Gravitational Waves

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Educator's guide from ligo.caltech.edu

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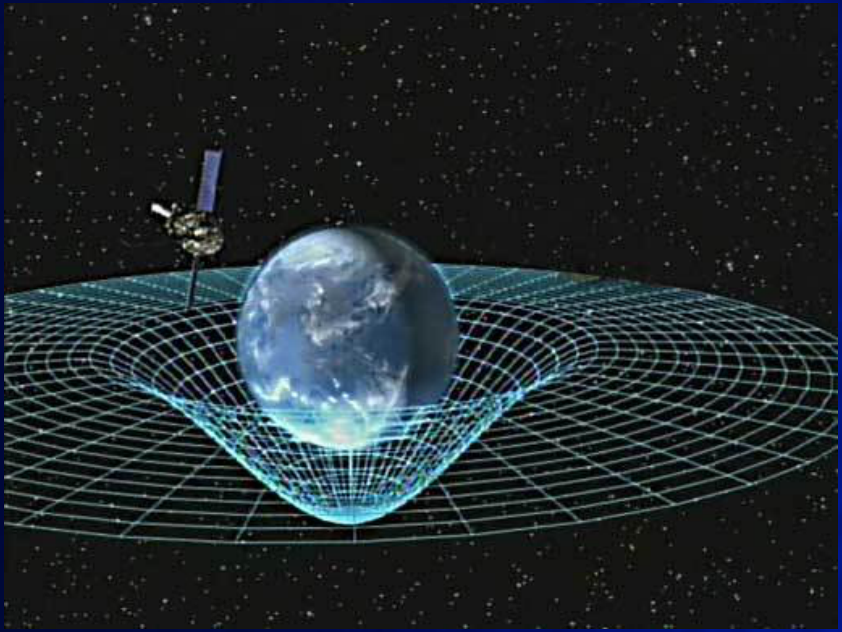
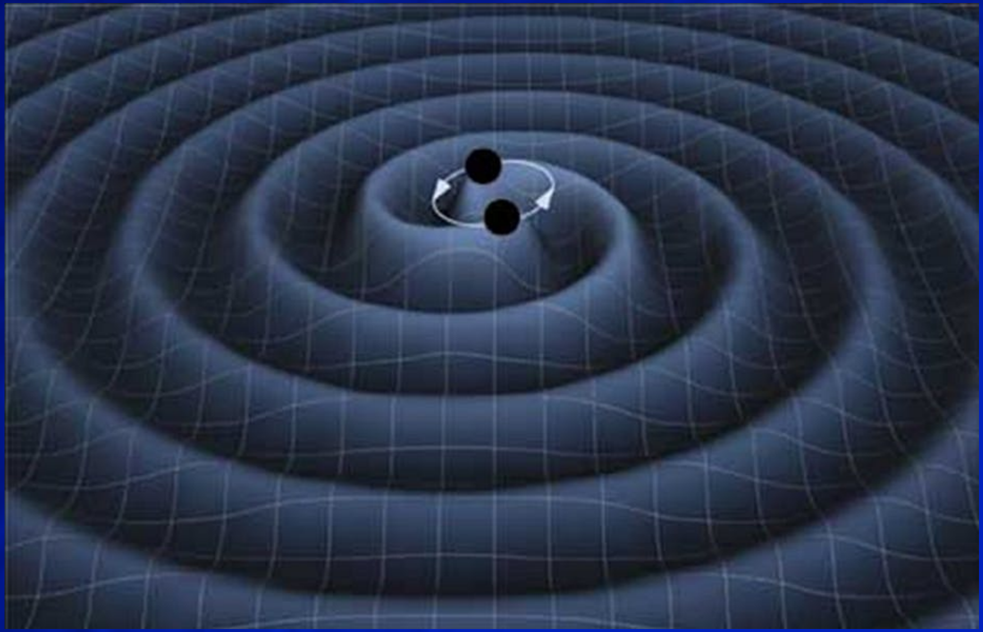
An interesting example of signal processing tools used in science is the observation of theoretically predicted gravitational waves.

Gravity waves are a fluid dynamics phenomena related for example, to buoyancy driven mixing at a layer boundary. Gravitational waves are astronomical disturbances (ripples) in space-time predicted by General Relativity primarily from rotating objects (e.g. a pair of black holes).





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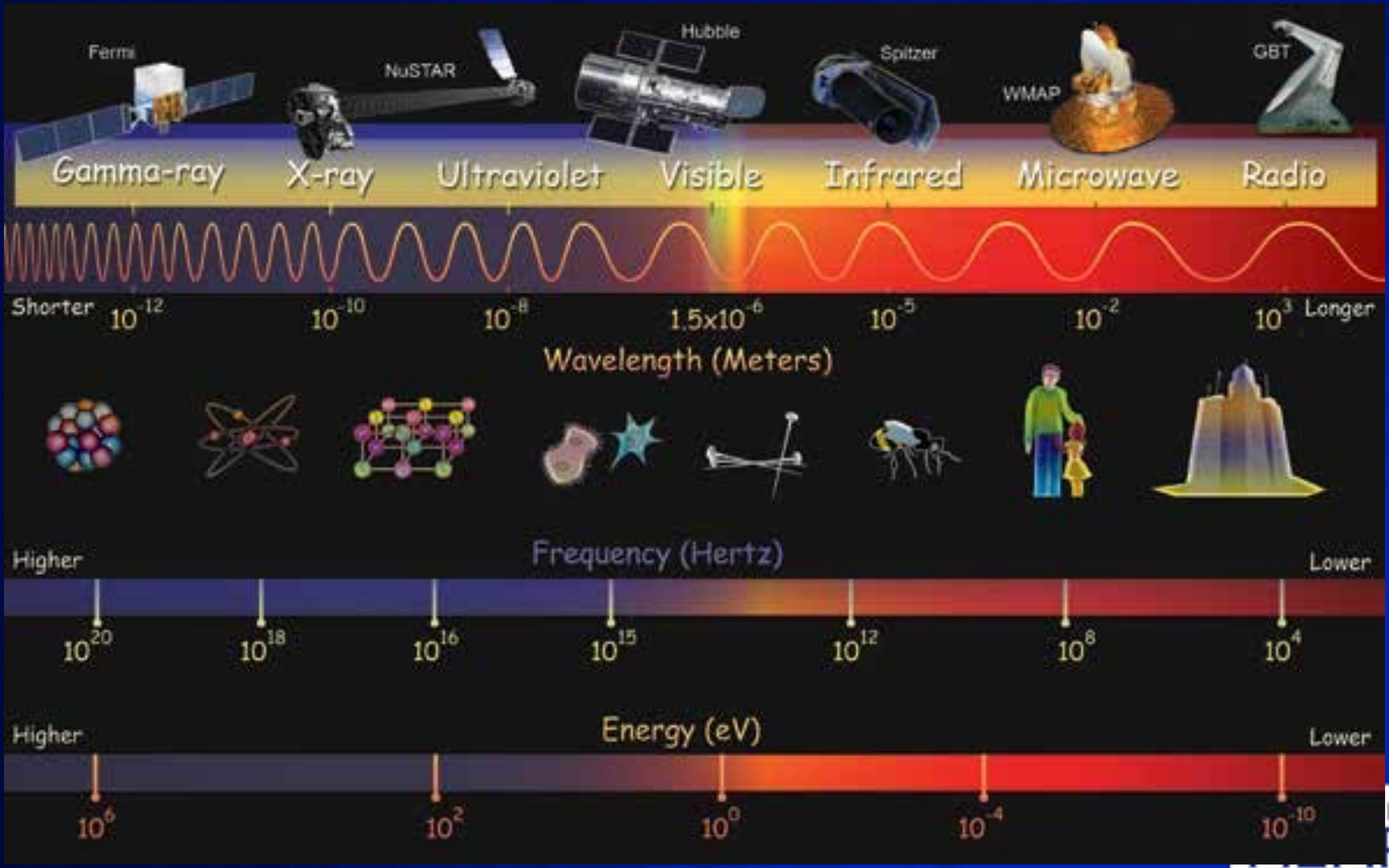
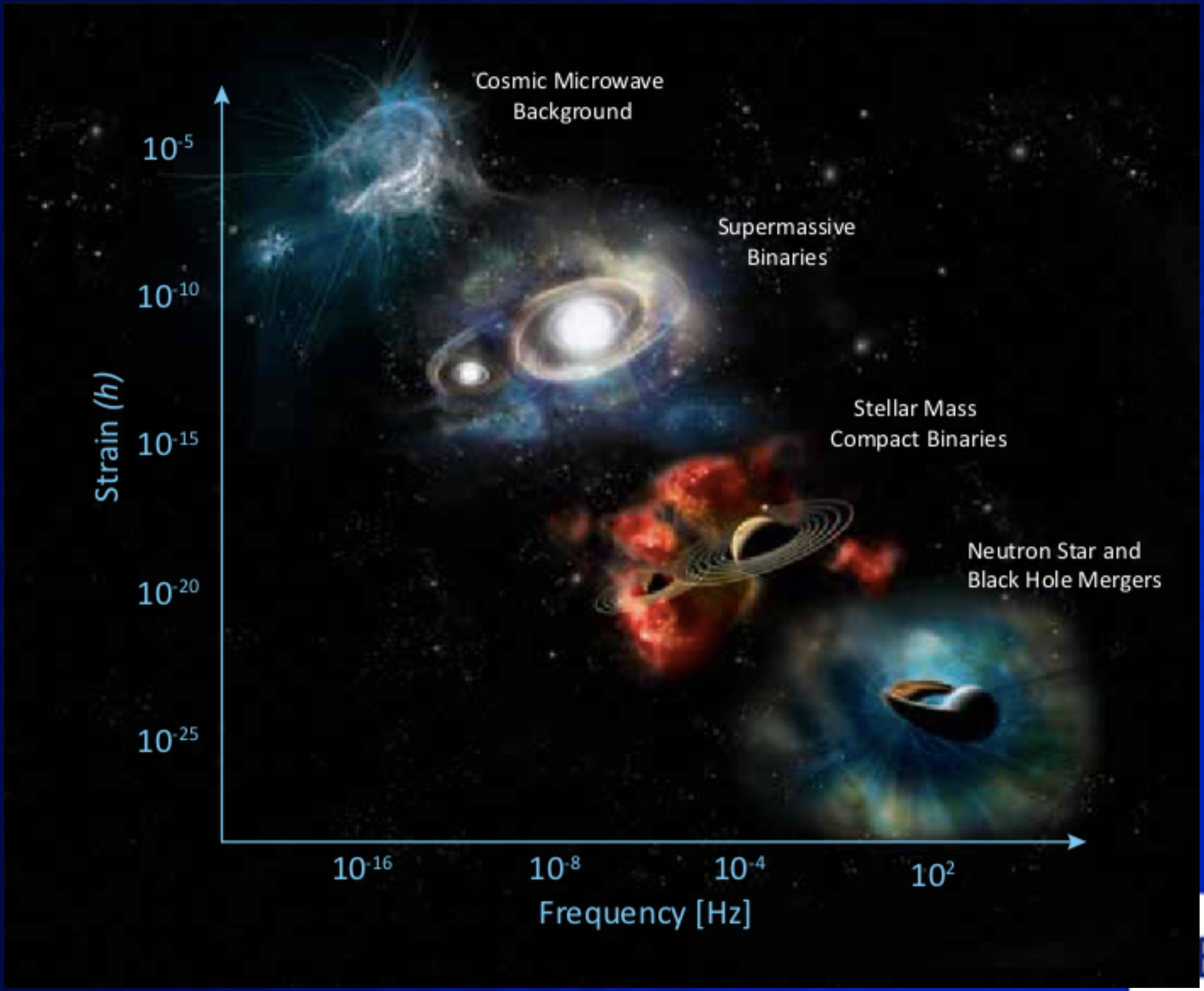
Astronomers traditionally rely on observations of electromagnetic radiation. Different phenomena are best observed in different frequency bands.



Likewise, different sources produce gravitational waves in different frequency bands.



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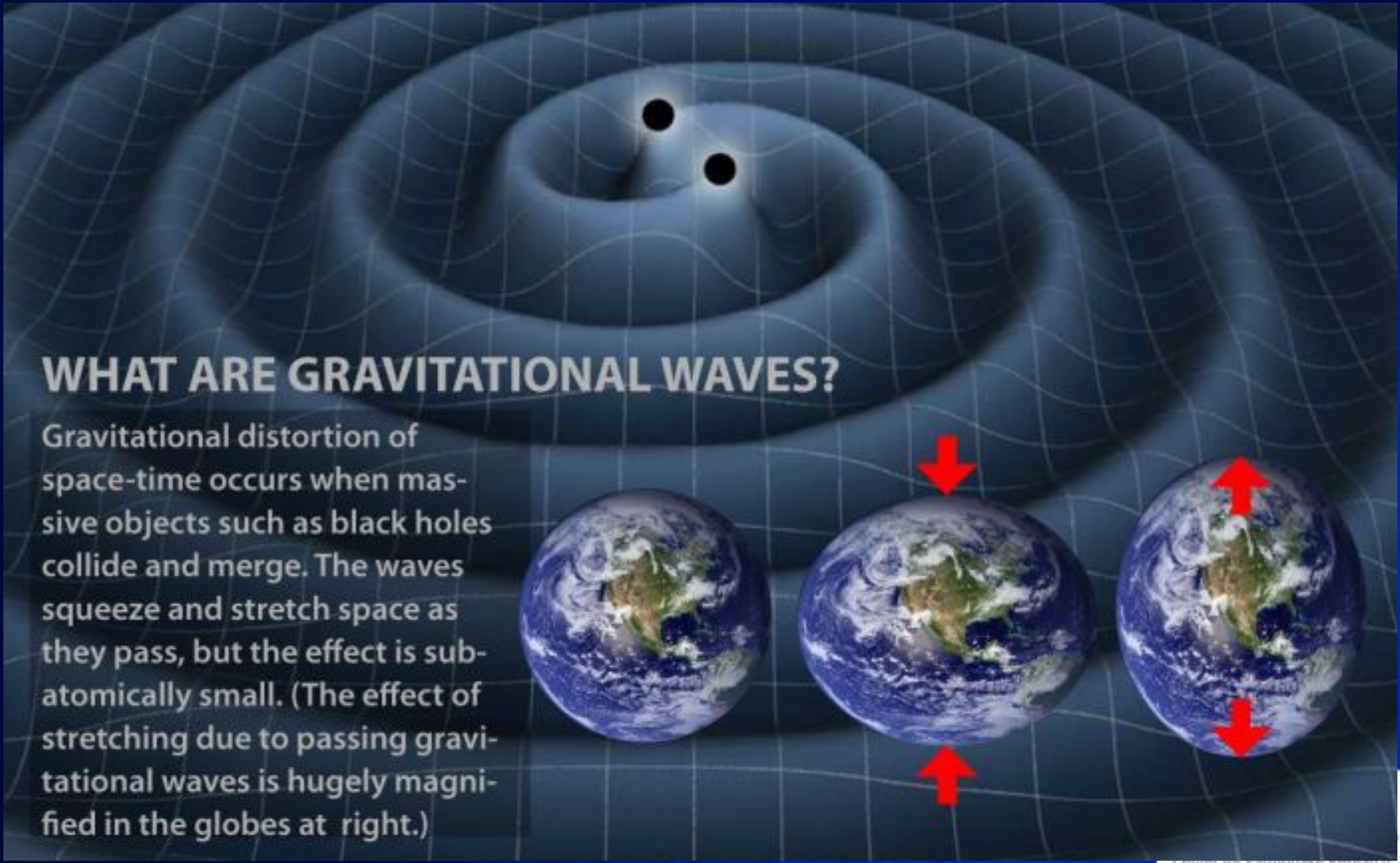
Gravitational waves can be observed by the effect of space-time distortion on the earth.



