f-k filters

Mitch Withers, Res. Assoc. Prof., Univ. of Memphis

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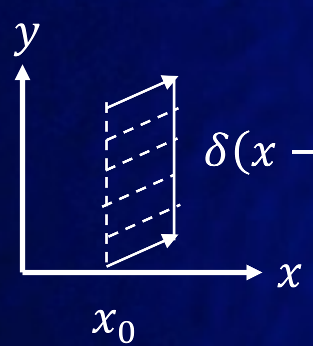

a line source sifts values of $f(x,y)$ at x_0 for all y.

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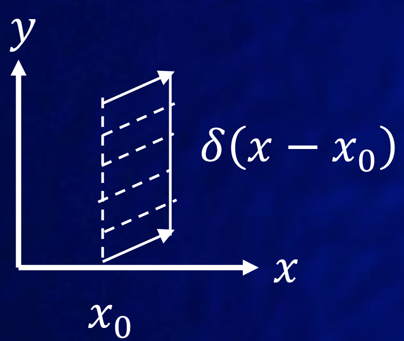

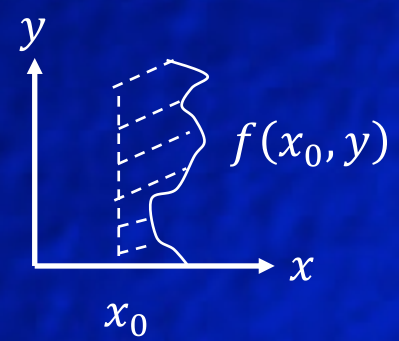

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The mathematics of the 2-d FT is ambivalent to our choice of units so one choice can be time and space, a seismic reflection line for example.

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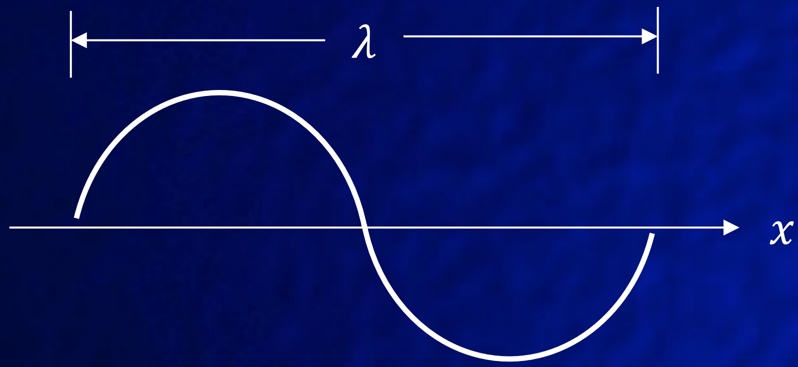
Where f is the familiar frequency in cycles/second and k is spatial frequency in cycles/length.



λ is the wavelength

$k = \frac{1}{\lambda}$ is the spatial frequency

Wavenumber is $k^* = 2\pi k = \frac{2\pi}{\lambda}$ similar to $\omega = 2\pi f$.

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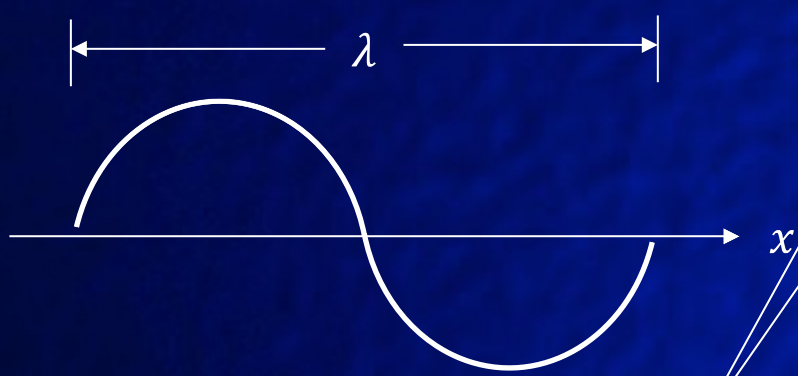

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We use k^* here to distinguish wavenumber from spatial frequency k though you may encounter k for both. Know which it is by context.

A few familiar properties in 2-d

$$F[g(x - x_0, t - t_0)] = G(k, f)e^{-i2\pi(kx_0 + ft_0)}$$

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Windowing

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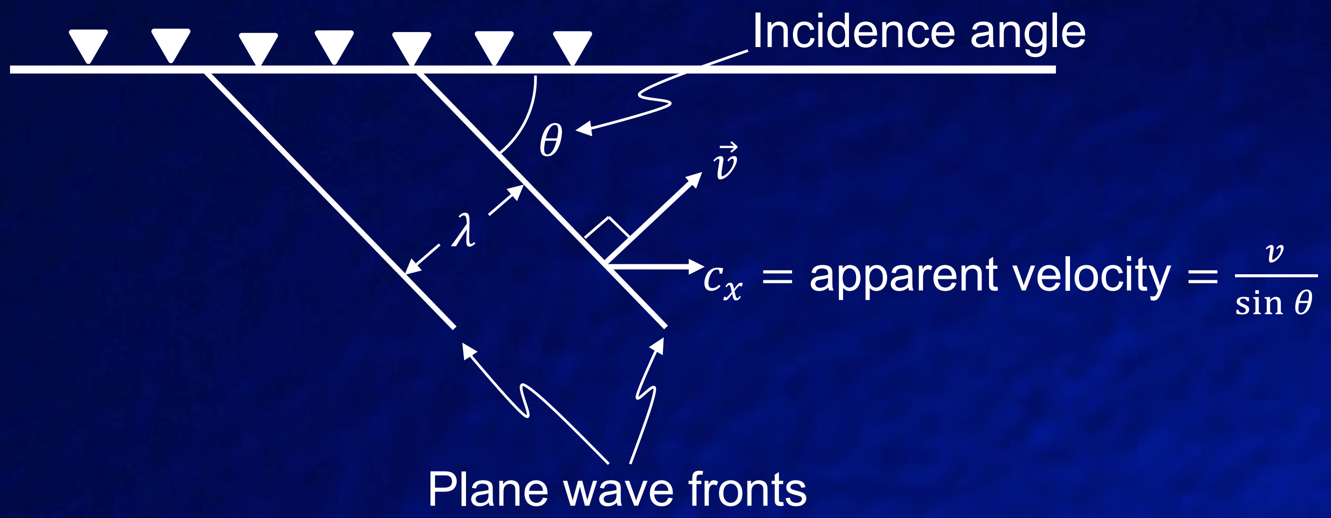

