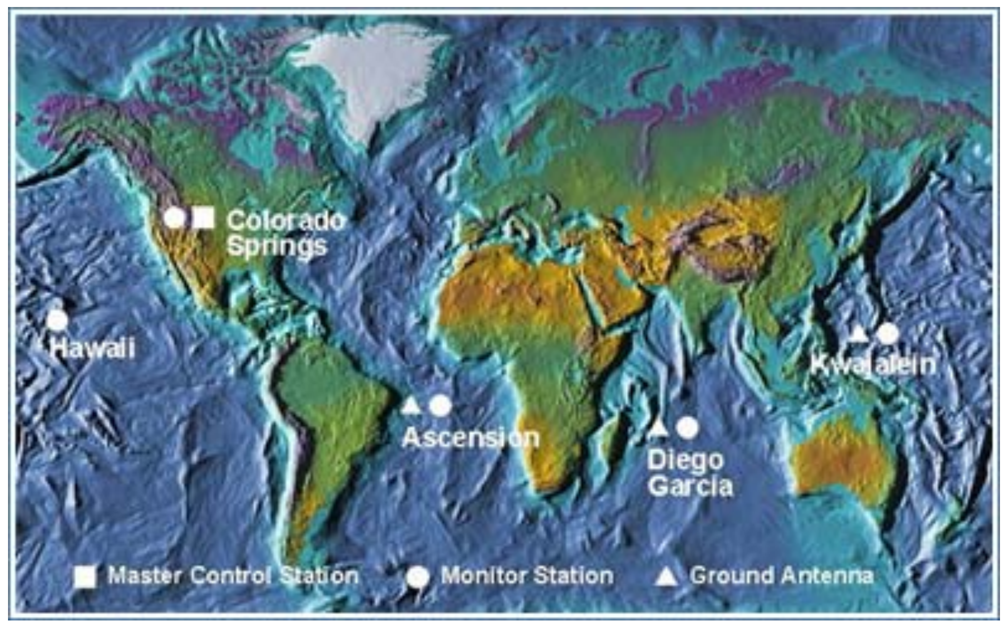
- Relativistic effects:
  - Clocks in orbit appear to run faster (38.3 microsec/day = 11.5 km/day!) => tuned at 10.22999999543 MHz before launching (g.)
  - Clocks speed is a function of orbit eccentricity (45 nsec = 14 m) => corrected at the data processing stage (s.):

$$\Delta t_R = -\frac{2}{c^2}\sqrt{a\mu} \quad e \quad \sin E$$

GPS control segment

- Monitor stations:
  - Monitor behavior of satellite orbit and clock, health of satellites
  - Uploads data to satellites according to orders from Master station.
- Master station:
  - Located at Falcon Air Force Base in Colorado Springs, Colorado
  - Calculates position and clock errors for each individual satellite based on information received from the monitor stations
  - Orders appropriate ground antennas to relay information back to satellites
  - Order maneuvers when necessary
  - Ensure clock synchronization = defines GPS time
- Computes and uploads **broadcast ephemerides** into the satellites





The GPS control segment

GPS broadcast ephemeris

- Distributed to users as part of the GPS signal in the navigation message included in the signal sent by the satellites.
- Following parameters are included:
  - Keplerian elements with periodic terms added to account for solar radiation and gravity perturbations.
  - Periodic terms are added for argument of perigee, geocentric distance and inclination.
  - Reference system is WGS84.
- Navigation message:
  - Updated every 2 hours
  - Considered valid from 2 hours before Time of Ephemeris (TOE) until 2 hours after TOE
  - Decoded by all GPS receivers from GPS signal
  - Distributed in ASCII format in *Receiver Independent Exchange* format (RINEX): [4-char][Day of year][Session].[yy]n (e.g. brdc0120.02n)

svprn	satellite PRN number
m_0	mean anomaly
t_{oe}	time of ephemeris
sqrt(a)	sqrt(semi-major axis)
Δn	variation of mean angular velocity
e	eccentricity
ω_0	argument of perigee
i_0	inclination
idot	rate of inclination
Ω_0	right ascension
Ωdot	rate of right ascension
C_{wc} C_{ws} C_{rc} C_{rs} C_{ic} C_{is}	correction coefficients