and Information

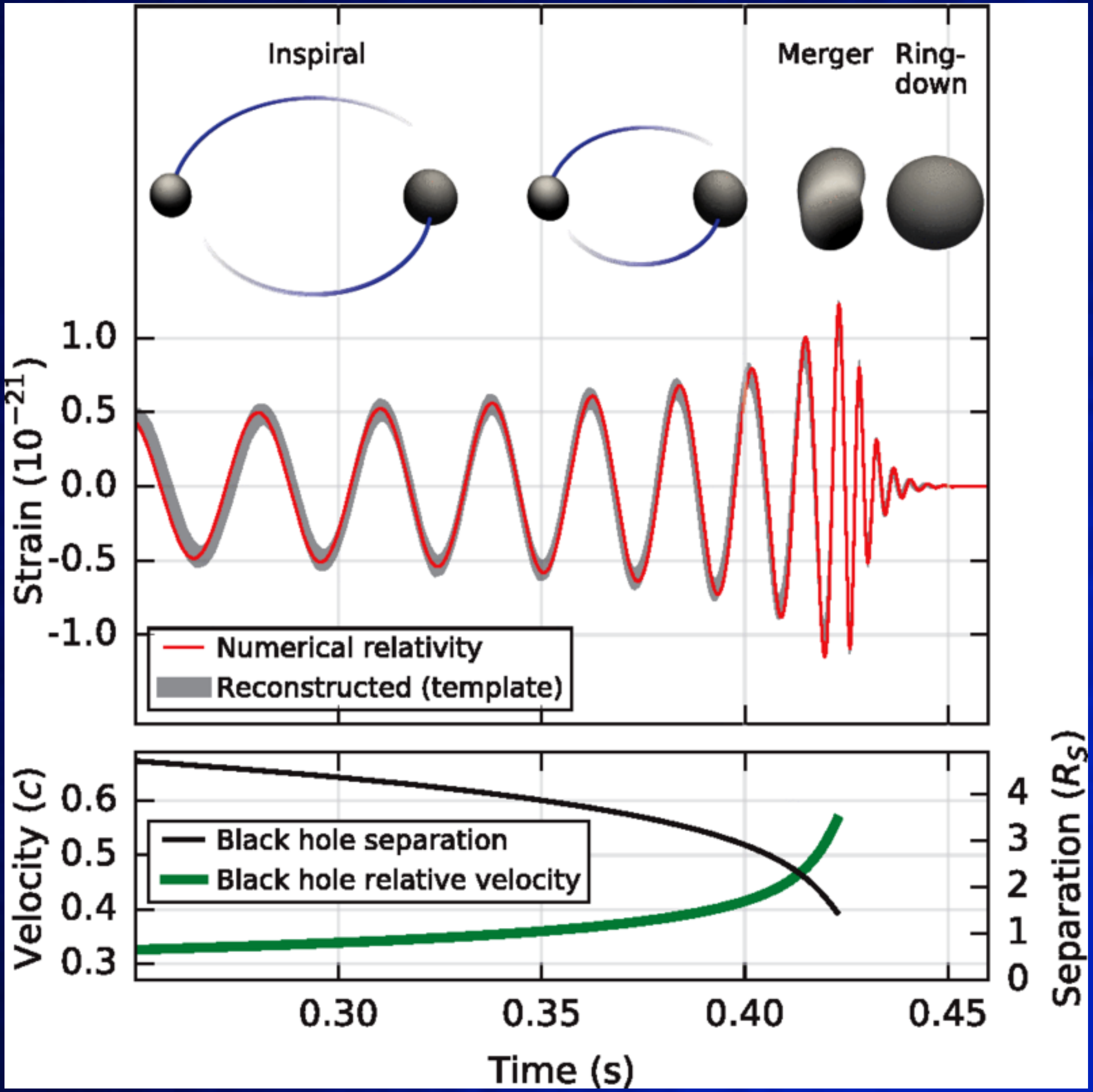
First observed on September 14, 2015 by the Laser Interferometer Gravitational-wave Observatory (LIGO).

LIGO (Laser Interferometer Gravitational-wave Observatory) is the world's largest gravitational wave observatory. LIGO consists of two laser interferometers located thousands of kilometers apart, one in Livingston, Louisiana and the other in Hanford, Washington. LIGO uses the physical properties of light and of space itself to detect gravitational waves. It was funded by the US National Science Foundation, and it is managed

by Caltech and MIT. Hundreds of scientists in the LIGO Scientific Collaboration, in many countries, contribute to the astrophysical and instrument science of LIGO. There are also other gravitational wave observatories in the world, including Virgo in Italy and GEO 600 in Germany.

Figure 9 LIGO Hanford and LIGO Livingston.
Credit: Caltech/MIT/LIGO

Signal Processing is used to separate the signal of interest from background "noise" of many different sources (including earthquakes). Knowing something about the signal can aid in filter design.

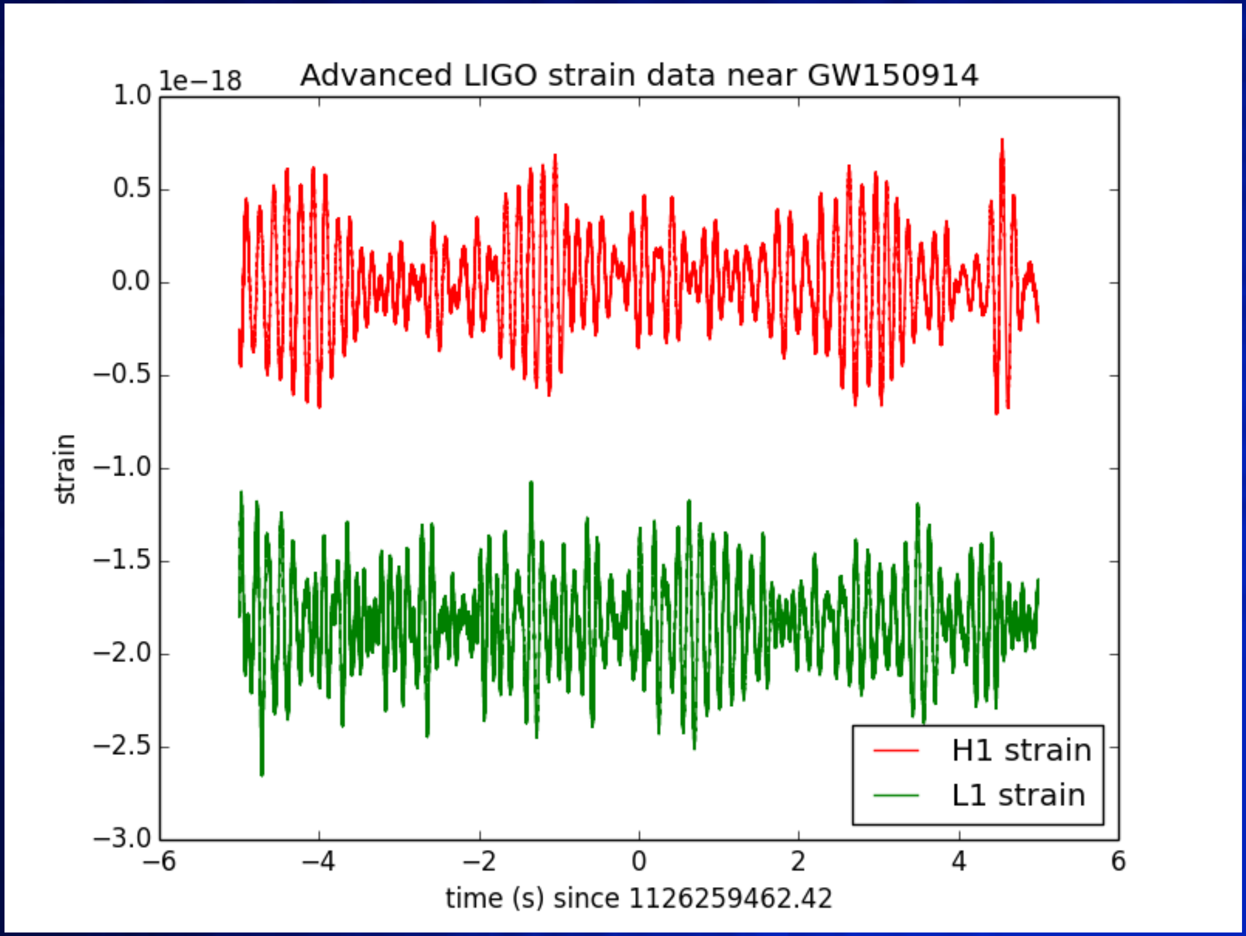


The following slides are material derived from the LIGO tutorial.

http://www.ceri.memphis.edu/people/mwithers/CERI7106/other/GW150914_tutorial.html

LIGO data for event GW150914 (Gravitational Wave observed on Sep. 14, 2015) are available online along with a python processing script to make the following figures.

The data are dominated by **low frequency noise**; there is no way to see a signal here, without some signal processing.



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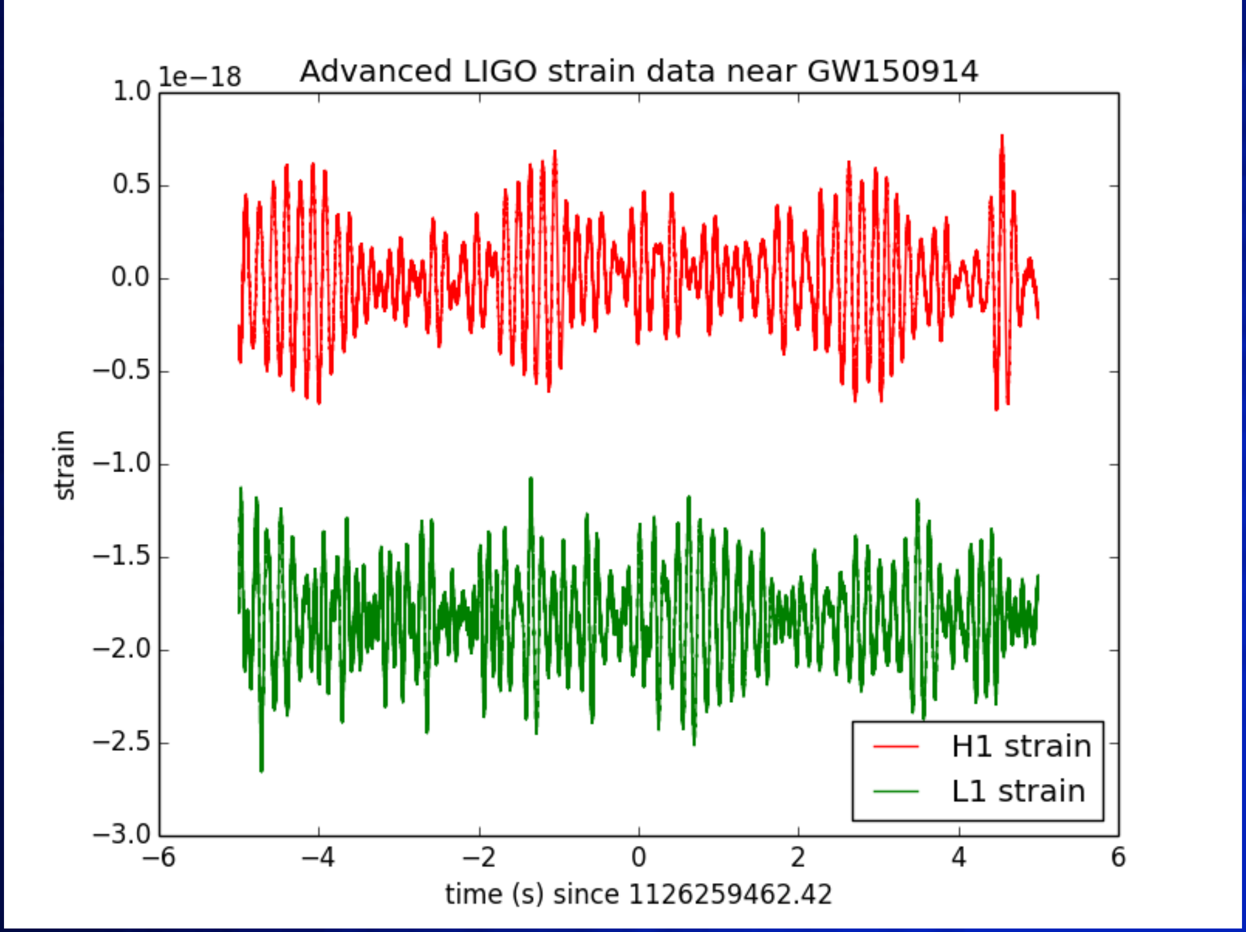
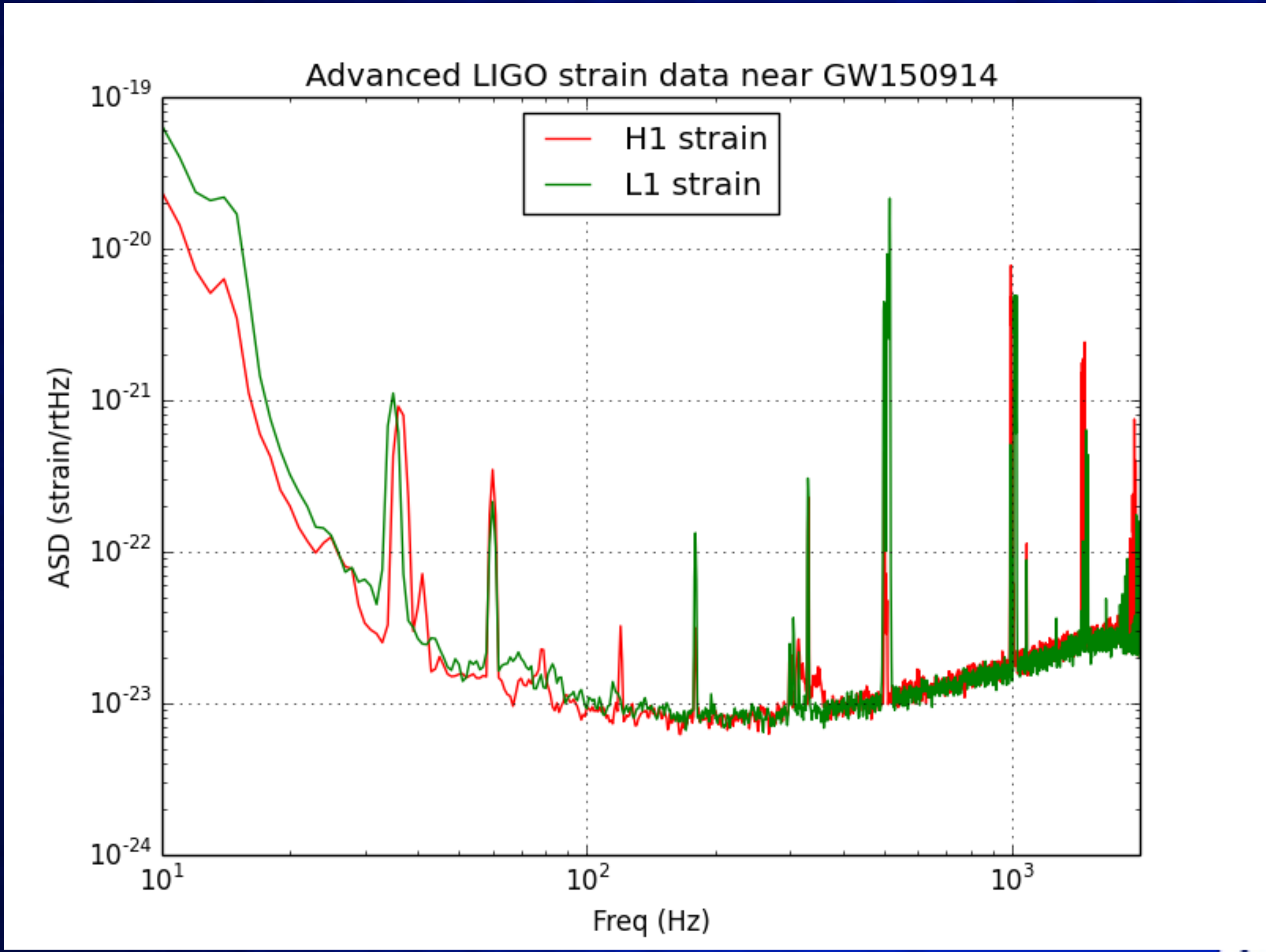
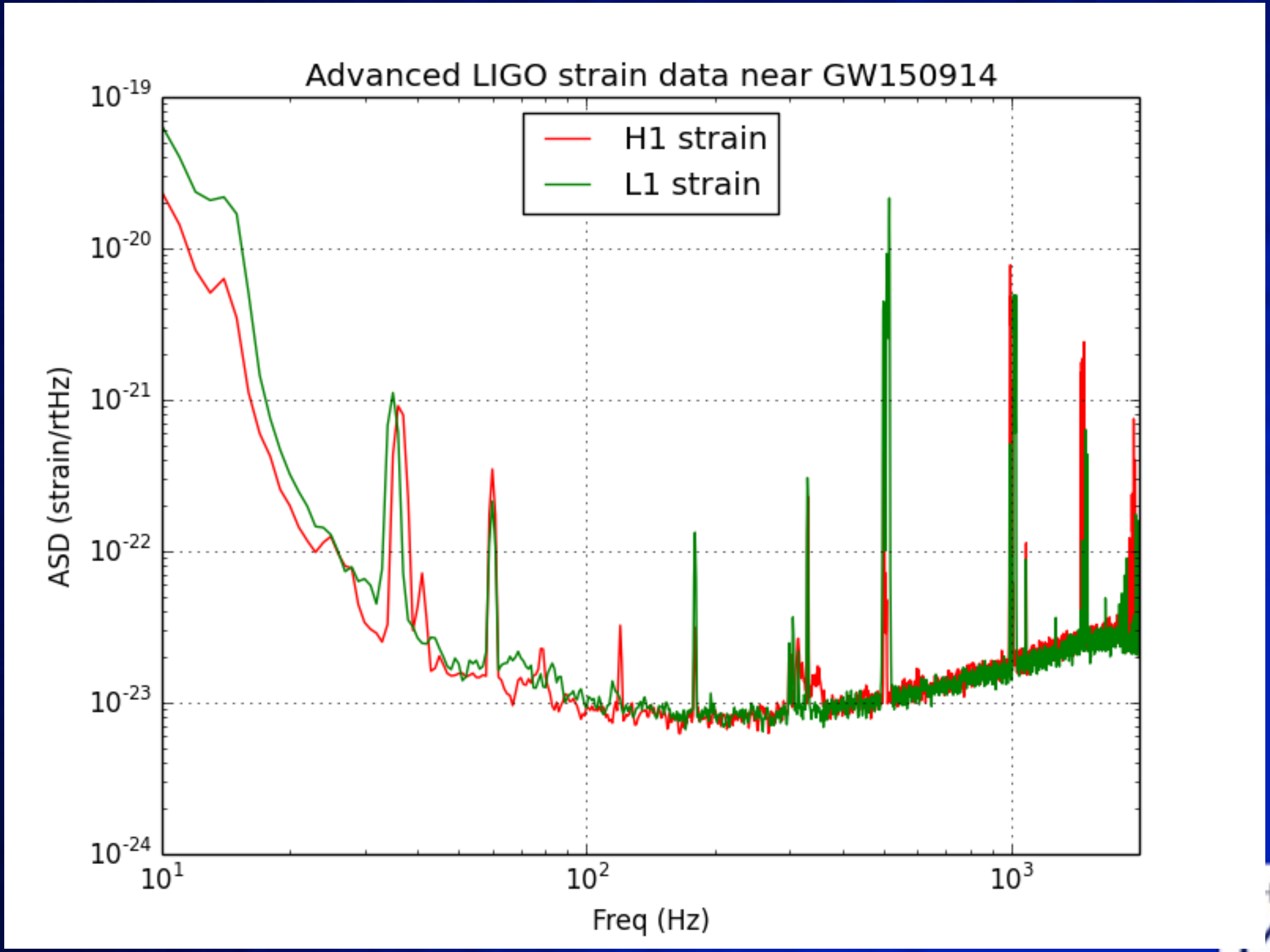
There are very low frequency oscillations that are putting the mean of the L1 strain at -2.0e-18 at the time around this event, so it appears offset from the H1 strain. These low frequency oscillations are essentially ignored in LIGO data analysis (see bandpassing, below).