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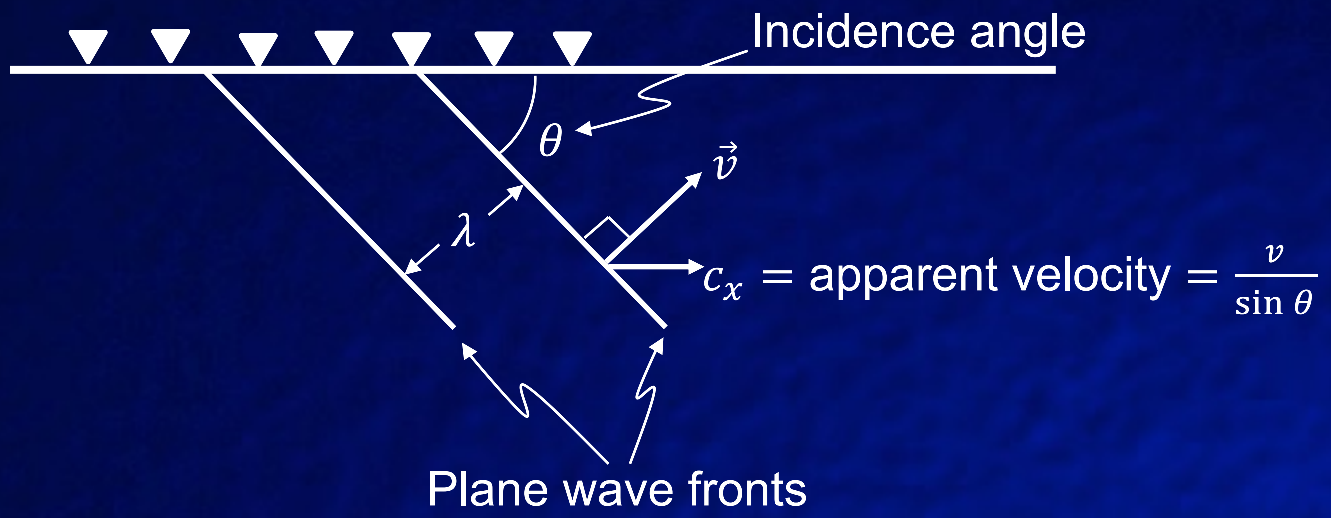
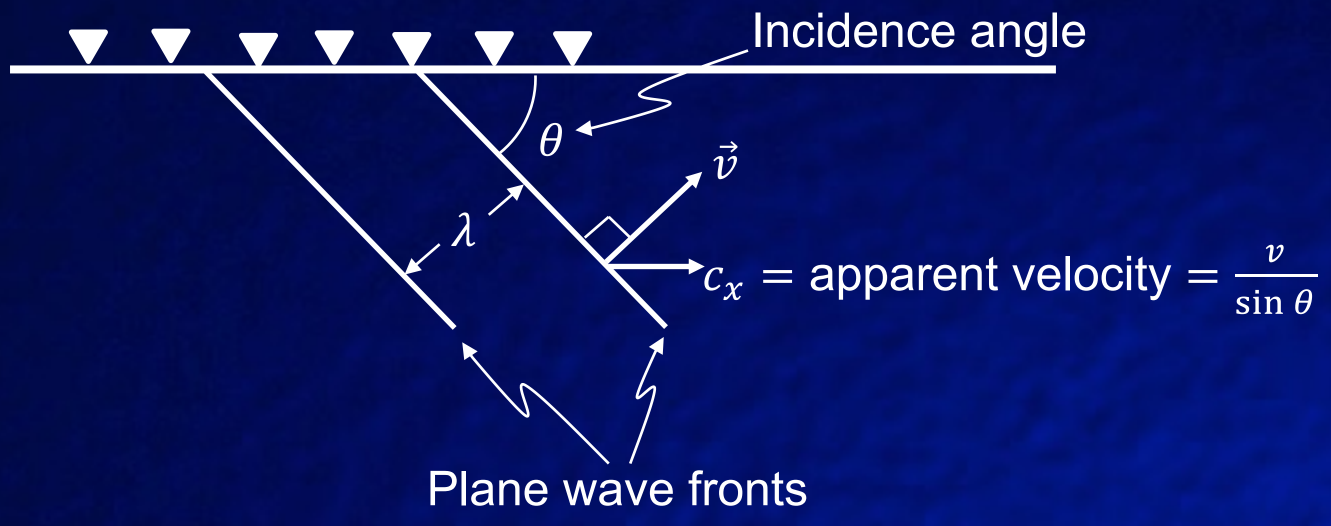


$\theta \rightarrow 90°$ Horizontally traveling wave (ground roll and surface waves)

$\theta \rightarrow 0°$ Vertically traveling wave (body waves and reflections)



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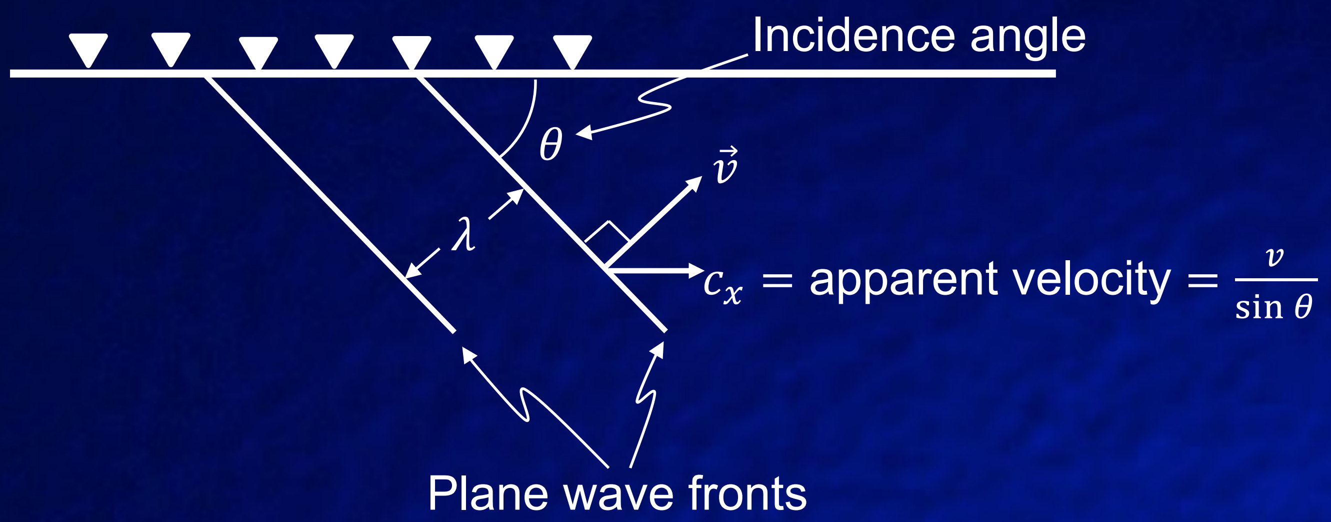