GPS broadcast ephemeris

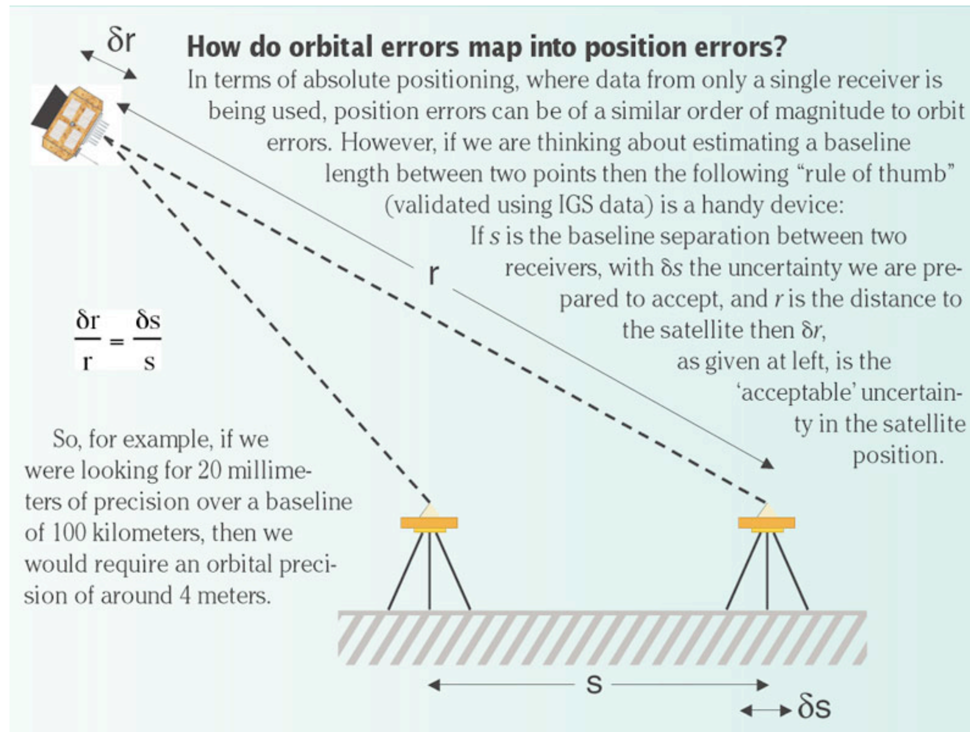
```
 2              NAVIGATION DATA                         RINEX VERSION / TYPE
comb_nav.pl         SOPAC      Fri Jan 11 04:00:16 GMT 2002 PGM / RUN BY / DATE
                 BROADCAST EPHEMERIS FILE                   COMMENT
                                                            COMMENT
      SOPAC - Scripps Orbit and Permanent Array Center      COMMENT
    IGPP - Institute of Geophysics and Planetary Physics    COMMENT
         SIO - Scripps Institution of Oceanography          COMMENT
       UCSD - University of California, San Diego           COMMENT
               email: archive@lox.ucsd.edu                  COMMENT
                  ftp://lox.ucsd.edu                        COMMENT
                  http://lox.ucsd.edu                       COMMENT
                                                            COMMENT
                                                            END OF HEADER
 1 02  1 10  0  0  0.0 2.108854241669D-04 1.591615728103D-12 0.000000000000D+00
    1.740000000000D+02-6.625000000000D+01 3.994452099249D-09 2.453913345123D+00
   -3.479421138763D-06 5.274268332869D-03 1.066364347935D-05 5.153708732605D+03
    3.456000000000D+05 1.378357410431D-07-2.987566298300D-01-2.235174179077D-08
    9.676350812908D-01 1.789375000000D+02-1.727933015620D+00-7.573886911362D-09
    3.507288949806D-10 0.000000000000D+00 1.148000000000D+03 0.000000000000D+00
    2.800000000000D+00 0.000000000000D+00-3.259629011154D-09 4.300000000000D+02
    3.455400000000D+05
 1 02  1 10  2  0  0.0 2.108966000378D-04 1.591615728103D-12 0.000000000000D+00
    1.750000000000D+02-7.146875000000D+01 3.928377918423D-09-2.779033653221D+00
   -3.810971975327D-06 5.274182534777D-03 1.071207225323D-05 5.153708749771D+03
    3.528000000000D+05 4.284083843231D-08-2.988106363767D-01 1.862645149231D-09
    9.676374409777D-01 1.748125000000D+02-1.728015465683D+00-7.532456614195D-09
    3.014411276615D-10 0.000000000000D+00 1.148000000000D+03 0.000000000000D+00
    4.000000000000D+00 0.000000000000D+00-3.259629011154D-09 4.310000000000D+02
    3.456000000000D+05
1 02  1 10  4  0  0.0 2.109077759087D-04 1.591615728103D-12 0.000000000000D+00
    1.760000000000D+02-6.781250000000D+01 3.973379792931D-09-1.728982466237D+00
   -3.568828105927D-06 5.274168215692D-03 1.092813909054D-05 5.153707971573D+03
    3.600000000000D+05 1.192092895508D-07-2.988646707189D-01 1.043081283569D-07
    9.676391291852D-01 1.698437500000D+02-1.727911795993D+00-7.616031523997D-09
    2.378670509746D-10 0.000000000000D+00 1.148000000000D+03 0.000000000000D+00
    4.000000000000D+00 0.000000000000D+00-3.259629011154D-09 4.320000000000D+02
    3.528000000000D+05
 1 02  1 10  6  0  0.0 2.109189517796D-04 1.591615728103D-12 0.000000000000D+00
    1.770000000000D+02-6.131250000000D+01 4.097670684432D-09-6.787591172017D-01
   -3.200024366379D-06 5.274690338410D-03 1.081638038158D-05 5.153708938599D+03
    3.672000000000D+05-1.154839992523D-07-2.989205995400D-01 6.146728992462D-08
    9.676398972172D-01 1.711250000000D+02-1.727978344137D+00-7.836397845996D-09
    2.517962026082D-10 1.000000000000D+00 1.148000000000D+03 0.000000000000D+00
    4.000000000000D+00 0.000000000000D+00-3.259629011154D-09 4.330000000000D+02
    3.646380000000D+05
…etc
```