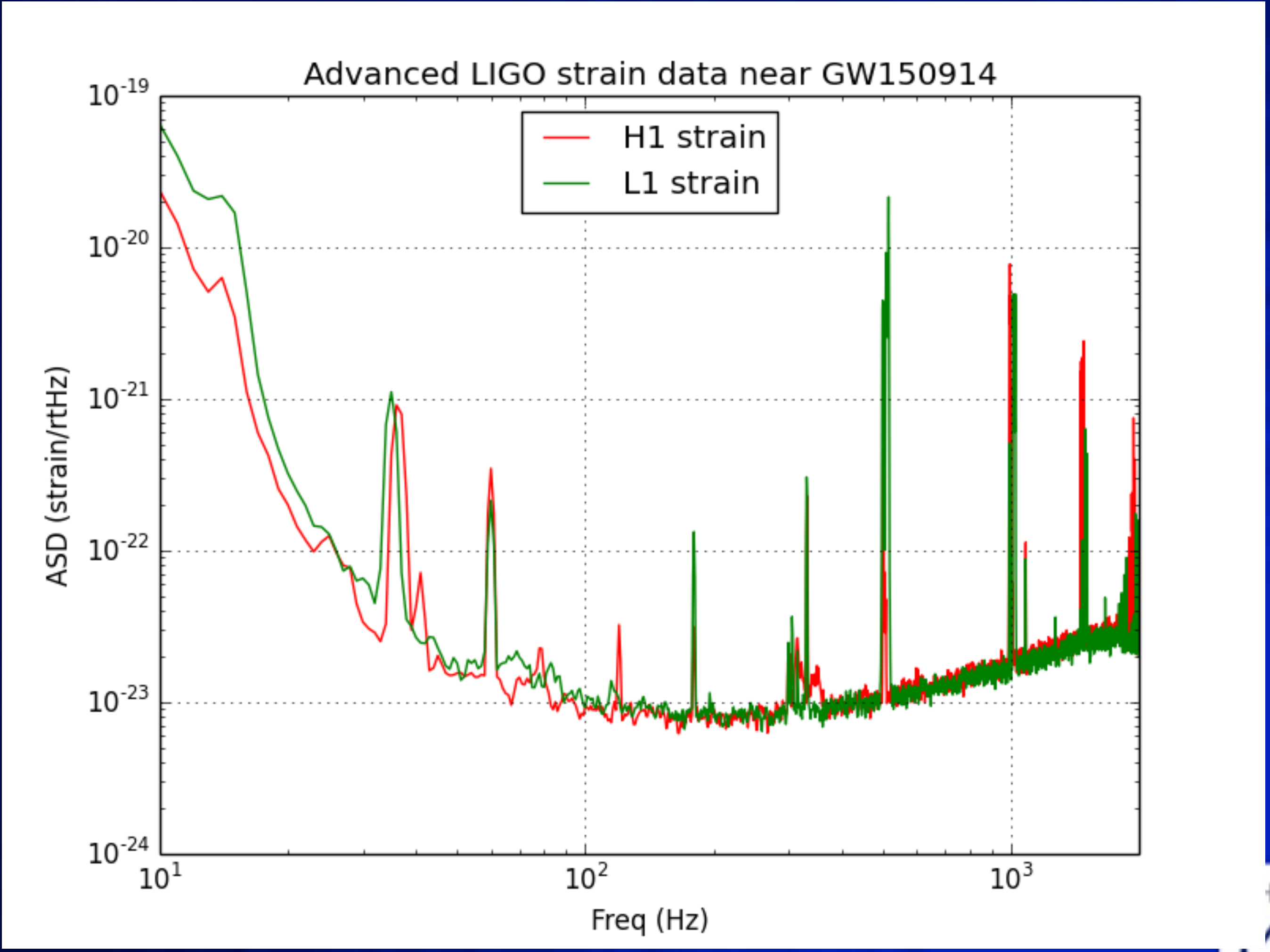
Plotting these data in the Fourier domain gives us an idea of the frequency content of the data. A way to visualize the frequency content of the data is to plot the amplitude spectral density, ASD.



The ASDs are the square root of the power spectral densities (PSDs), which are averages of the square of the fast fourier transforms (FFTs) of the data.



They are an estimate of the "strain-equivalent noise" of the detectors versus frequency, which limit the ability of the detectors to identify GW signals.



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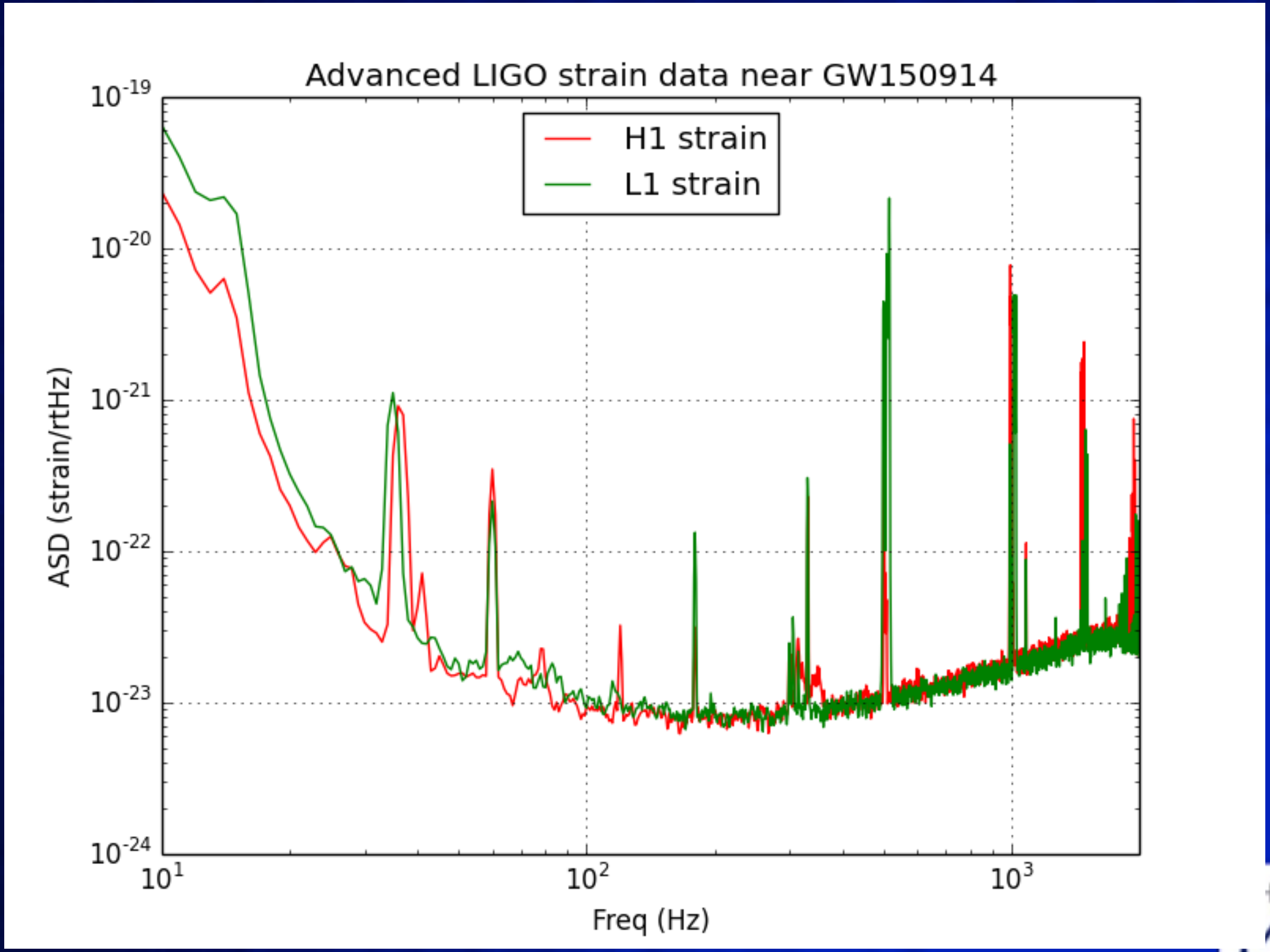
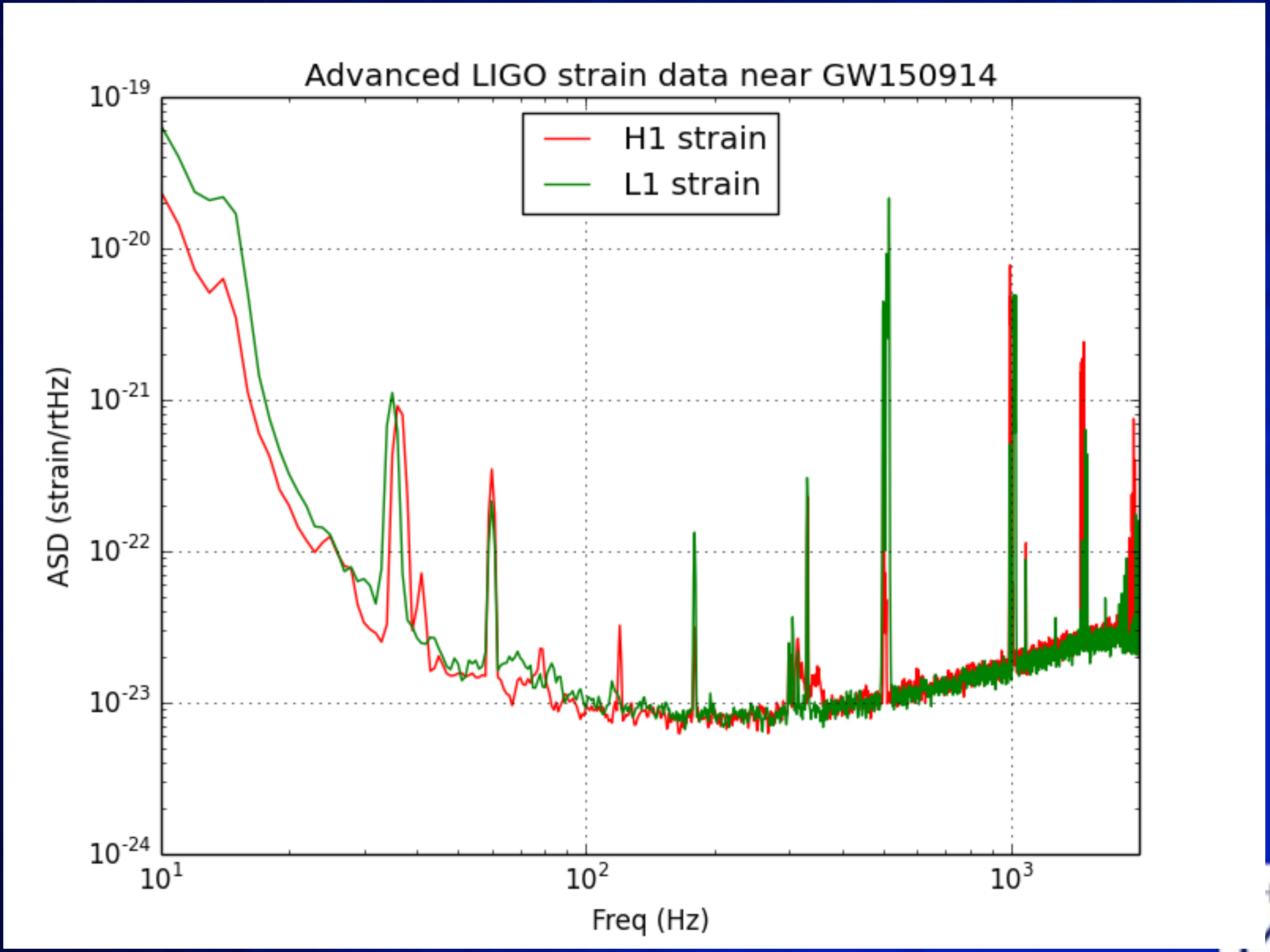
They are in units of strain/sqrt(Hz). So, if you want to know the root-mean-square (rms) strain noise in a frequency band, integrate (sum) the squares of the ASD over that band, then take the square-root.