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A horizontal seismic line measures c_x & k_x not v & k.

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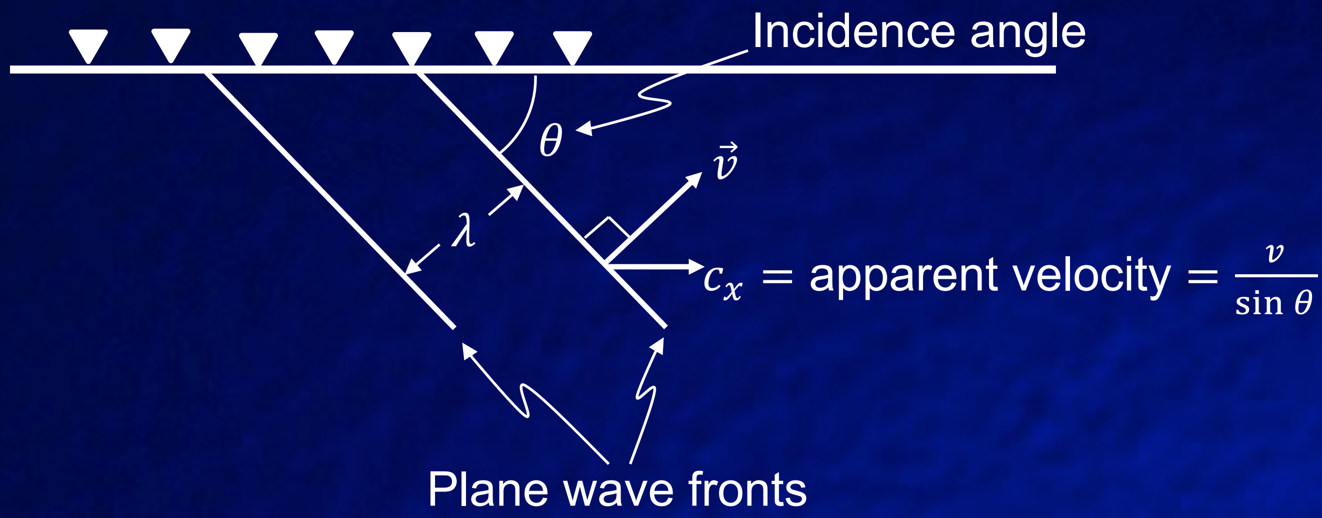

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Slope in t-d plot ($\left(\frac{\Delta t}{\Delta x}\right)^{-1}$)

Slope in f-k plot ($\frac{\Delta f}{\Delta k}$)

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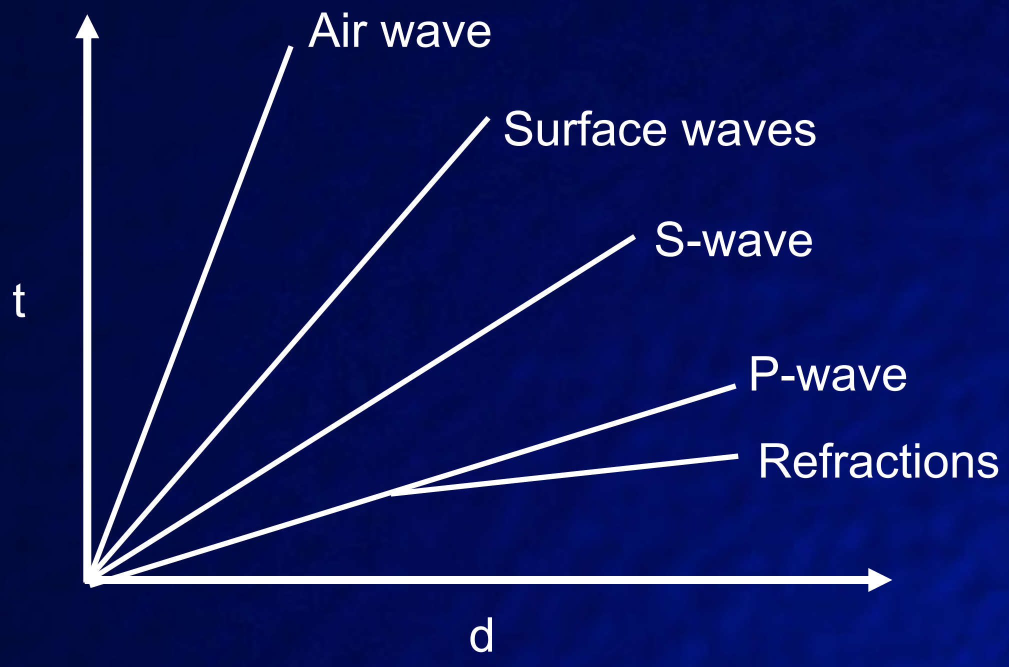
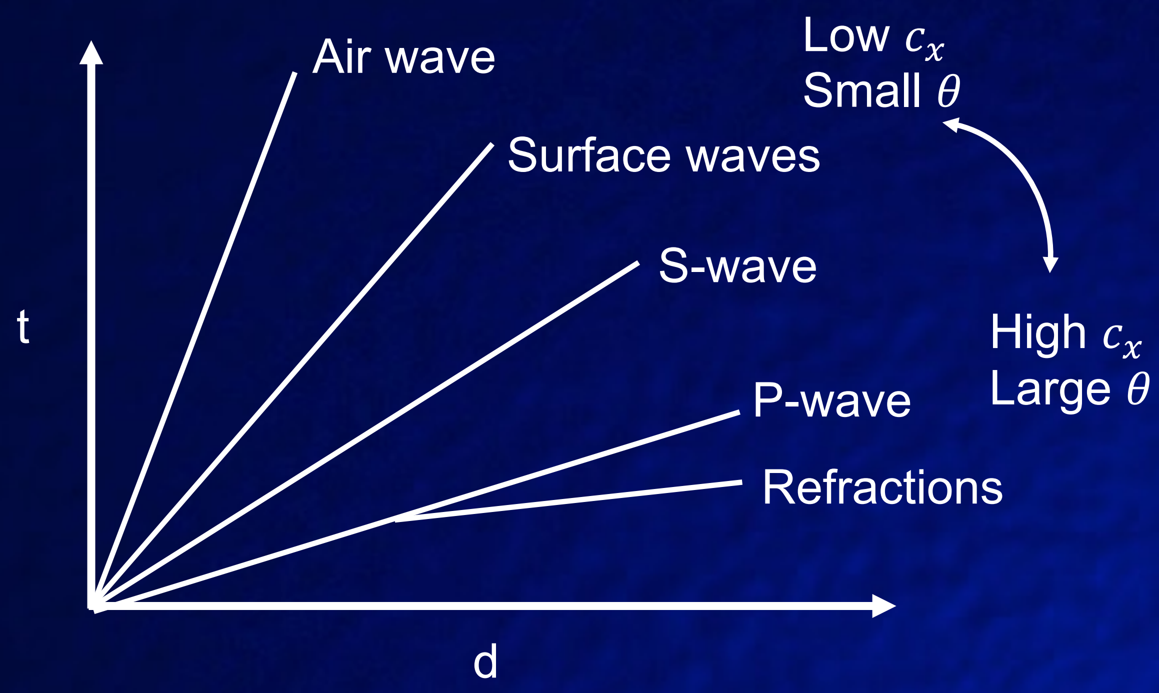

