FIR Filters

Mitch Withers, Res. Assoc. Prof., Univ. of Memphis

See Aster and Borchers, Time Series Analysis, chapter 5.

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Ut quod ali cibus est aliis fuat acre venenum.
What is food to one, is to others bitter poison.
Titus Lucretius Carus (c. 99 BC – 55 BC), Roman poet and philosopher.

An alternative interpretation: One person’s noise is another person’s signal.

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5. Creative effects in audio and video

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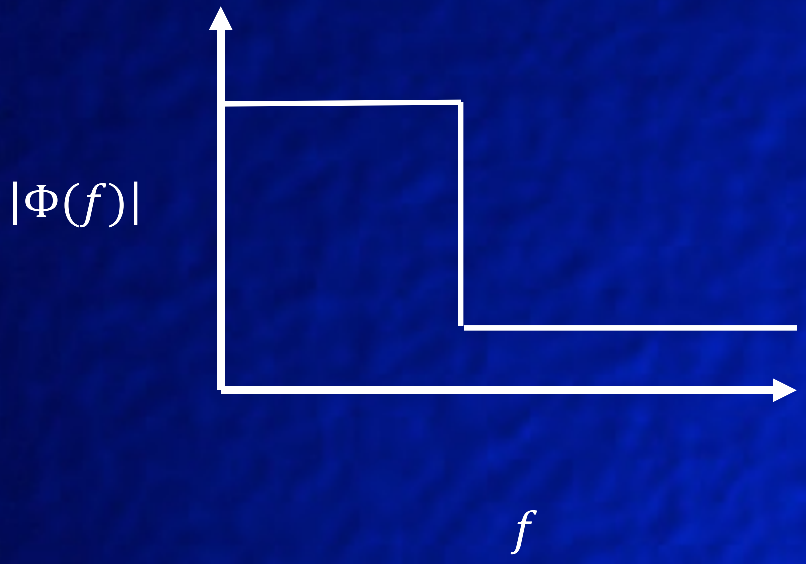

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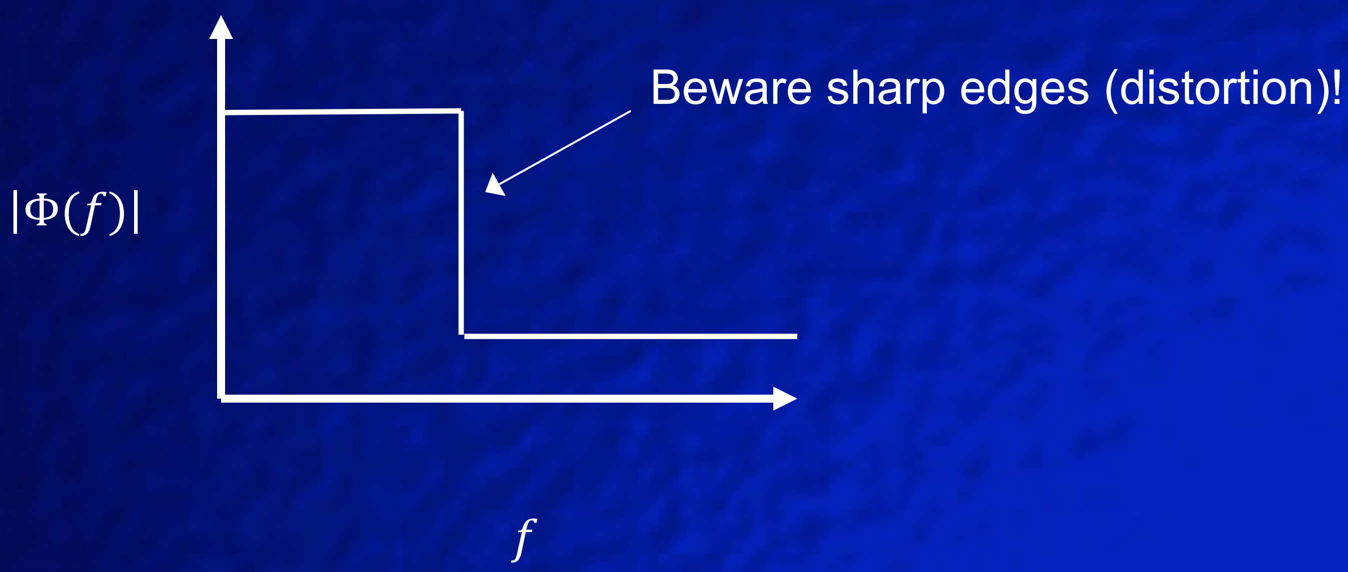