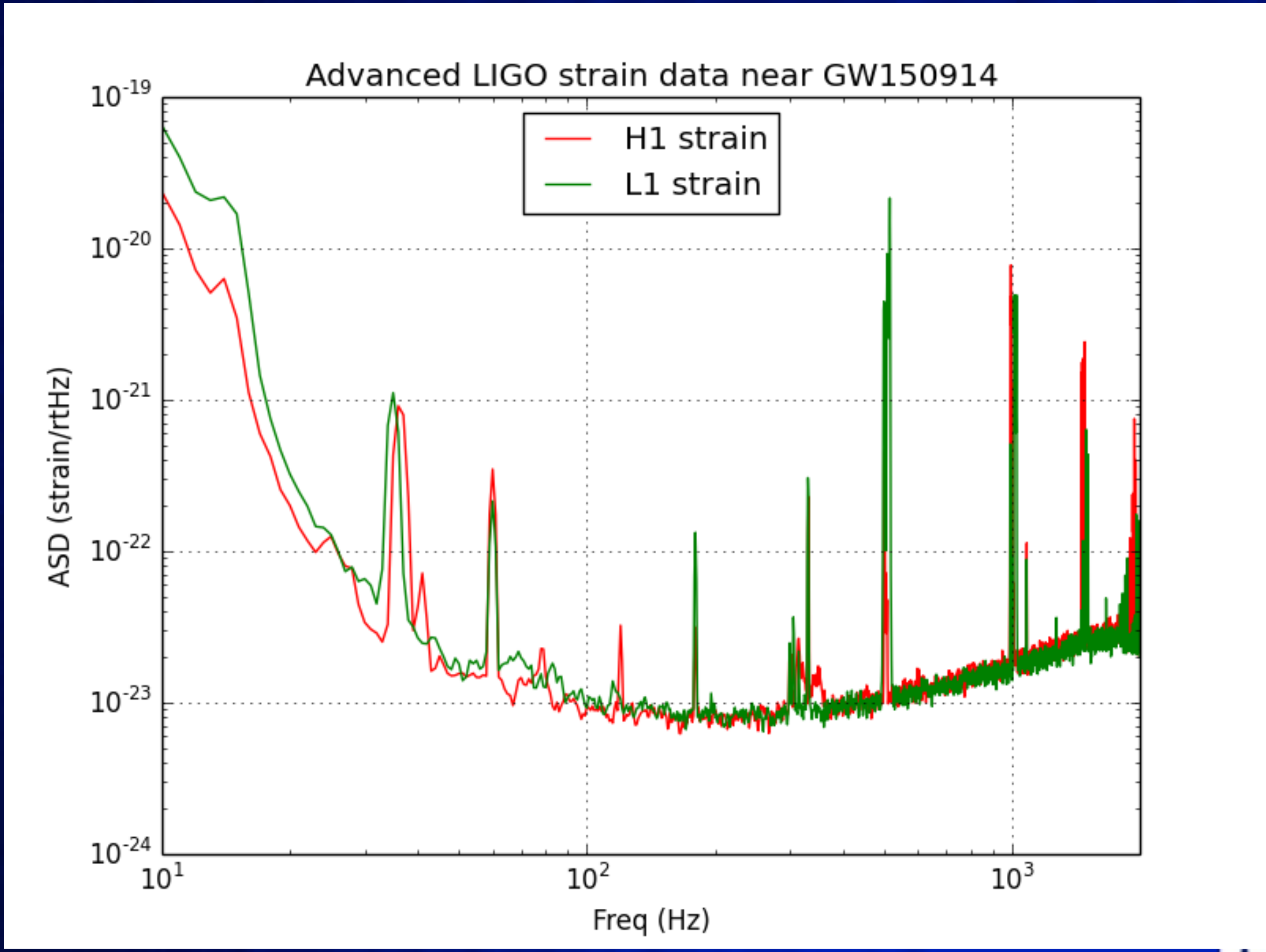
There's a signal in these data!

For the moment, let's ignore that, and assume it's all noise.



NOTE that we only plot the data between fmin = 10 Hz and fmax = 2000 Hz.

Below fmin, the data **are not properly calibrated**. That's OK, because the noise is so high below fmin that LIGO cannot sense gravitational wave strain from astrophysical sources in that band.



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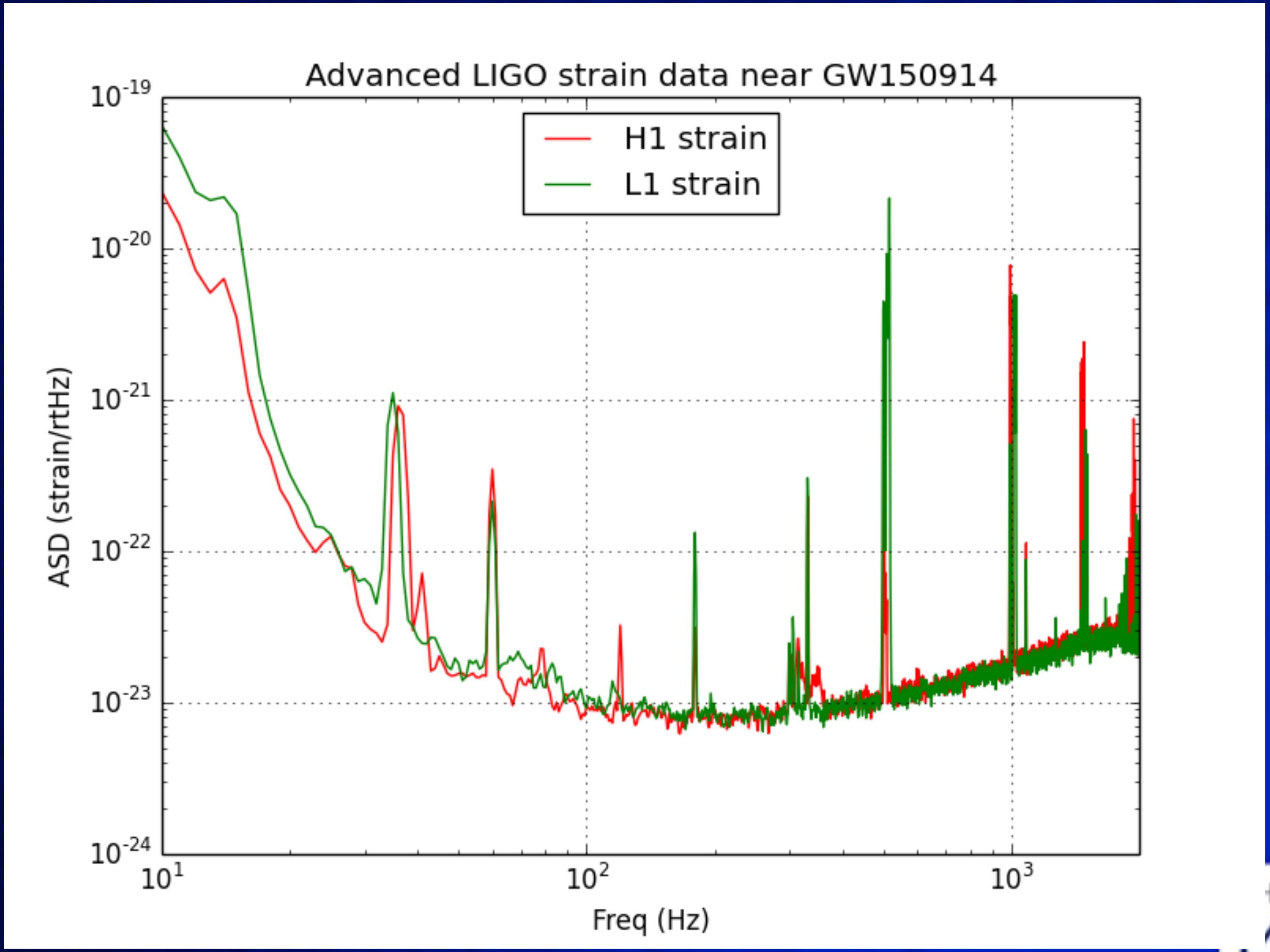
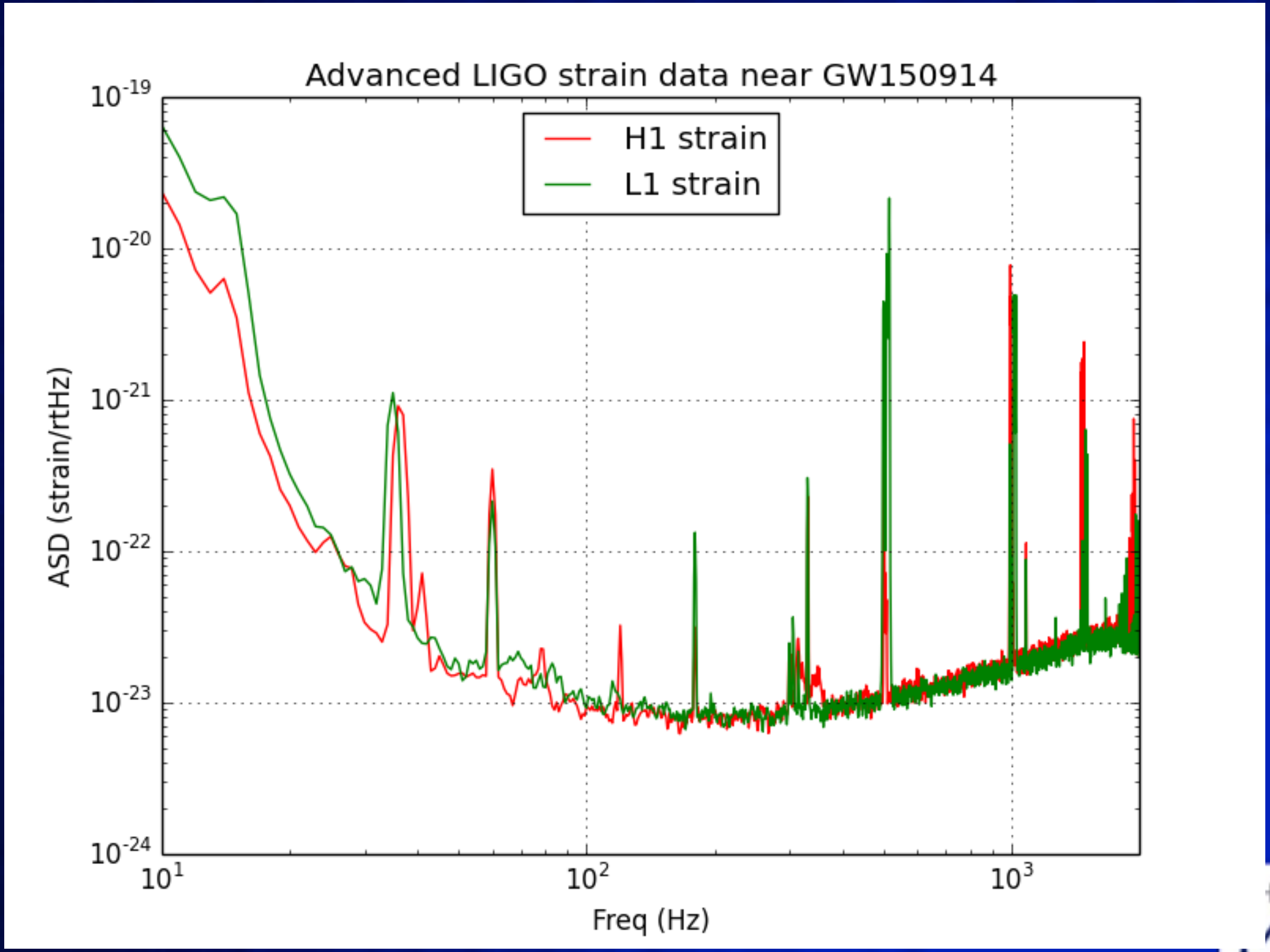
The sample rate is fs = 4096 Hz (2^12 Hz), so the data cannot capture frequency content above the Nyquist frequency = fs/2 = 2048 Hz. That's OK, because GW150914 only has detectable frequency content in the range 20 Hz - 300 Hz.

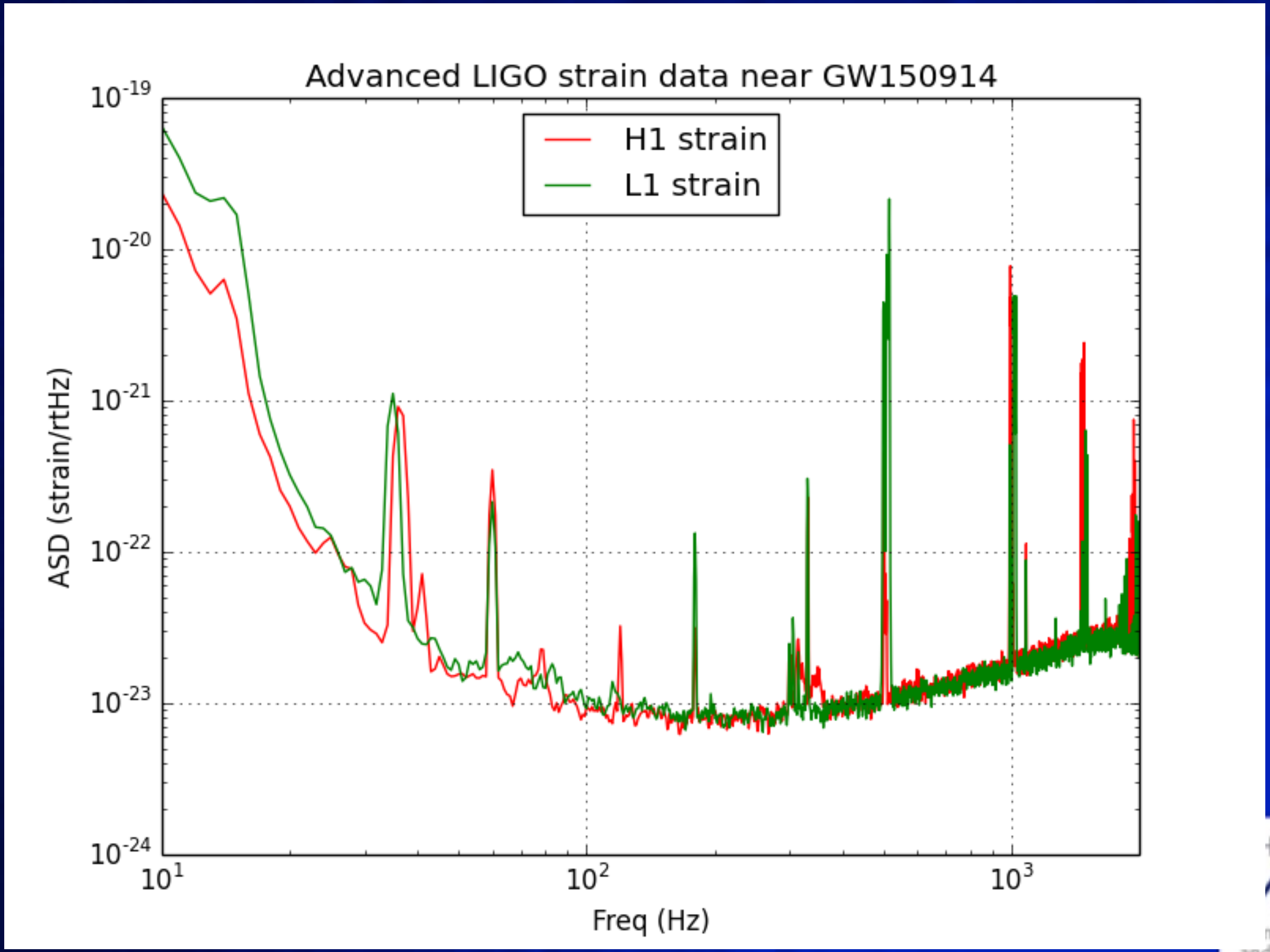


You can see strong spectral lines in the data; they are all of instrumental origin. Some are engineered into the detectors (mirror suspension resonances at ~500 Hz and harmonics, calibration lines, control dither lines, etc) and some (60 Hz and harmonics) are unwanted. We'll return to these, later.



You can't see the signal in this plot, since it is relatively weak and less than a second long, while this plot averages over 32 seconds of data. So this plot is entirely dominated by instrumental noise.

Later on in this tutorial, we'll look at the data sampled at the full 16384 Hz (2^14 Hz).