Air wave
Surface waves
S-wave
P-wave
Refractions
t
d


Air wave
Surface waves
S-wave
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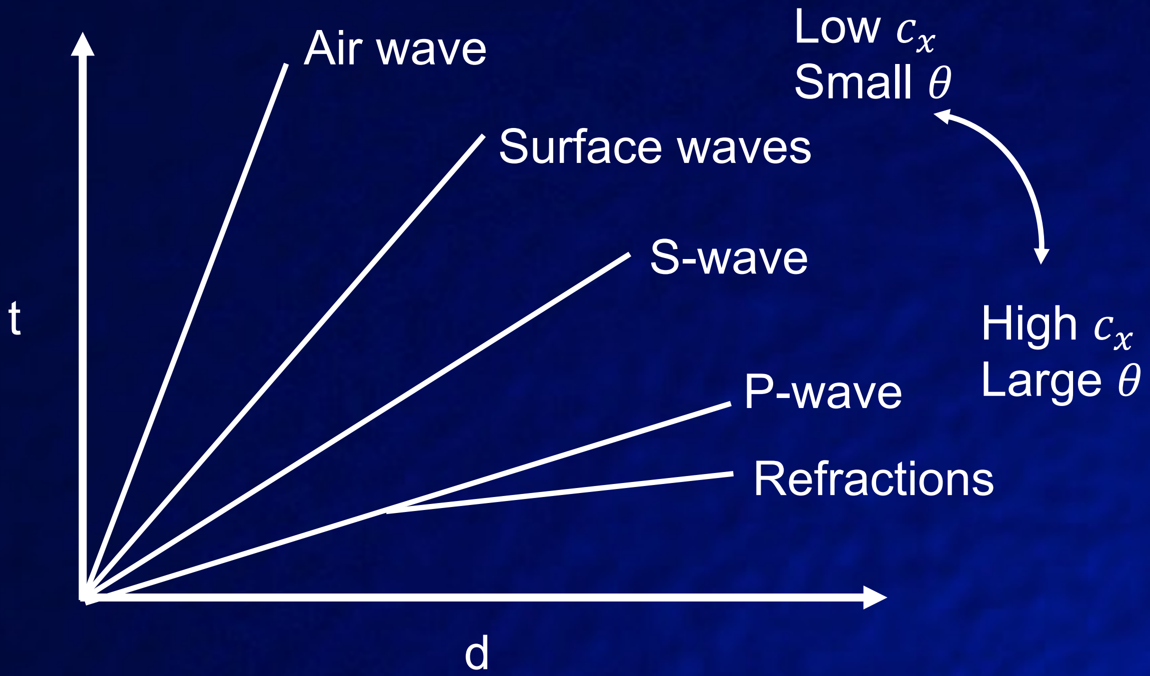
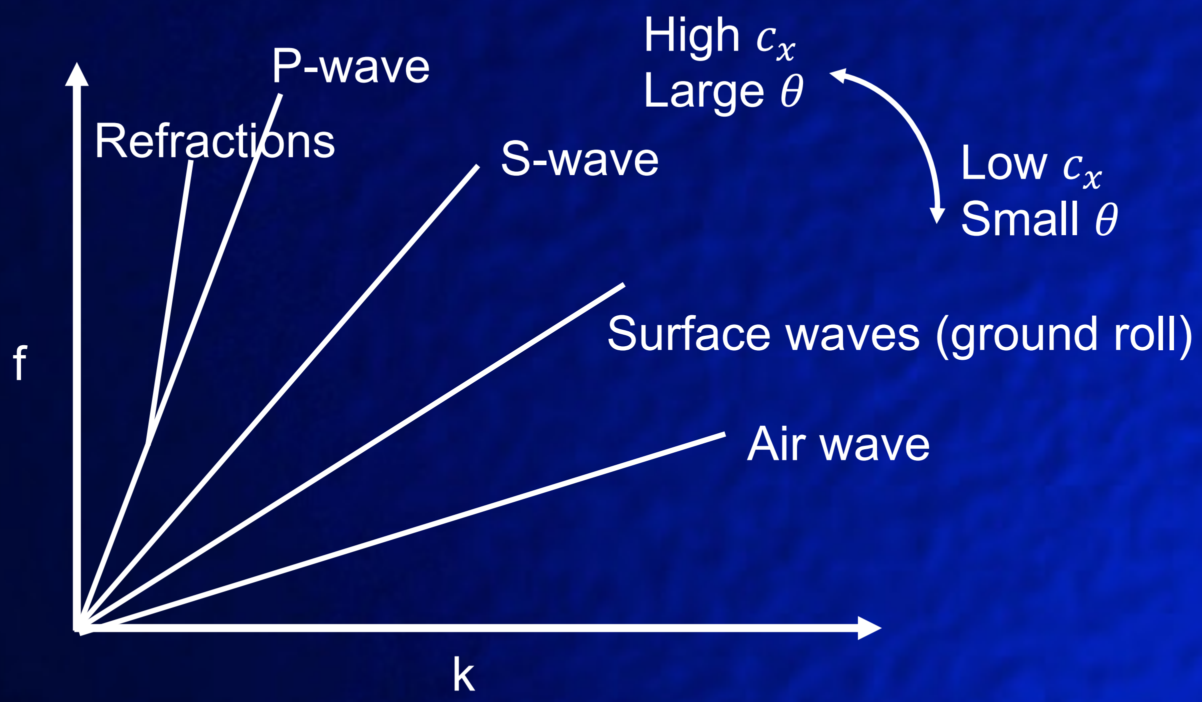
Refractions
t
d
Low c_x
Small θ
High c_x
Large θ