TABLE A4
NAVIGATION MESSAGE FILE - DATA RECORD DESCRIPTION

OBS. RECORD	DESCRIPTION	FORMAT
PRN / EPOCH / SV CLK	- Satellite PRN number - Epoch: Toc - Time of Clock year (2 digits) month day hour minute second - SV clock bias (seconds) - SV clock drift (sec/sec) - SV clock drift rate (sec/sec2)	I2, 5I3, F5.1, 3D19.12
BROADCAST ORBIT - 1	- IODE Issue of Data, Ephemeris - Crs (meters) - Delta n (radians/sec) - M0 (radians)	3X,4D19.12
BROADCAST ORBIT - 2	- Cuc (radians) - e Eccentricity - Cus (radians) - sqrt(A) (sqrt(m))	3X,4D19.12
BROADCAST ORBIT - 3	- Toe Time of Ephemeris (sec of GPS week) - Cic (radians) - OMEGA (radians) - CIS (radians)	3X,4D19.12
BROADCAST ORBIT - 4	- i0 (radians) - Crc (meters) - omega (radians) - OMEGA DOT (radians/sec)	3X,4D19.12
BROADCAST ORBIT - 5	- IDOT (radians/sec) - Codes on L2 channel - GPS Week # (to go with TOE) - L2 P data flag	3X,4D19.12
BROADCAST ORBIT - 6	- SV accuracy (meters) - SV health (MSB only) - TGD (seconds) - IODC Issue of Data, Clock	3X,4D19.12
BROADCAST ORBIT - 7	- Transmission time of message (sec of GPS week, derived e.g. from Z-count in Hand Over Word (HOW) - spare - spare - spare	3X,4D19.12