The data in the ASD were very strongly "colored" - noise fluctuations were much larger at low and high frequencies and near spectral lines, reaching a roughly flat ("white") minimum in the band around 80 to 300 Hz.

We can "whiten" the data (dividing it by the noise amplitude spectrum, in the fourier domain), suppressing the extra noise at low frequencies and at the spectral lines, to better see the weak signals in the most sensitive band.

Whitening is always one of the first steps in astrophysical data analysis (searches, parameter estimation).

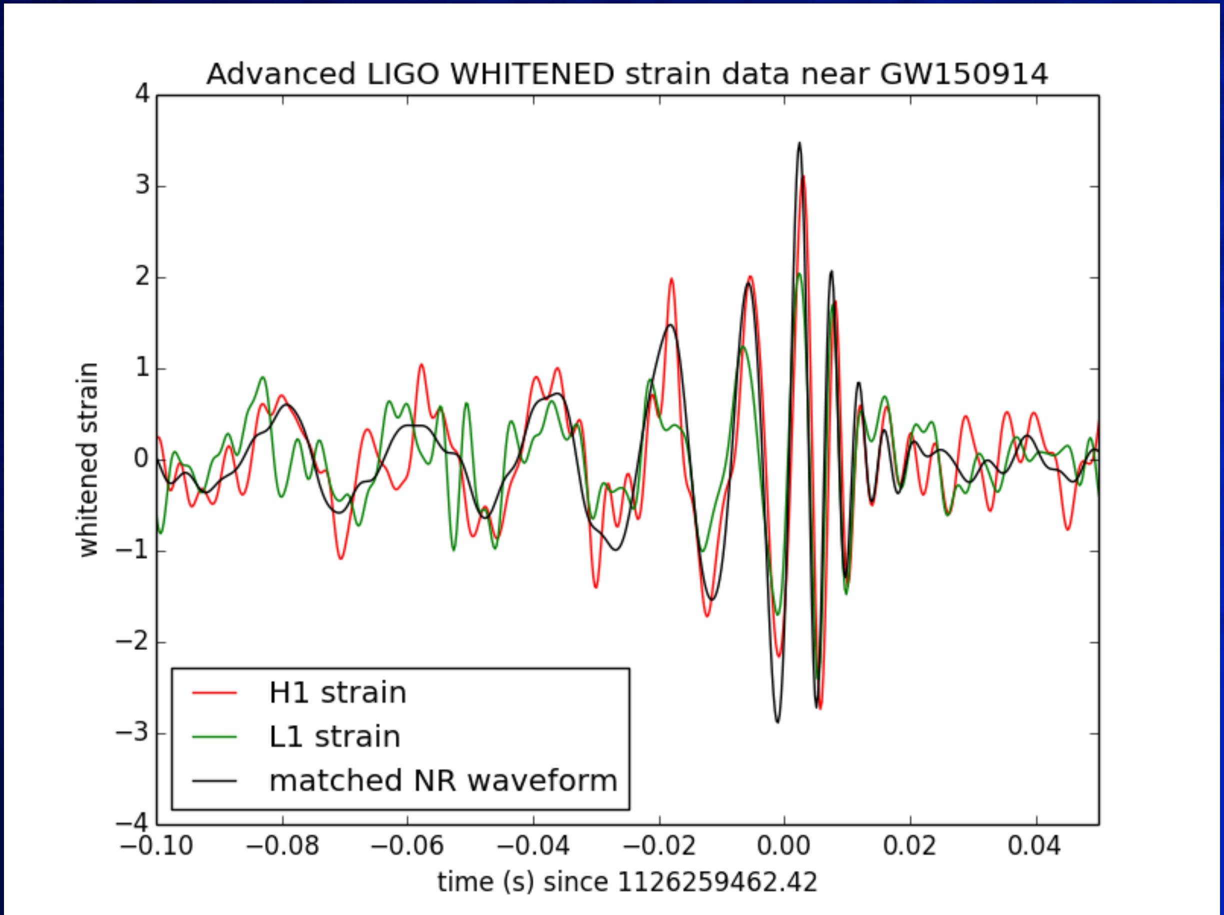
Whitening requires no prior knowledge of spectral lines, etc; only the data are needed.

Recall the ASD was just the square root of the PSD which used Welch's method to average over several windows

The resulting time series is no longer in units of strain; now in units of "sigmas" away from the mean.

The signal is now clearly visible in the whitened and bandpassed data. The "DC" offset between H1 and L1 data visible in the first plot is no longer visible here; the bandpassing (4-pole butterworth) cuts off frequency components below around 20 Hz and above 300 Hz.

The signal is visible as an oscillation sweeping from low to high frequency from -0.10 seconds to 0, then damping down into the random noise.



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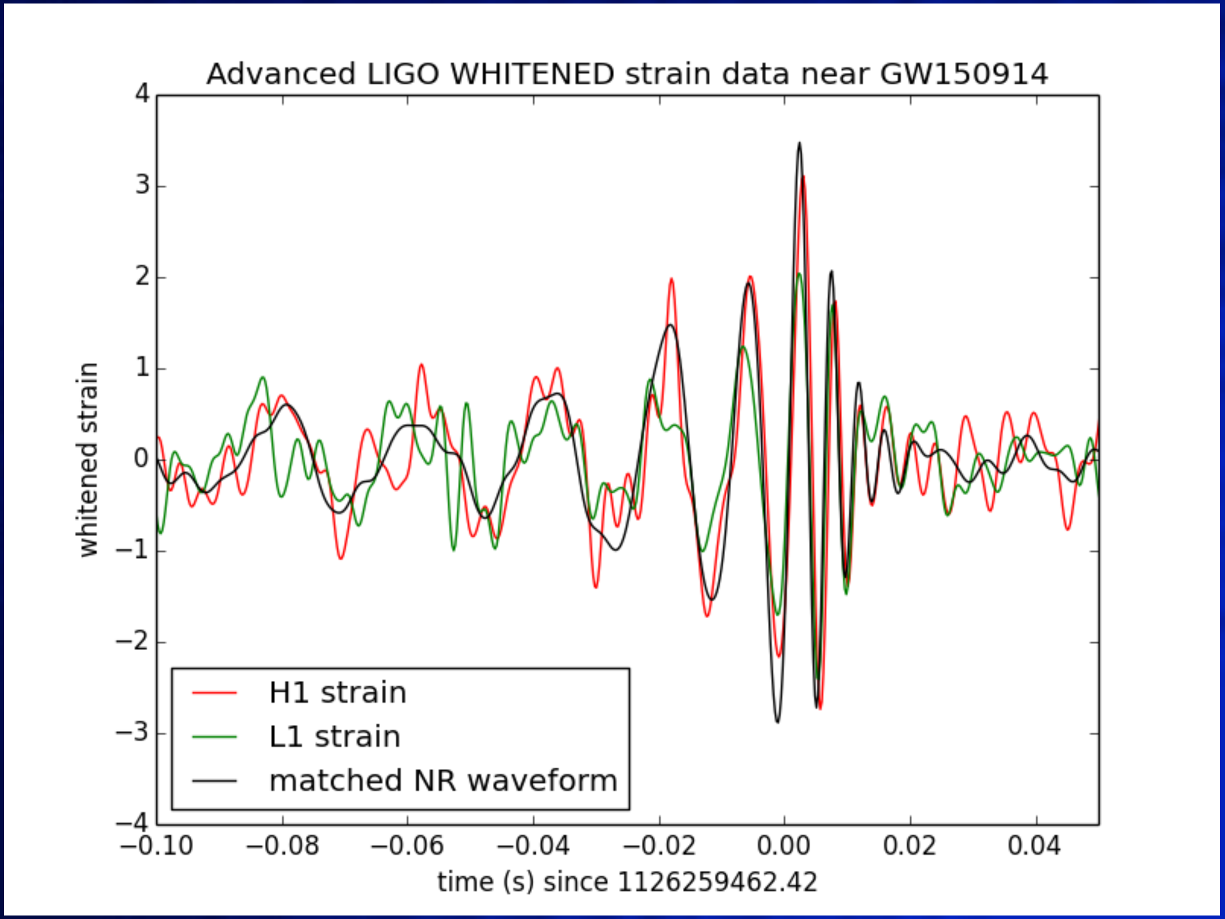
The signal looks roughly the same in both detectors. We had to shift the L1 data by 7 ms to get it to line up with the data from H1, because the source is roughly in the direction of the line connecting H1 to L1, and the wave travels at the speed of light, so it hits L1 7 ms earlier. Also, the orientation of L1 with respect to H1 means that we have to flip the sign of the signal in L1 for it to match the signal in H1.



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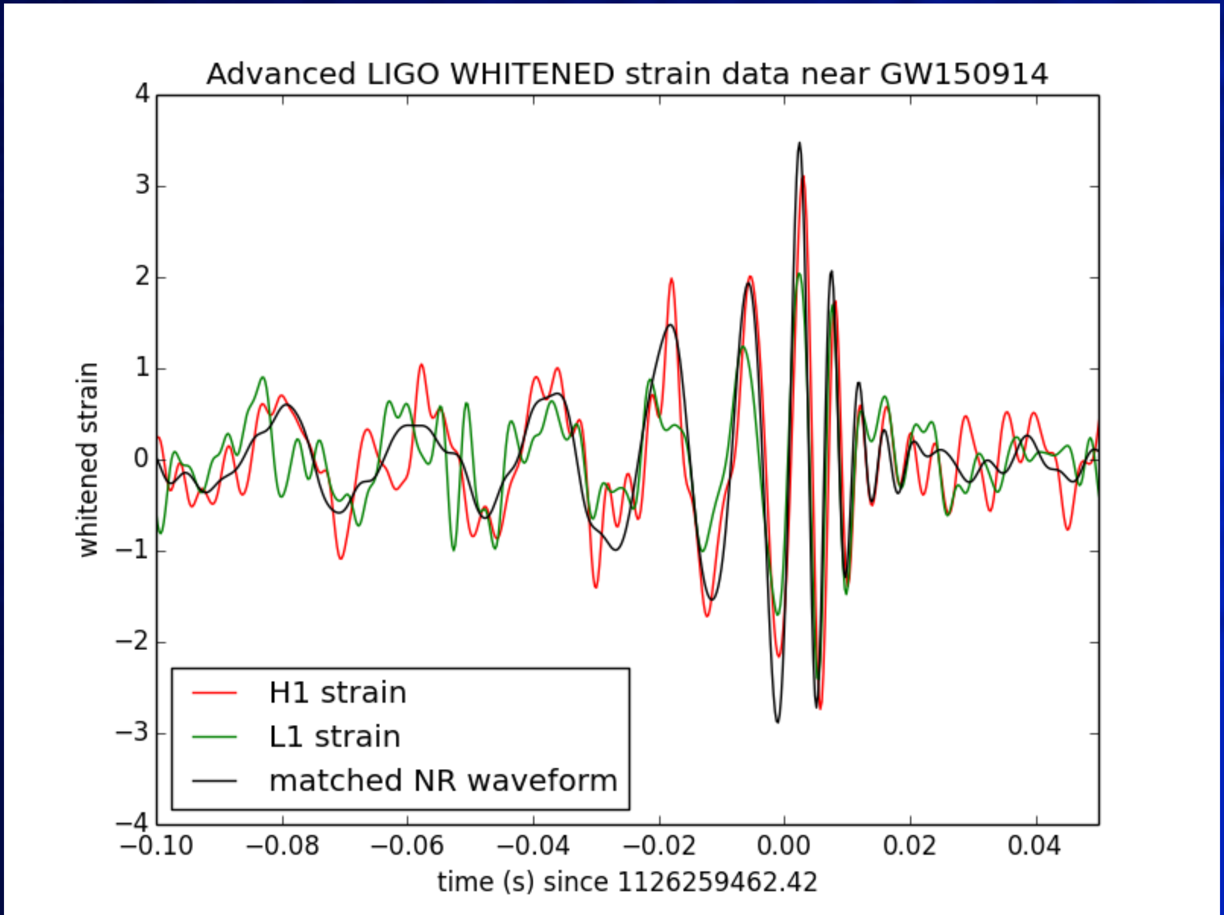
It's exactly the kind of signal we expect from the inspiral, merger and ringdown of two massive black holes, as evidenced by the good match with the numerical relativity (NR) waveform, in black.



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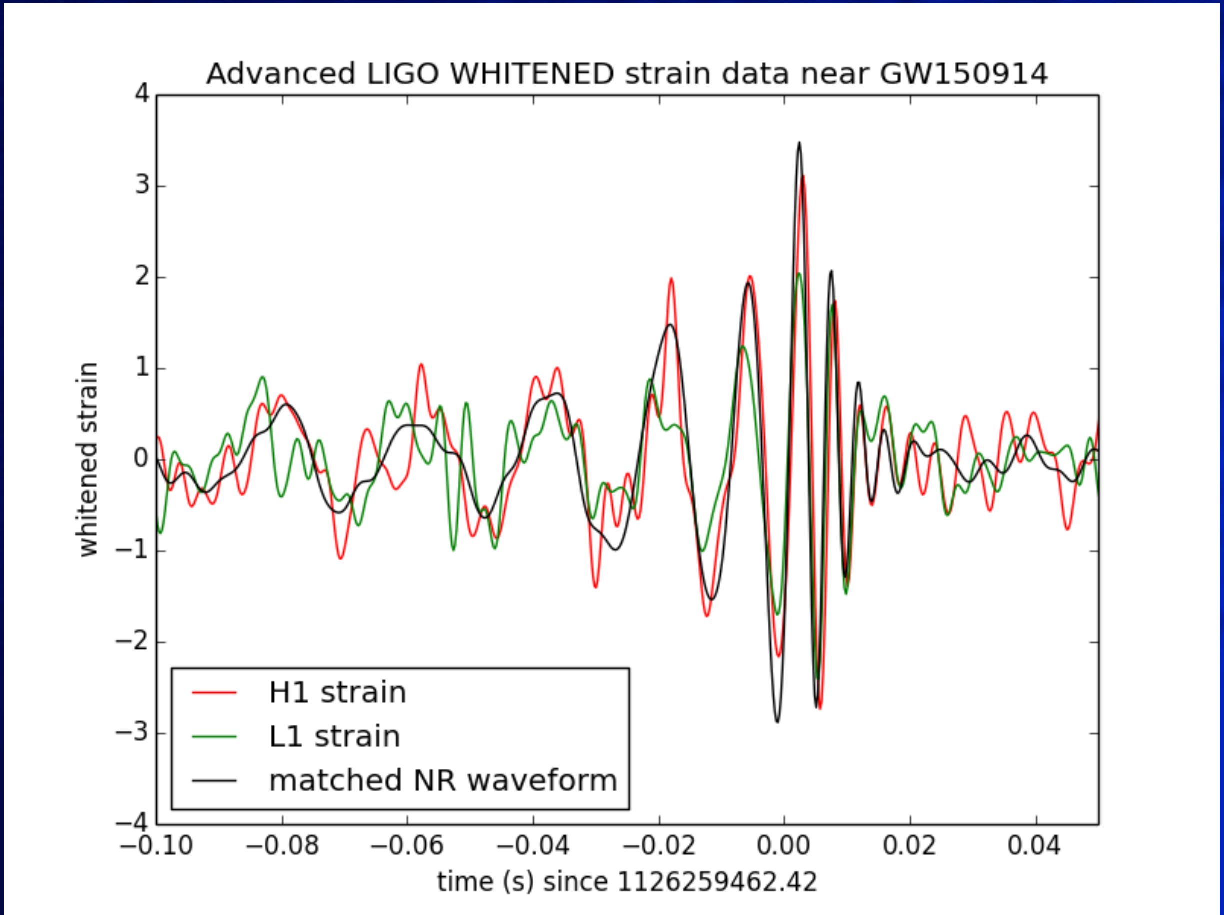
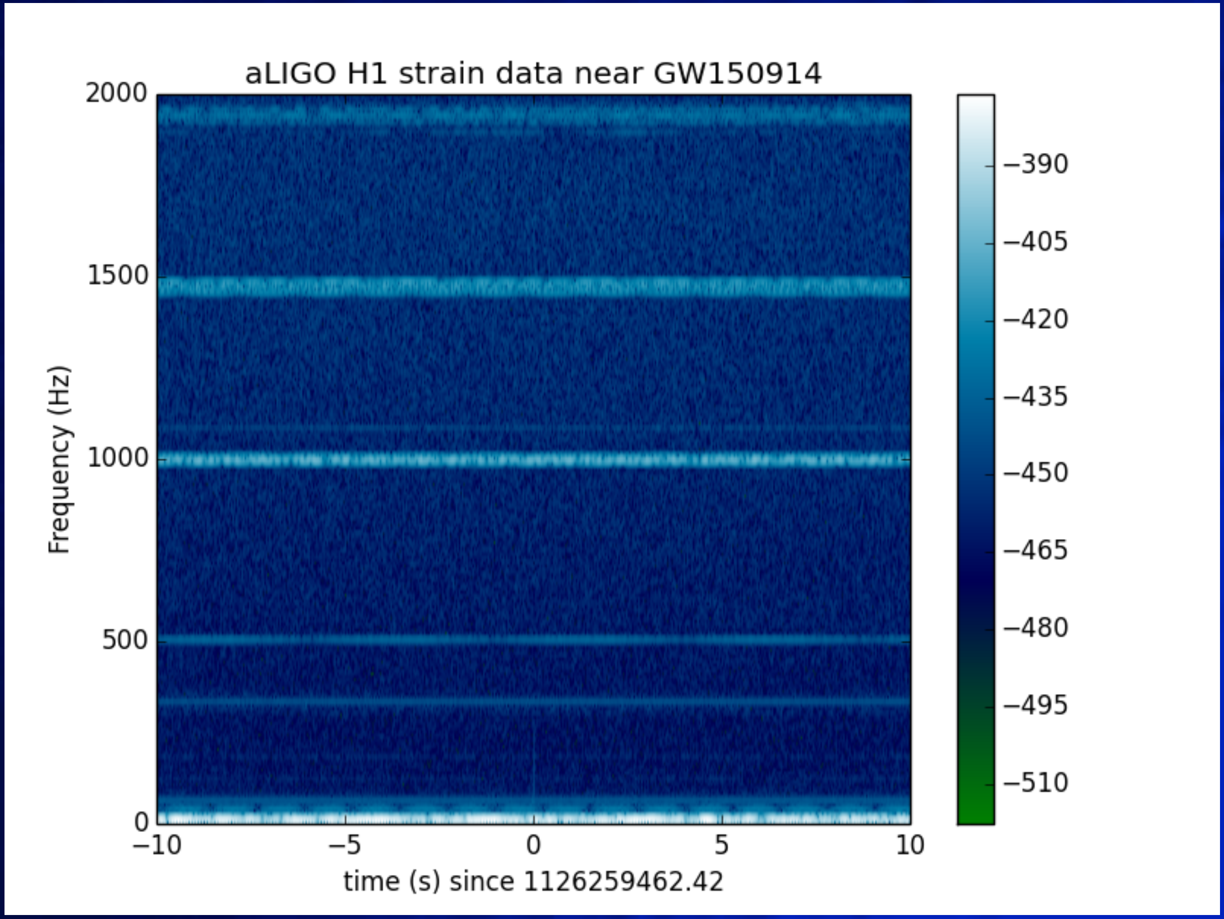
LIGO uses a rather elaborate software suite to match the data against a family of such signal waveforms ("templates"), to find the best match. This procedure helps LIGO to "optimally" separate signals from instrumental noise, and to infer the parameters of the source (masses, spins, sky location, orbit orientation, etc) from the best match templates.