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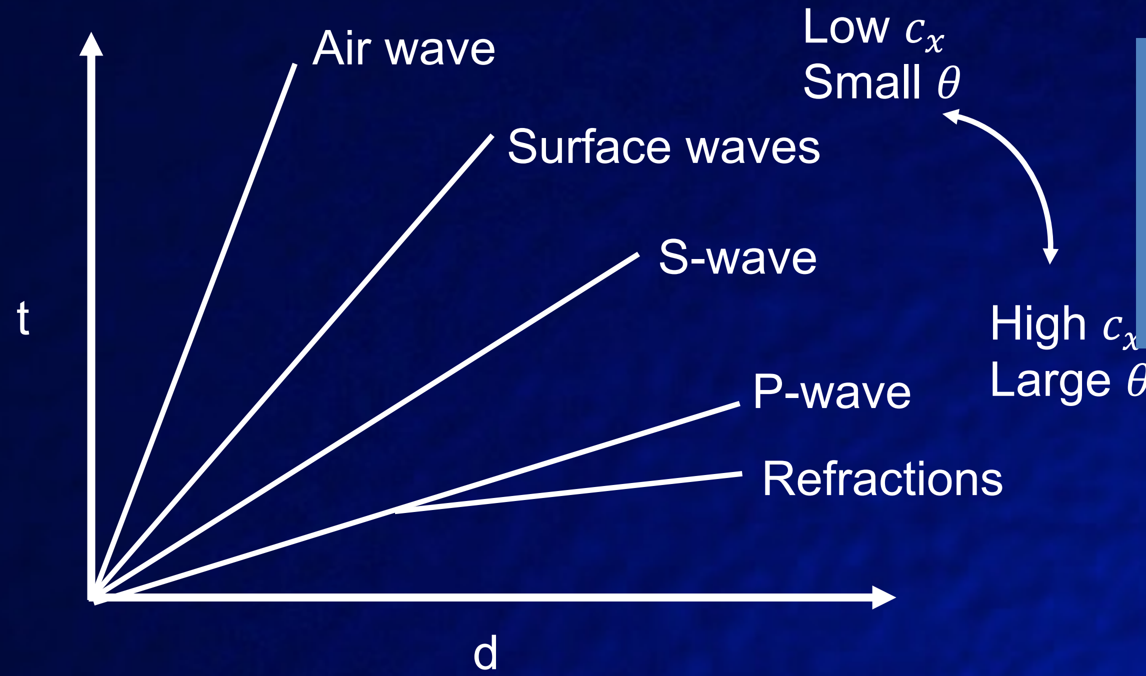
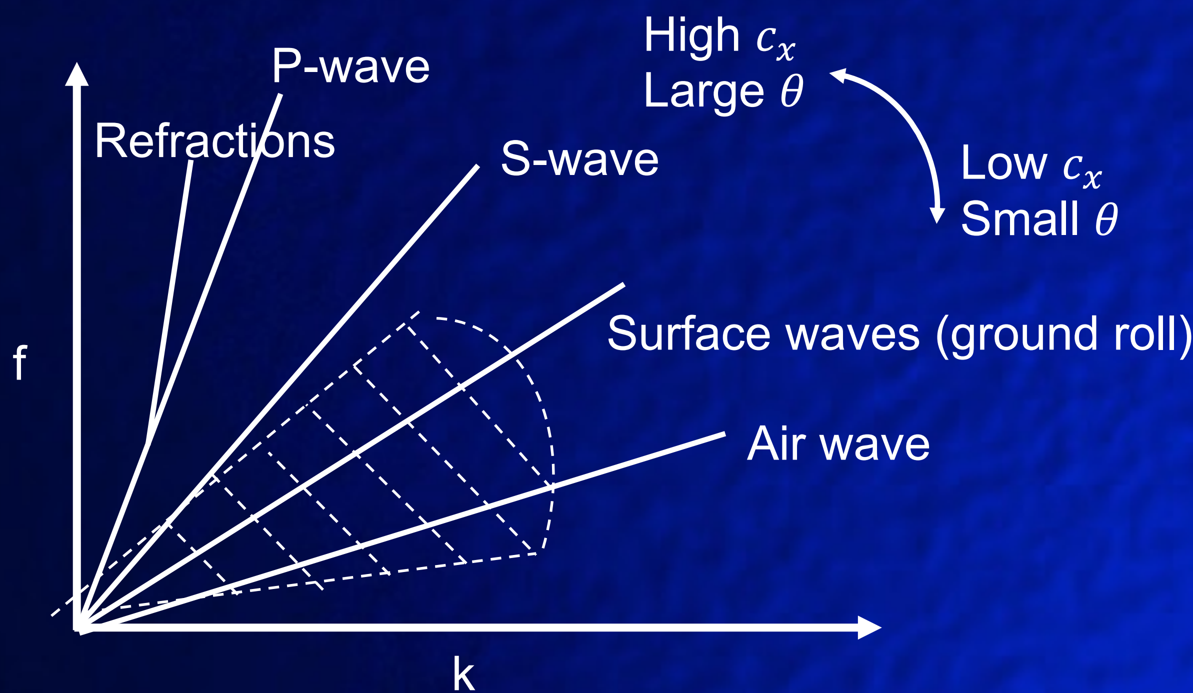
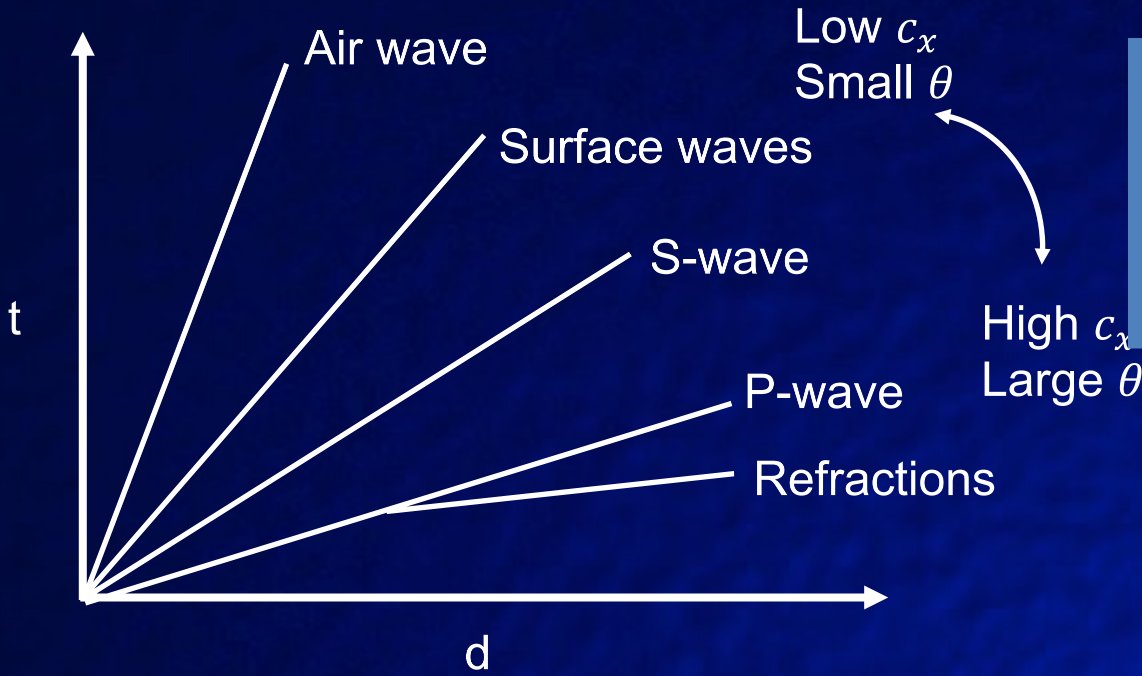
We can apply a fan filter to remove ground roll (surface waves) and keep signals of interest based on incidence angle and horizontal velocity.