GPS broadcast ephemeris

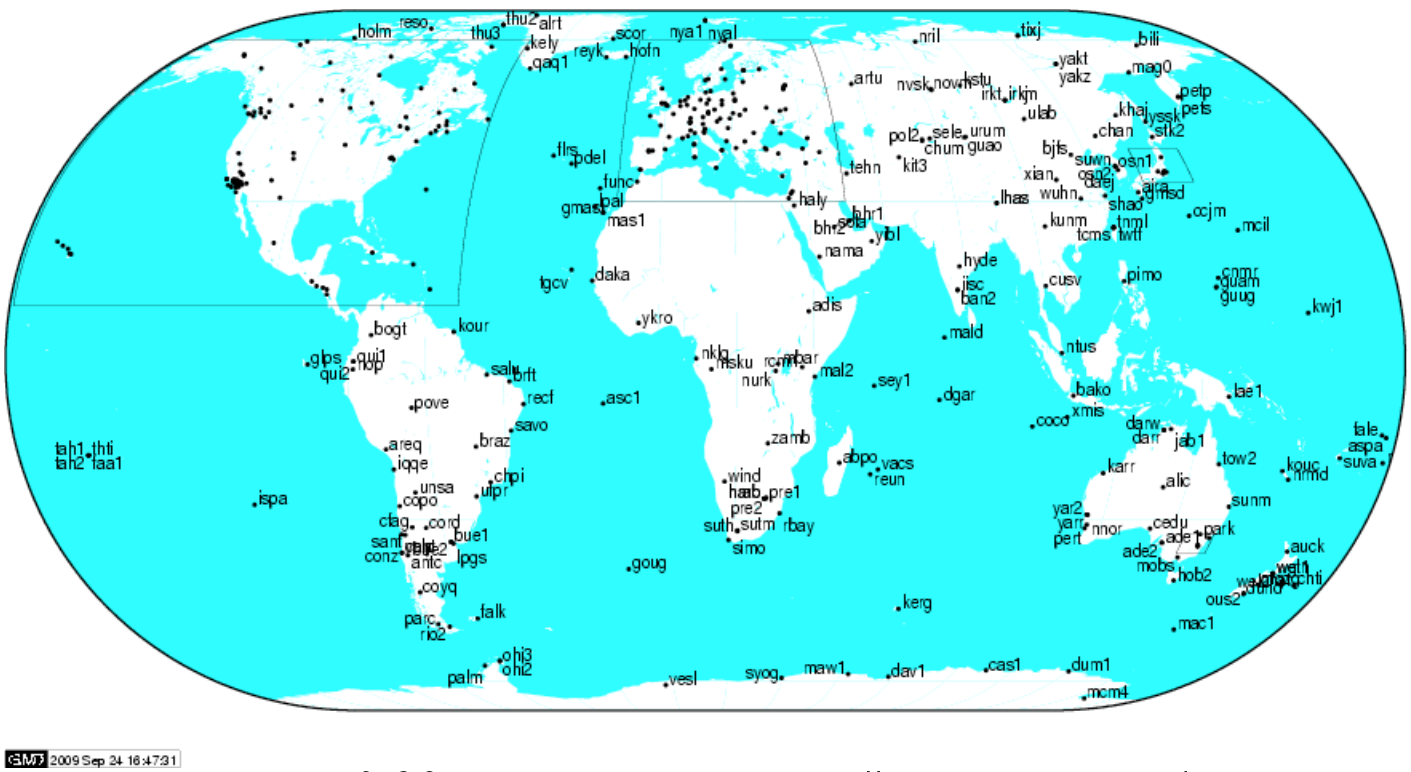
- Accuracy of broadcast ephemeris:
  - ~ 10 m
  - Can be degraded by the DoD to 300 m
  - $\delta s/s = \delta r/r$ => if δr = 200 m (r = 20 000 km):
    - s = 100 km => δr = 1 m, not sufficient for geophysical applications
    - s = 1 km => δr = 1 cm, sufficient for surveying
- Broadcast ephemerides are not accurate enough for most geophysical applications.
- Requirements for geophysical applications (mm – cm level):
  - Accurate orbits: e.g. 10 cm orbit error => 0.5 mm error on a 100 km baseline
  - Independent from the DoD



For a given satellite orbit error, δr, the error in the estimated baseline vector length, δs, in relative GPS positioning, is roughly proportional to the ratio of baseline length to the distance to the satellite.