Now plot a short time-frequency spectrogram around GW150914.

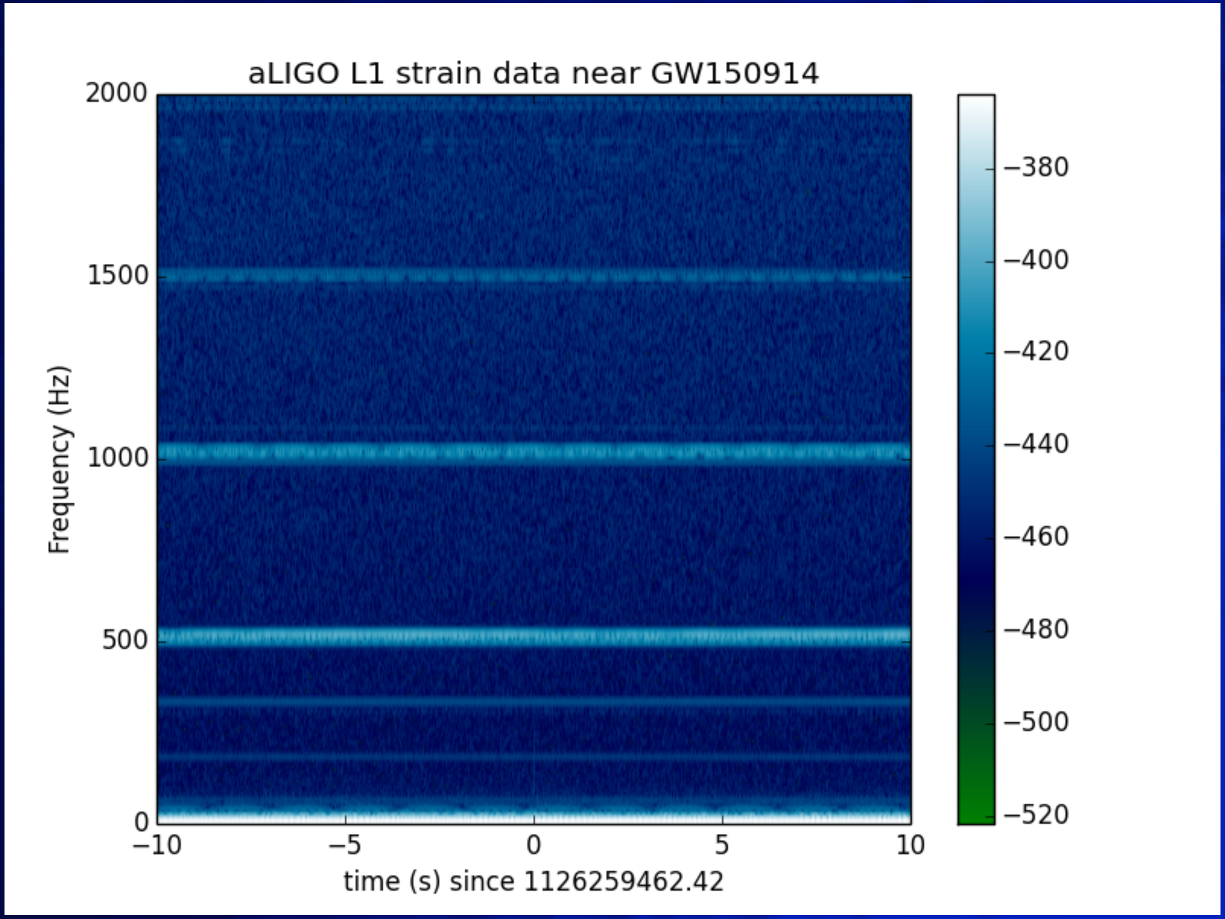
Choose a short FFT time interval, in this case 1/8 of a second, with a lot of overlap to resolve short-time features and also choose a blackman window to minimize spectral leakage

The is the spectrogram for the H1 data



Now plot the L1 spectrogram

The spectrograms have lots of excess power below ~20 Hz, as well as strong spectral lines at 500, 1000, 1500 Hz (also evident in the ASDs). The lines at multiples of 500 Hz are the harmonics of the "violin modes" of the fibers holding up the mirrors of the LIGO interferometers.



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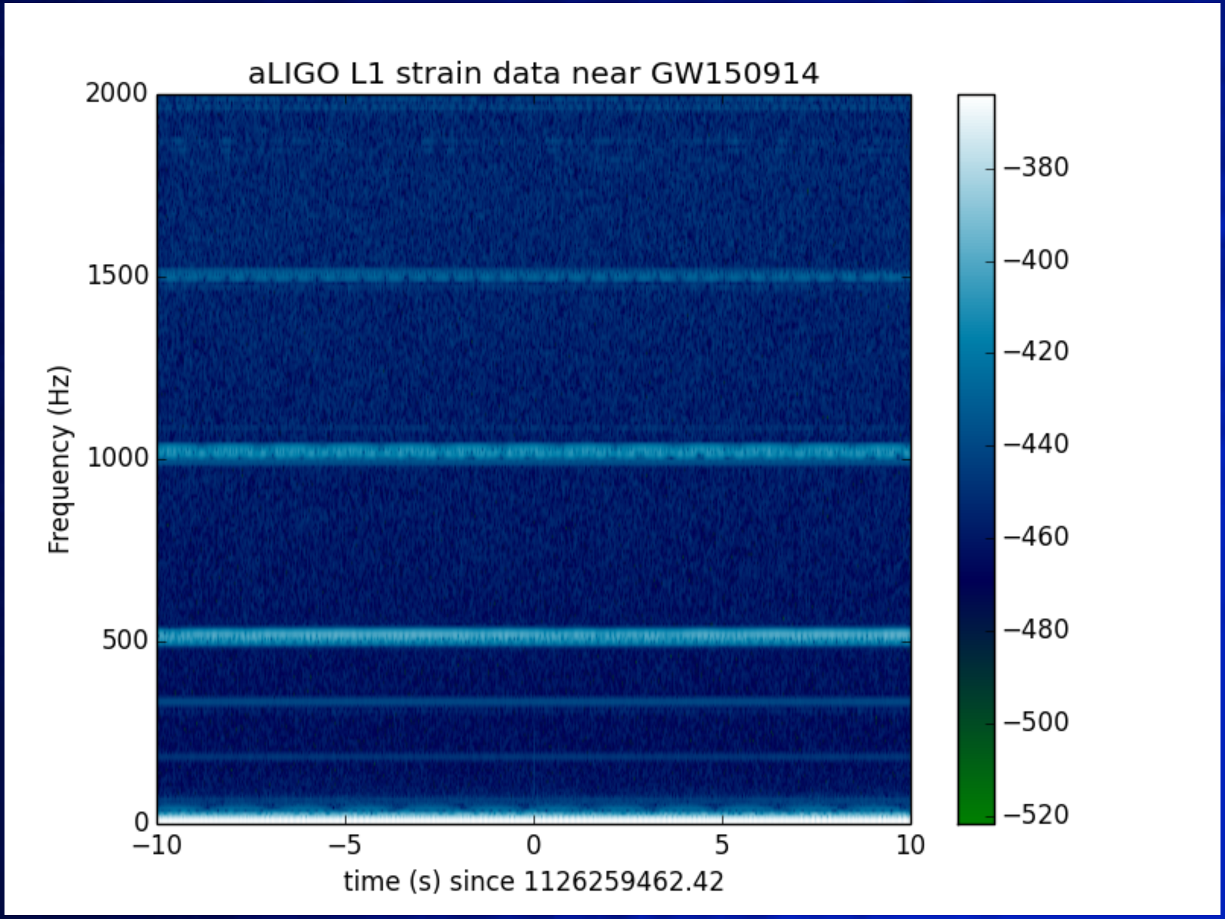
The signal is just barely visible here, at time=0 and below 500 Hz. We need to zoom in around the event time, and to the frequency range from [20, 400] Hz, and use the whitened data generated above.



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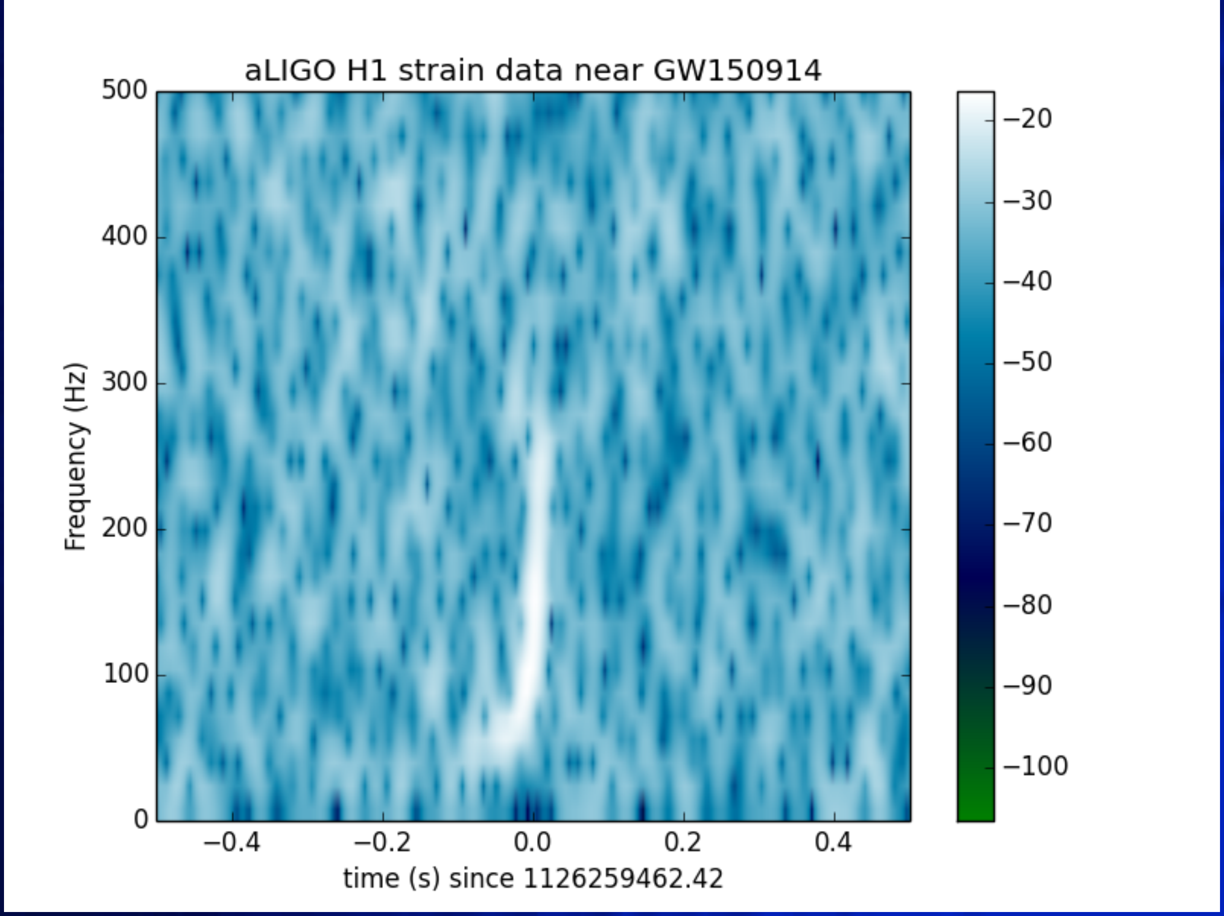
Now plot the spectrogram for the whitened data zooming in to the signal

Use a 1/16 second window with 15 s overlap and again choose the blackman window

This is for the H1 data



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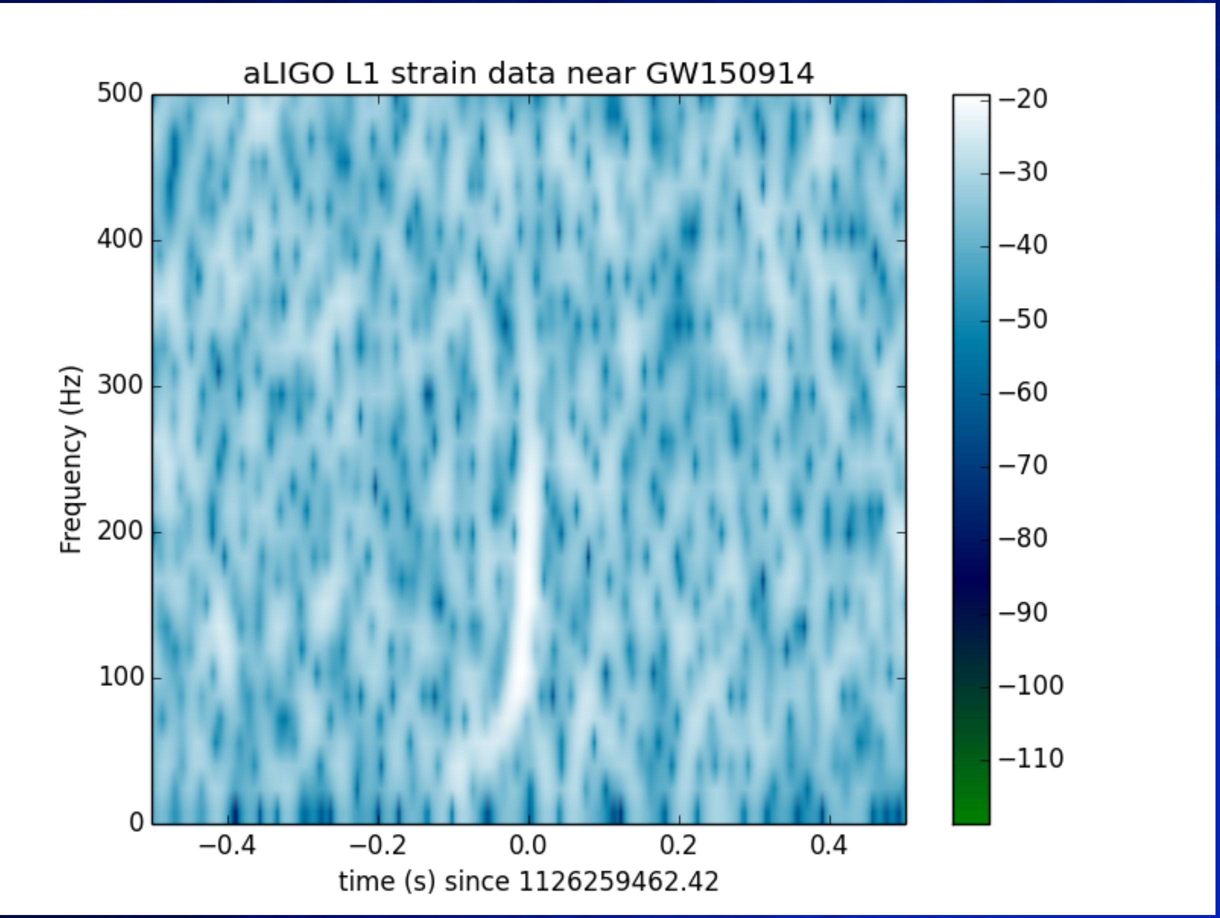
and now the spectragram for the L1 whitened data
See the smudge between -0.2 and 0 seconds? That's our signal!