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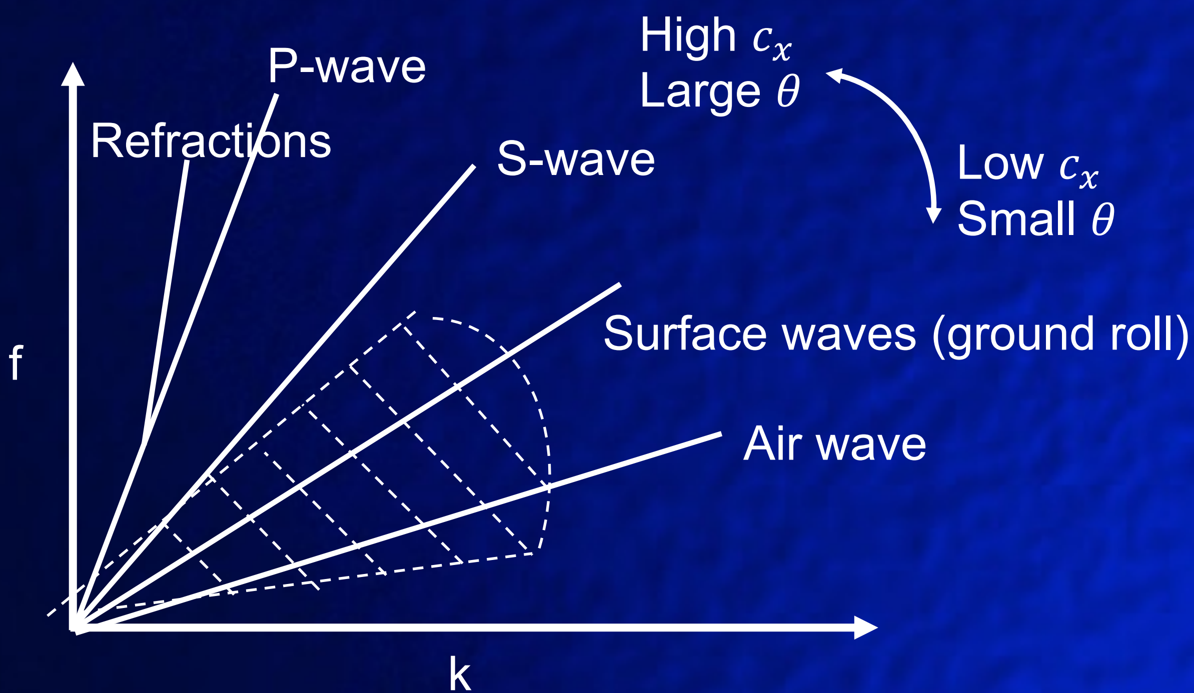
But be careful, all the rules on windowing apply.