IGS: The International GNSS Service for Geodynamics

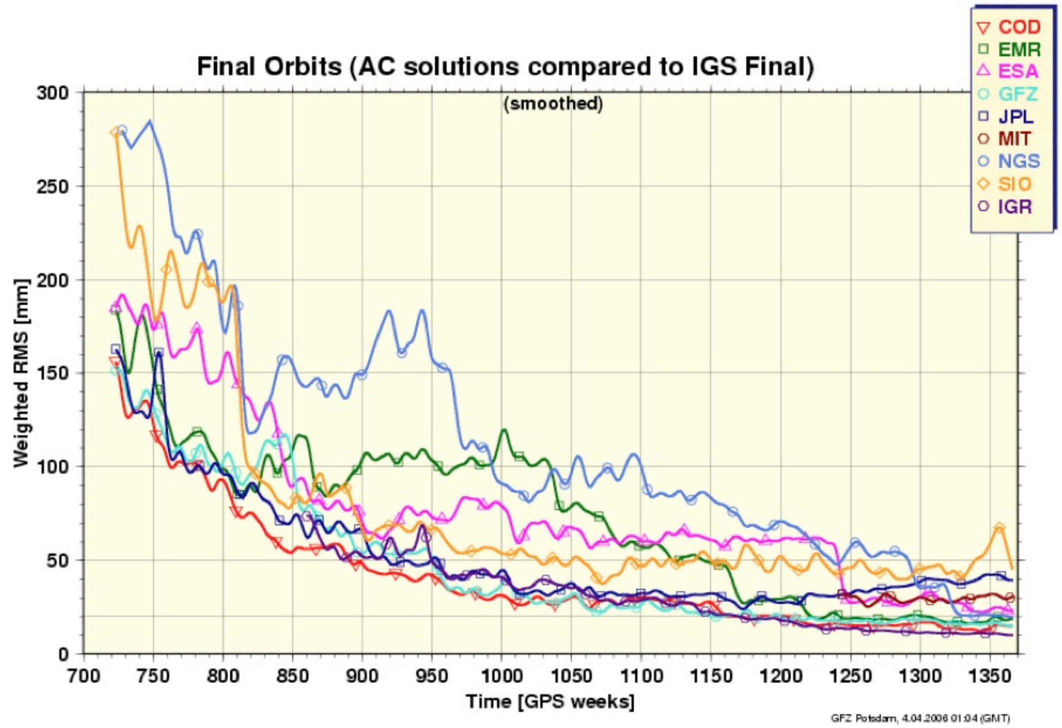
- International service of the IAG (International Association of Geodesy)
- Coordinates data archiving and processing of a global control network of >350 dual-frequency permanent GPS stations
- Test campaign in 1992, routine operations since 1994
- Provides precise GPS products, in particular satellite orbits



Map of IGS station distribution - http://igscb.jpl.nasa.gov/

IGS orbits

- Core network of globally distributed, high-quality, continuous GPS stations
- Data processed by IGS analysis centers
- Analysis center coordinator produces weighted average = final IGS orbit
- Process takes 2 weeks (to ensure best station distribution and solution quality)
- IGS also provides satellite clock corrections
- Other products:
  - Rapid
  - Predicted



Graph courtesy Analysis Coordinator G. Gendt, GFZ Potsdam

Orbits Type	Accuracy/clock accuracy	Latency	Updates	Sample interval
Broadcast	~260 cm/~7 ns	Real-time		daily
Final	< 5 cm/0.1 ns	14 days	Weekly	15 min
Rapid	5 cm/0.2 ns	17 hours	Daily	15 min
Predicted (ultra-rapid)	~10 cm/~7 ns	Real-time	Twice daily	15 min

Orbit comparison: IGS - broadcast