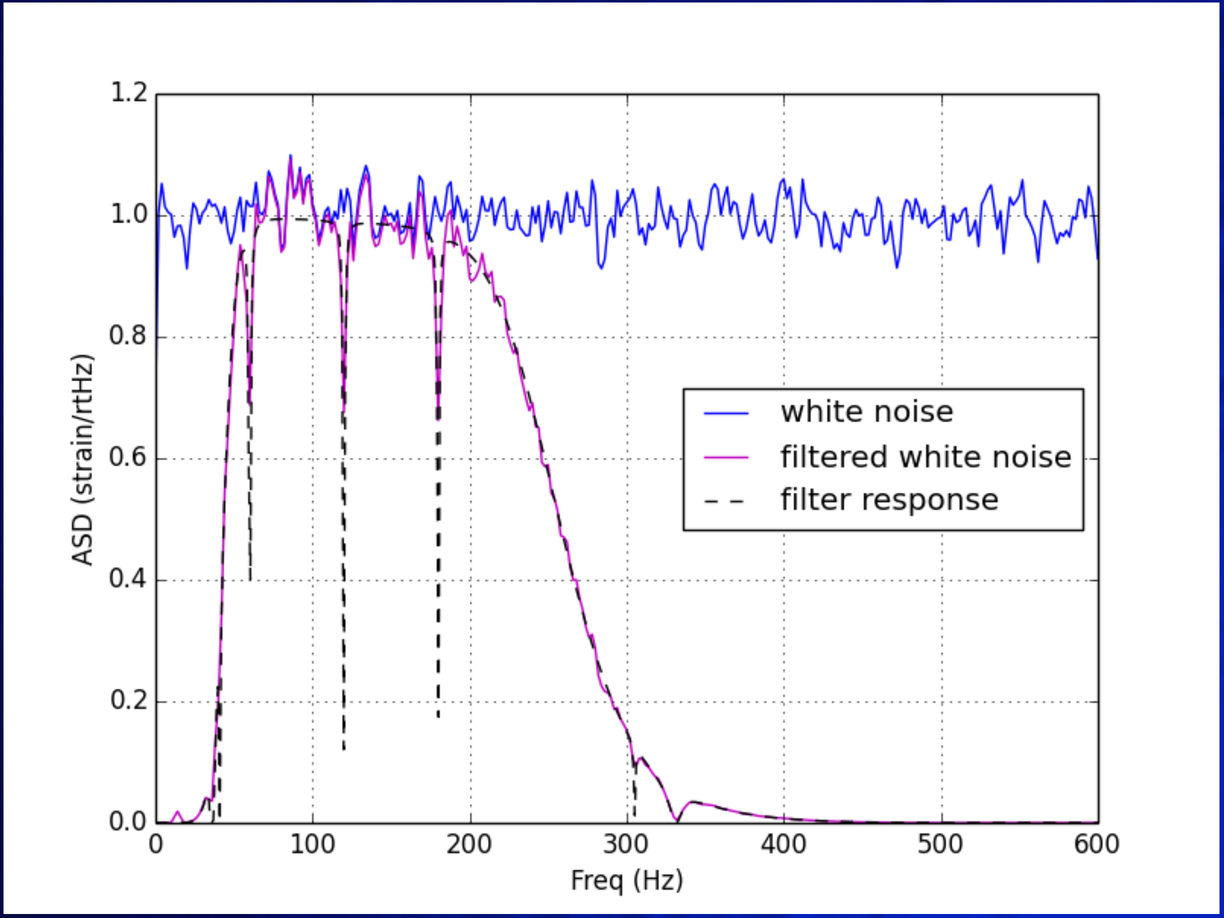
You can see it 'chirping' from lower to higher frequency over a small fraction of a second.



Now let's filter the signal in the time domain, using bandpassing to reveal the signal in the frequency band [40 , 300 Hz], and notching of spectral lines to remove those noise sources from the data.

Frequencies of notches at known instrumental spectral line frequencies.

You can see these lines in the ASD above, so it is straightforward to make this list.



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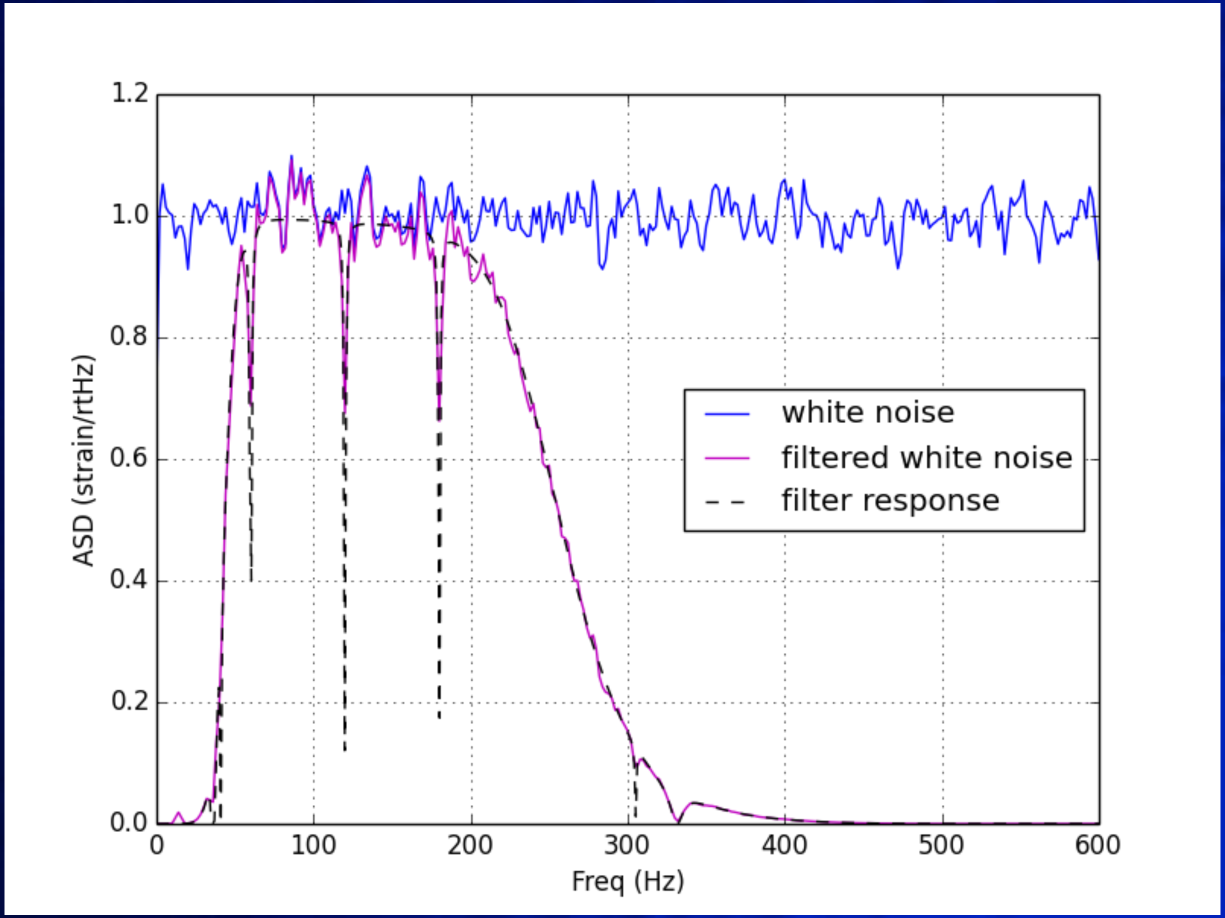
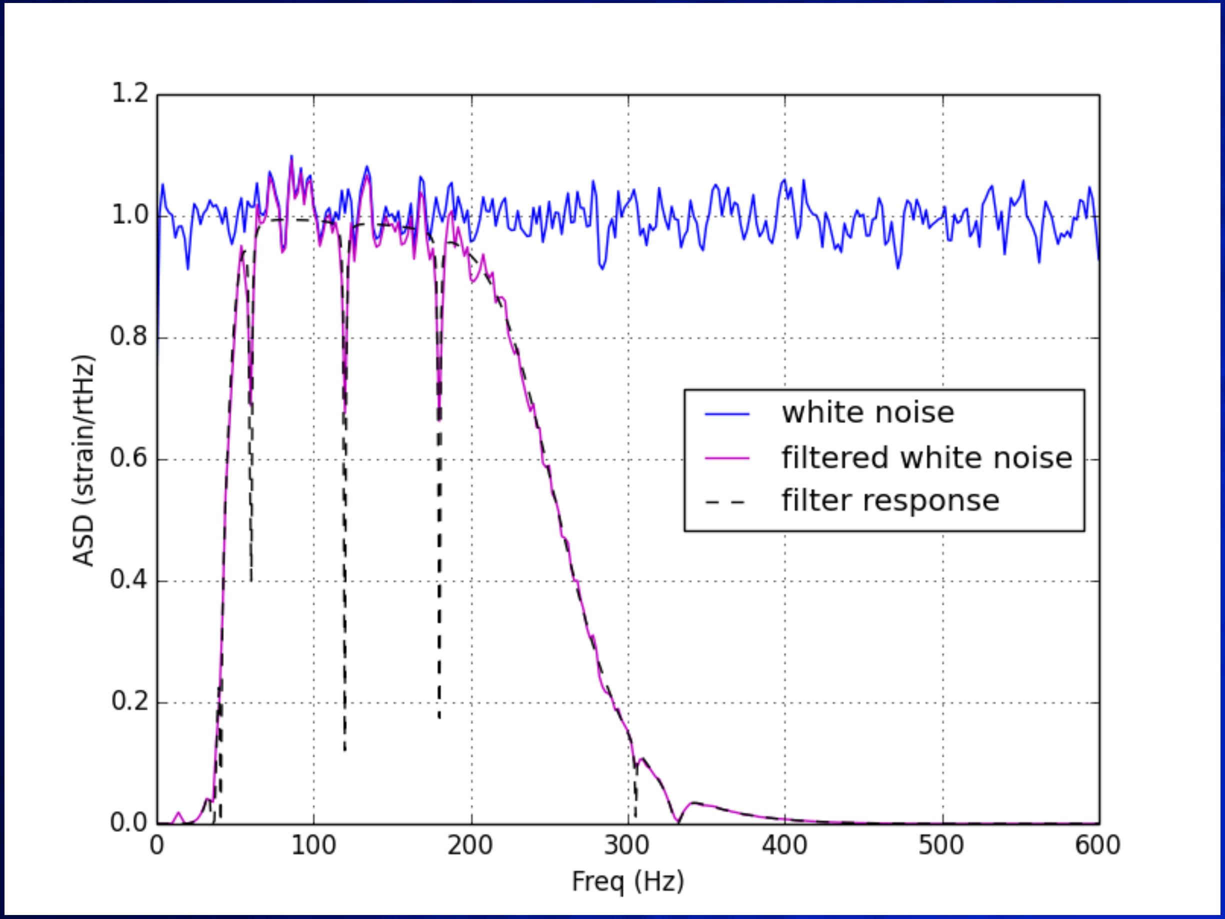
To visualize the effect of this filter, let's generate "white" gaussian noise, and filter it.

We "double pass" the filtering using filtfilt, so we square the filter response

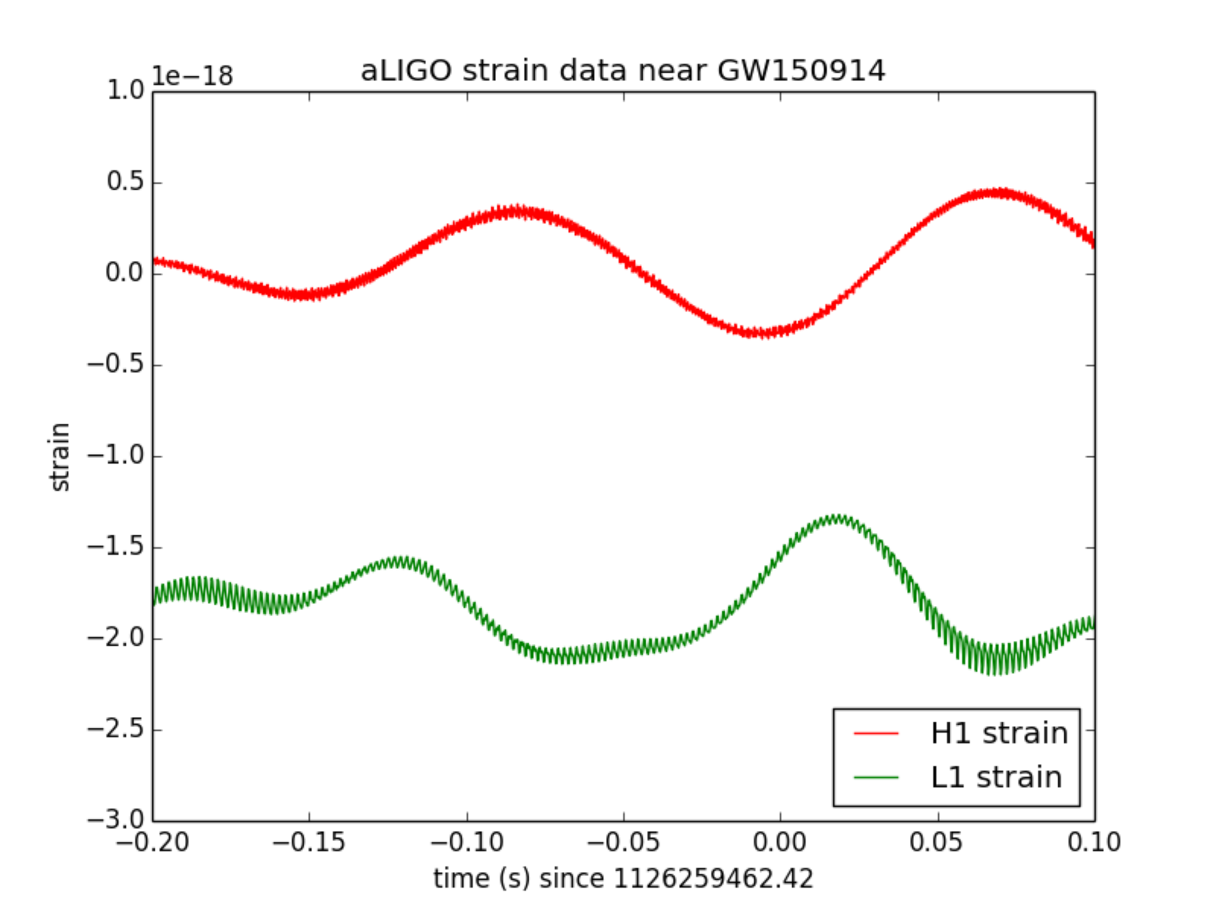
And as we learned before, this produces a zero phase filter response and hence, has no time shifts



From the above, you can see that the gaussian noise (blue) is "white" - it is flat in frequency (all the way up to Nyquist frequency of 2048 Hz, but we've cut it off at 600 Hz to see the effect of filtering). You can see in the filtered data (magenta) the effects of the bandpassing and the notches.