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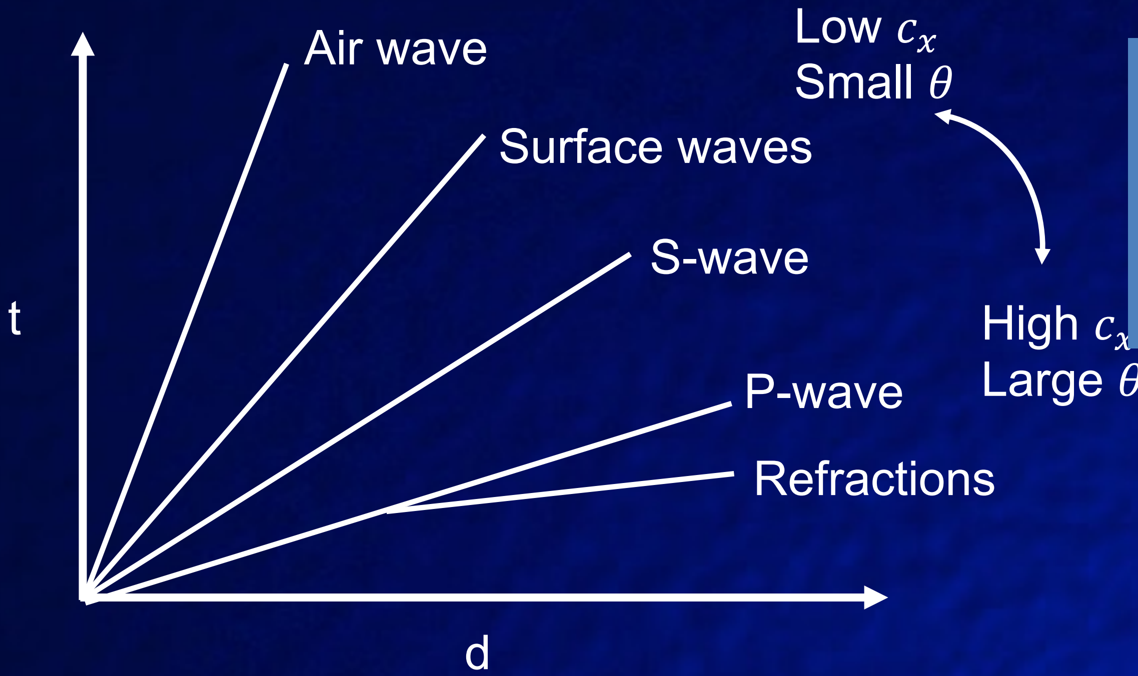

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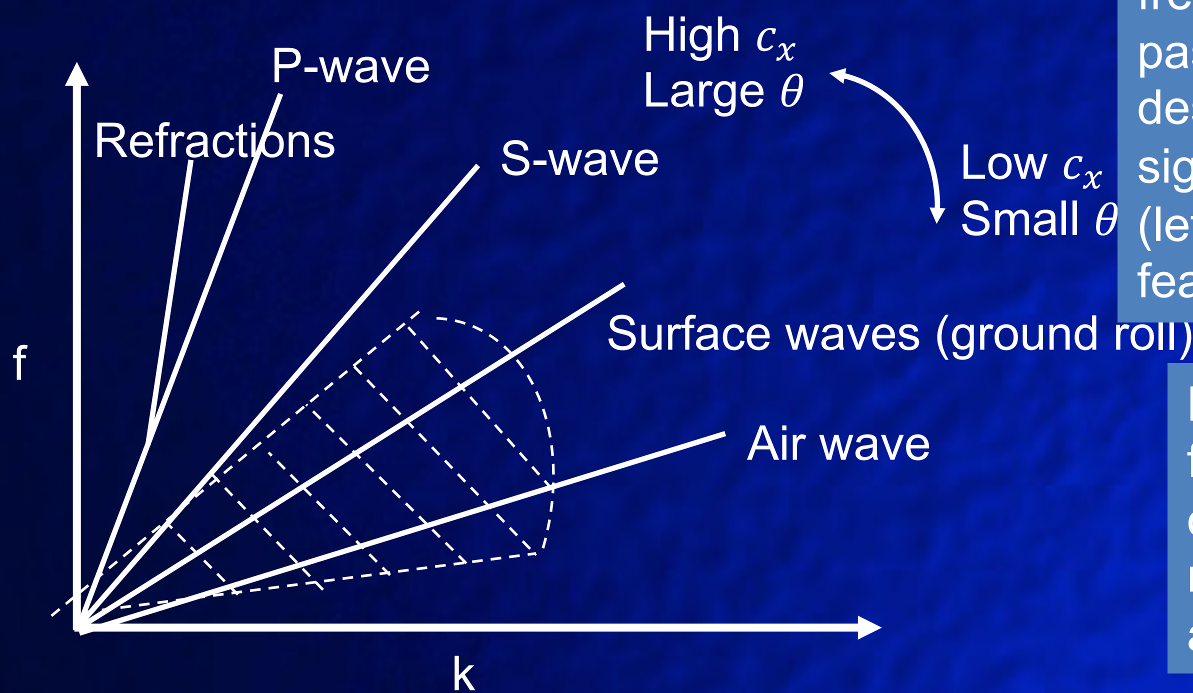
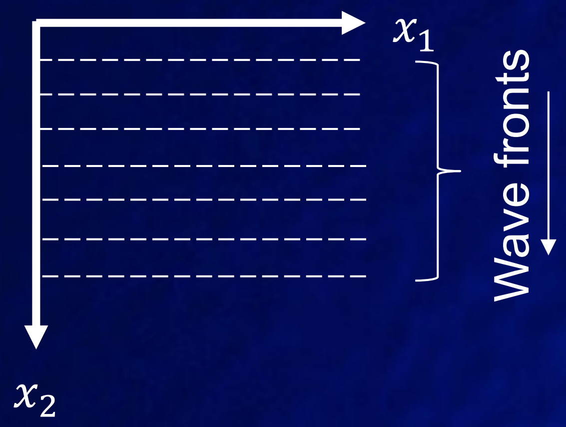


Can still filter specific frequencies (e.g. low pass, high pass). Filters can also be designed to remove updip signals (right side) or downdip (left side) and many other features.

Many additional possible filters introduce greater opportunity for misinterpreting filter artifacts as structure.

Consider a rectangular surface array recording of a horizontal plane wave traveling in the x_2 direction.