Unfiltered data.



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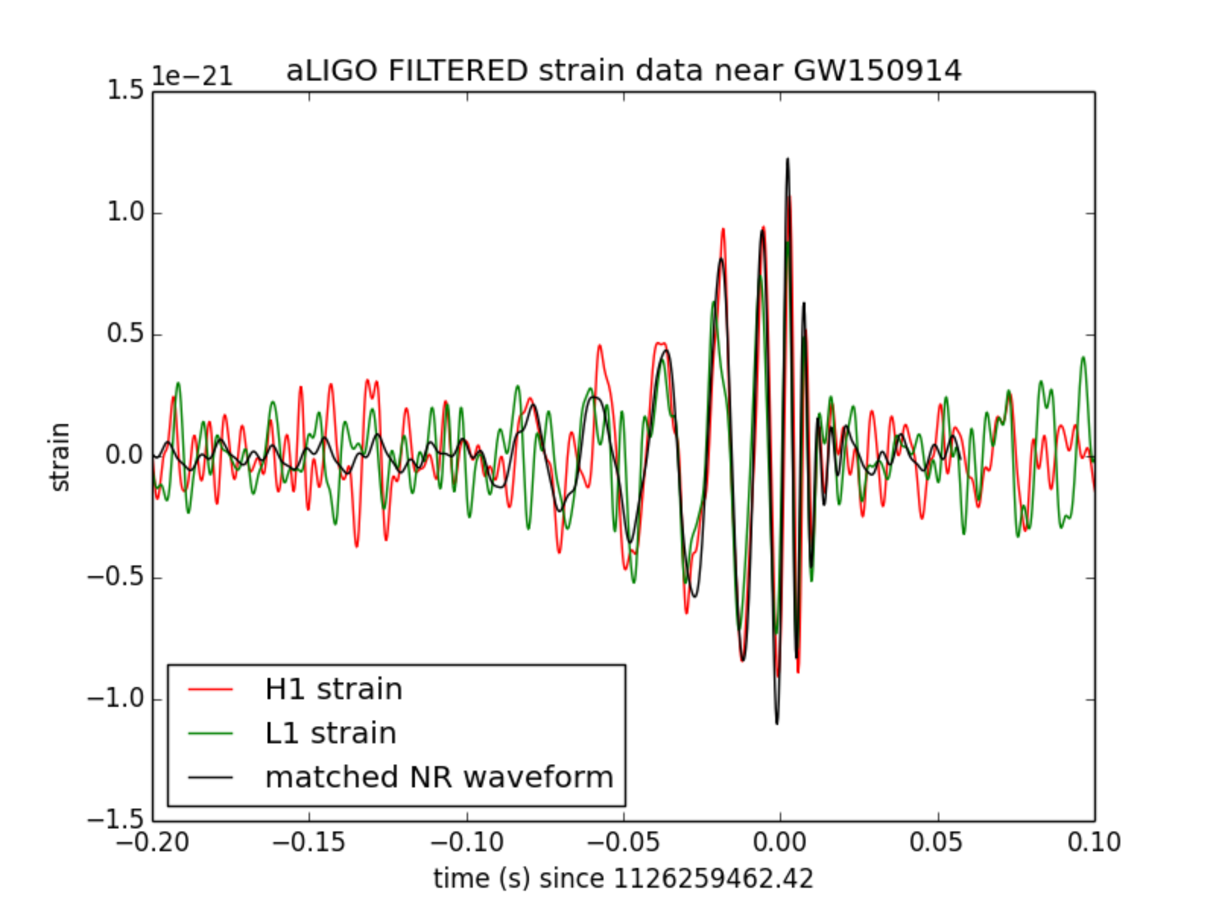
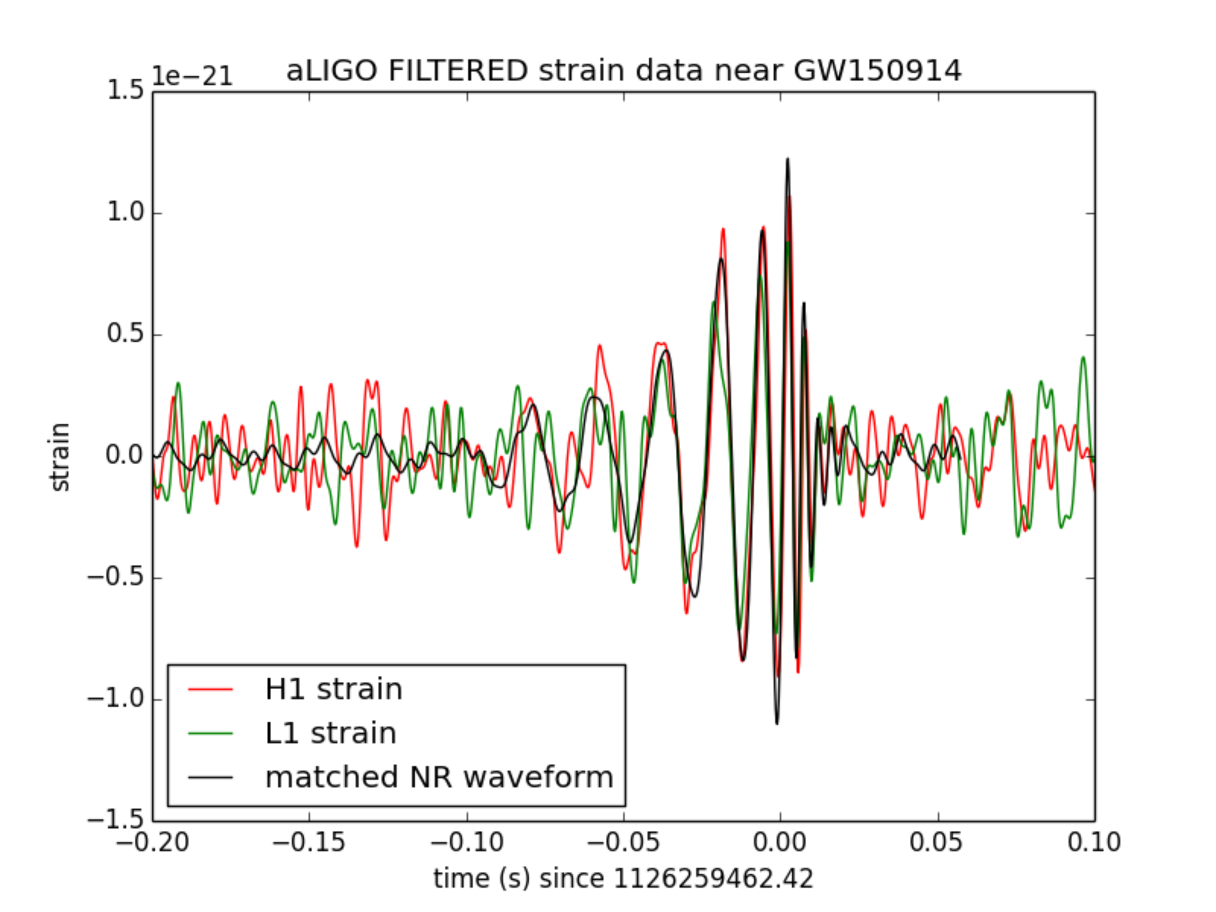
First, shift L1 by 7 ms, and invert. See the GW150914 detection paper for why!

We also have to shift the NR template by 2 ms to get it to line up properly with the data



The filtered data peak at around 1.e-21, 1000 times smaller than the scale in the first plot. The "DC" offset between H1 and L1 data visible in the first plot is no longer visible here; the bandpassing cuts off frequency components below around 40 Hz.



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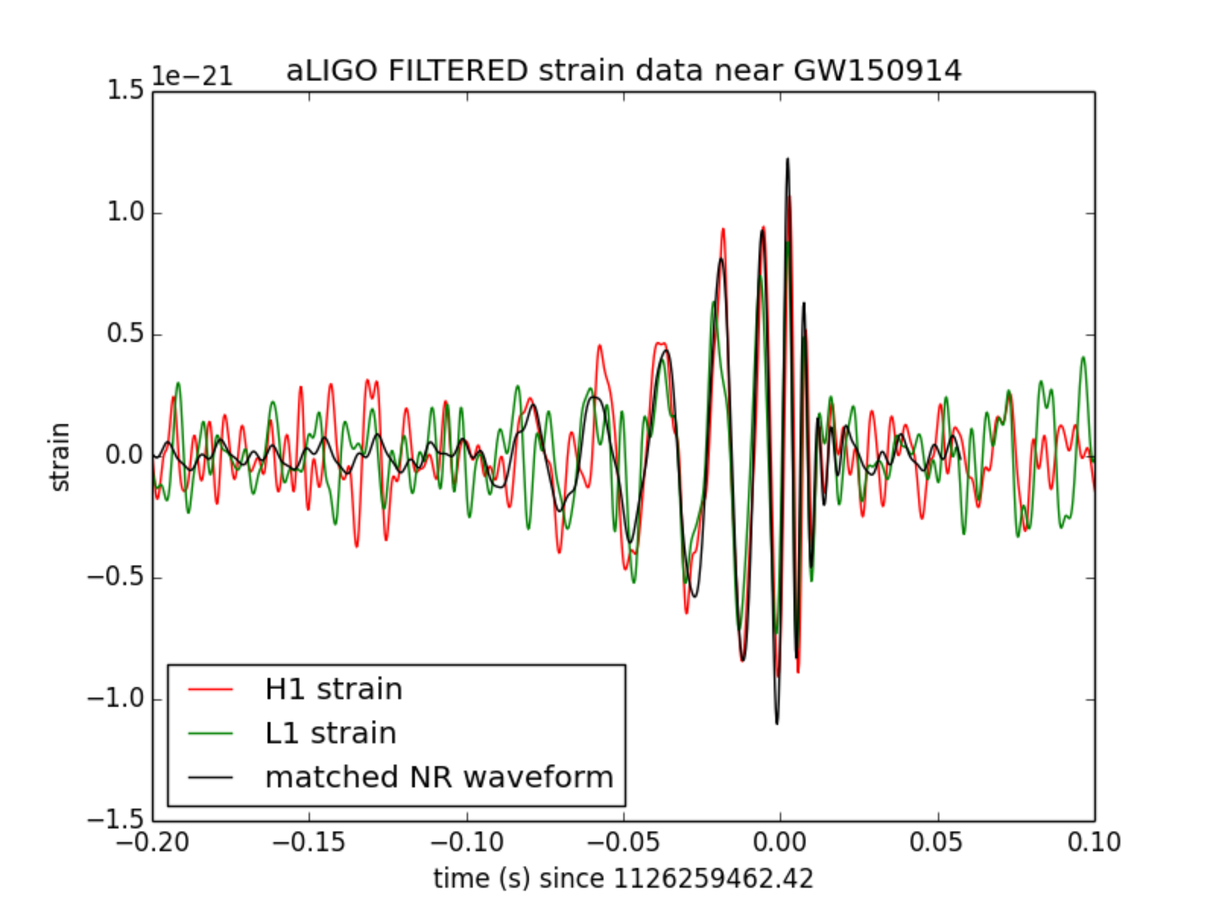
Now, as with whitening, the signal is visible as an oscillation sweeping from low to high frequency from -0.10 seconds to 0, then damping down into the random noise. Again, it looks roughly the same in both detectors, after shifting and flipping the L1 data with respect to H1. It's exactly the kind of signal we expect from the inspiral, merger and ringdown of two massive black holes.



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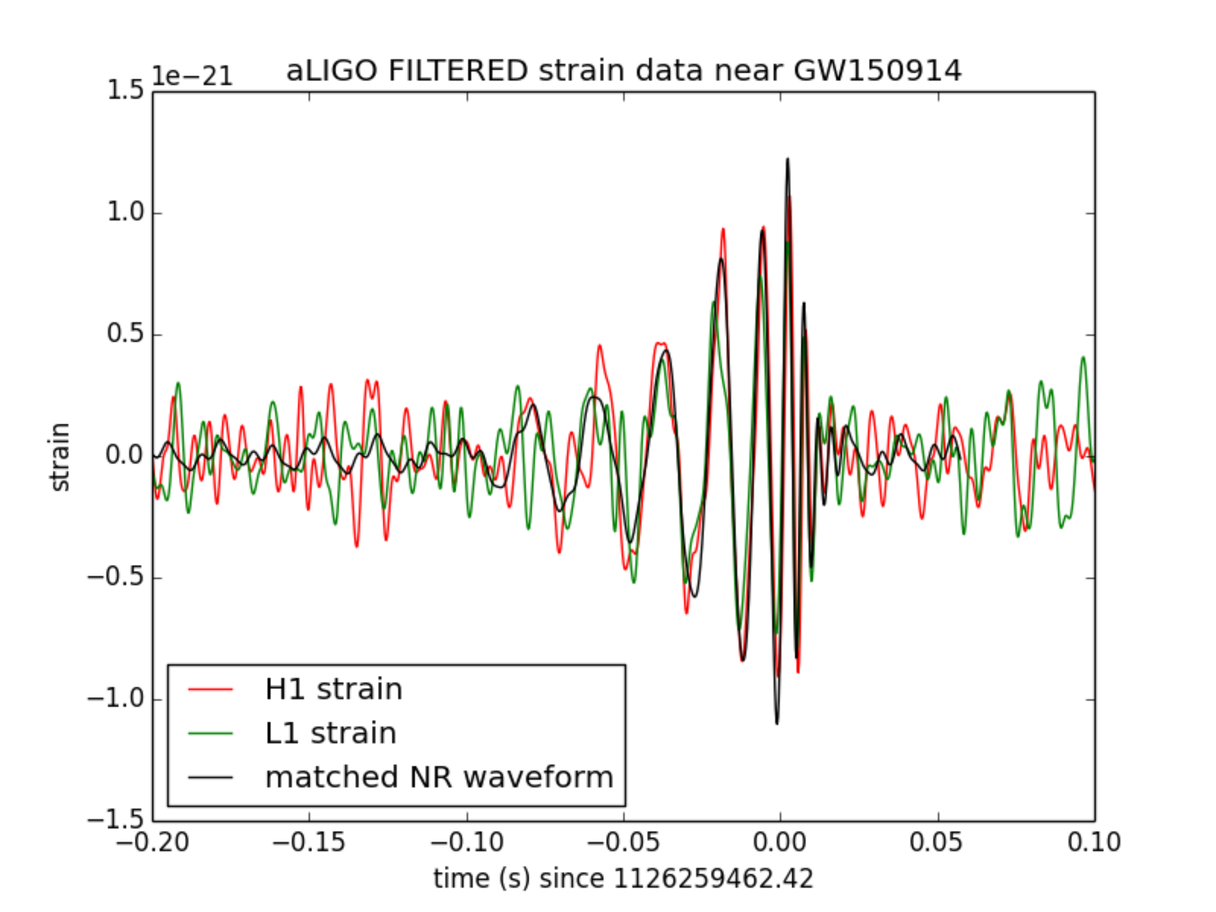
And as with whitening, the NR waveform looks, by eye, to be a good match to the data in both detectors; the signal is consistent with the waveform predicted from General Relativity.



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