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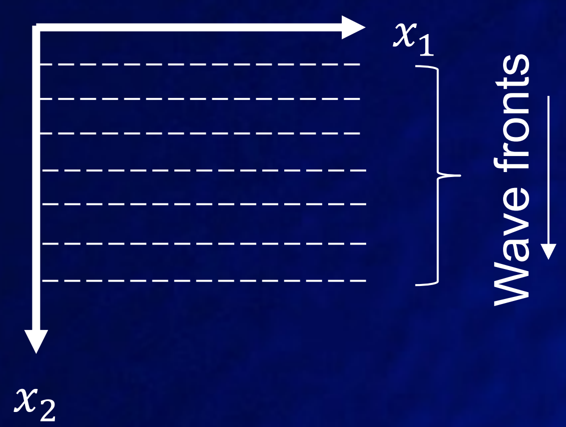

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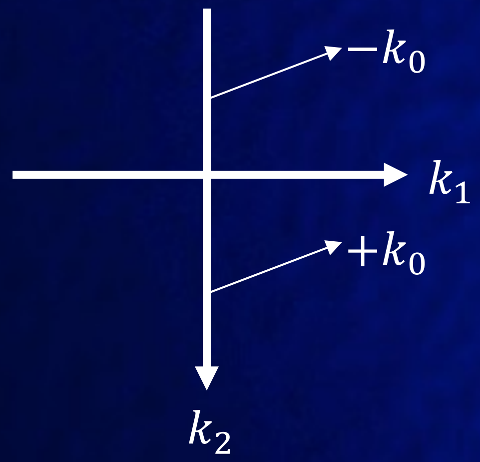
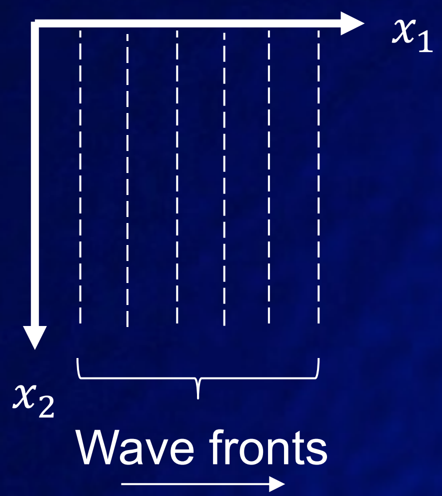
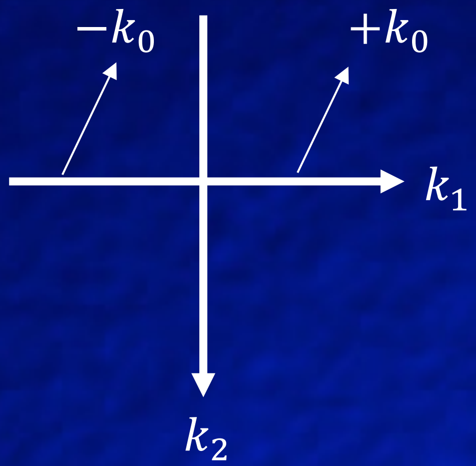


The FT of a constant frequency sine wave is a delta function. When traveling in the x_2 direction the impulses lie on the k_2 axis.

Similarly





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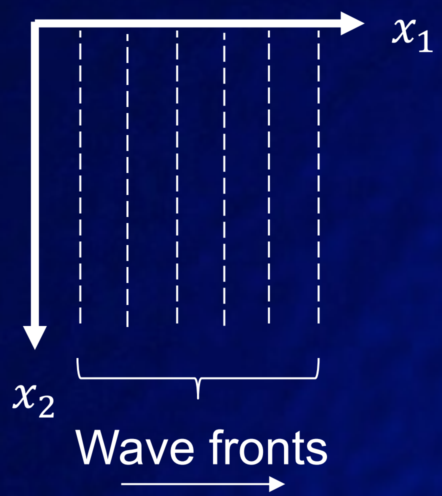

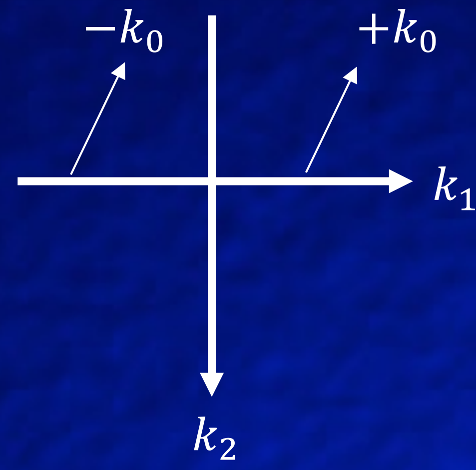

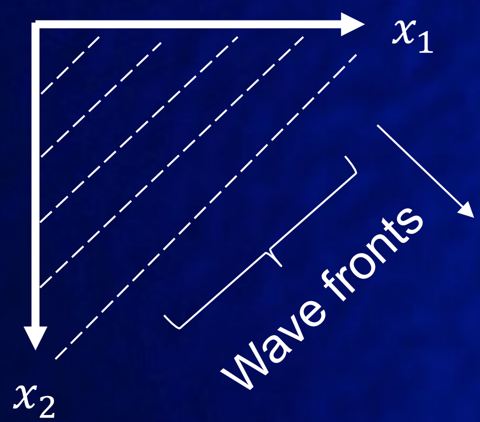

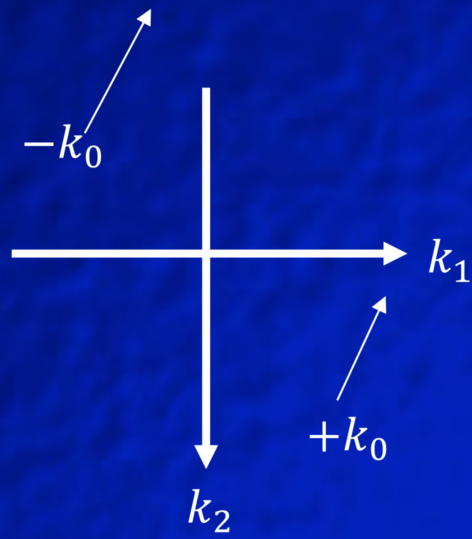

There is a great deal more that can be done with F-K analysis (e.g. reflection seismology) and with arrays (they don't always have to be rectangular or evenly spaced; array analysis) and it would be easy to spend an entire semester on just one of those topics.