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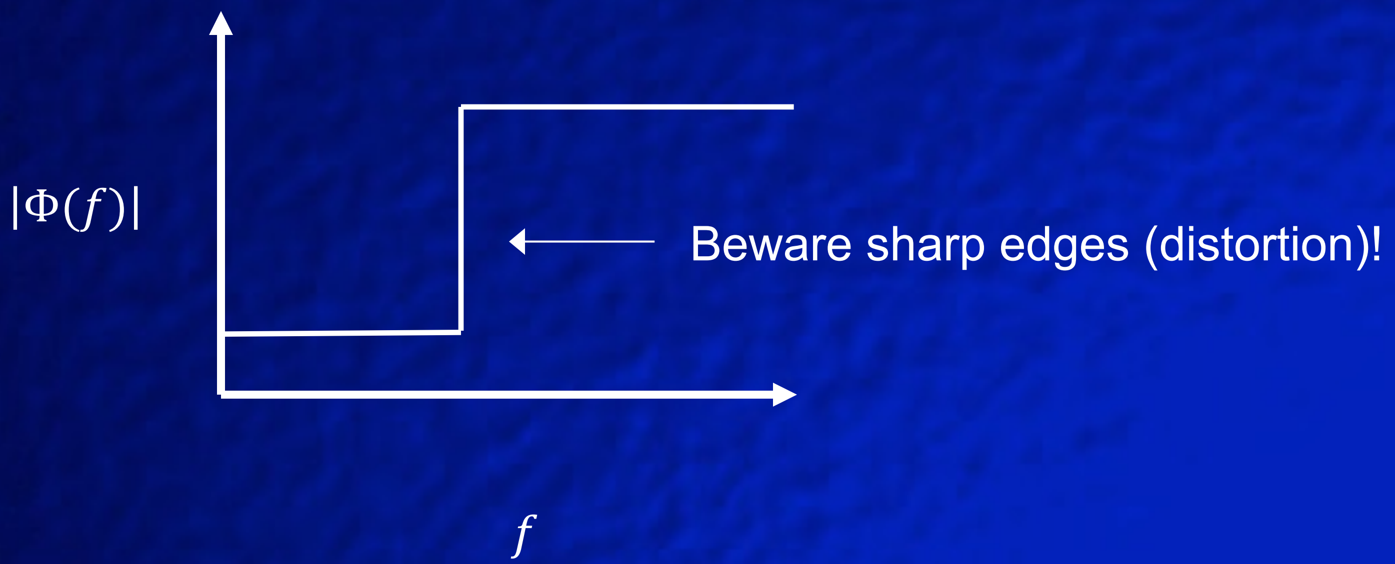

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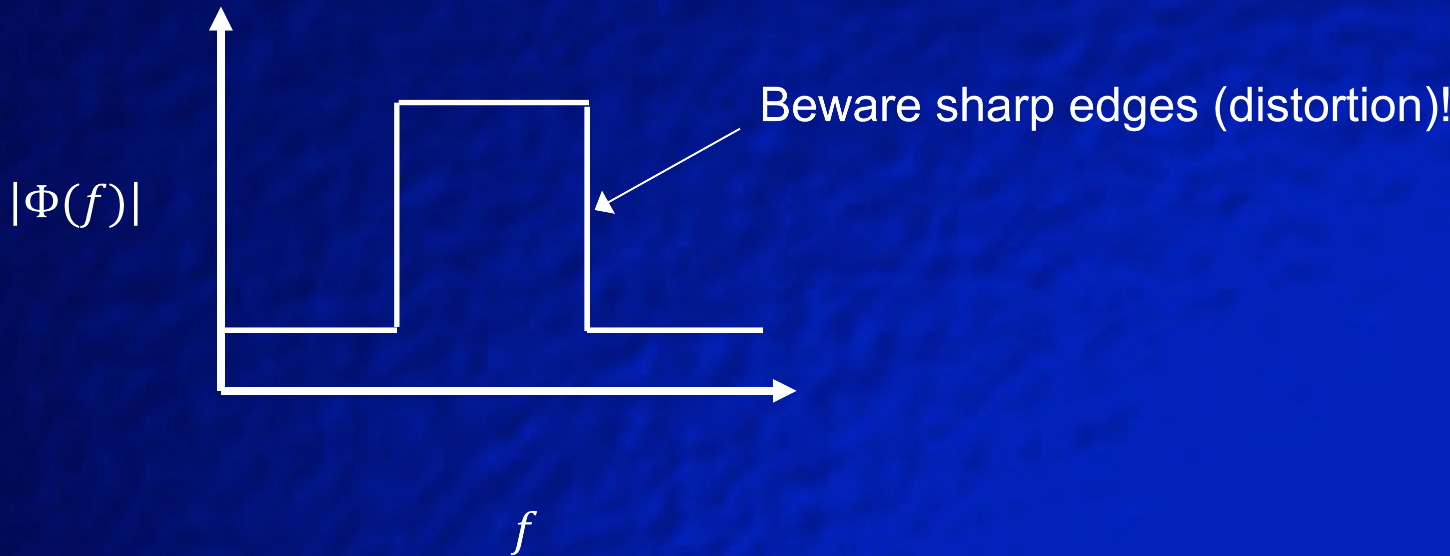
A band pass filter keeps frequencies within a specified range and rejects those outside the range.



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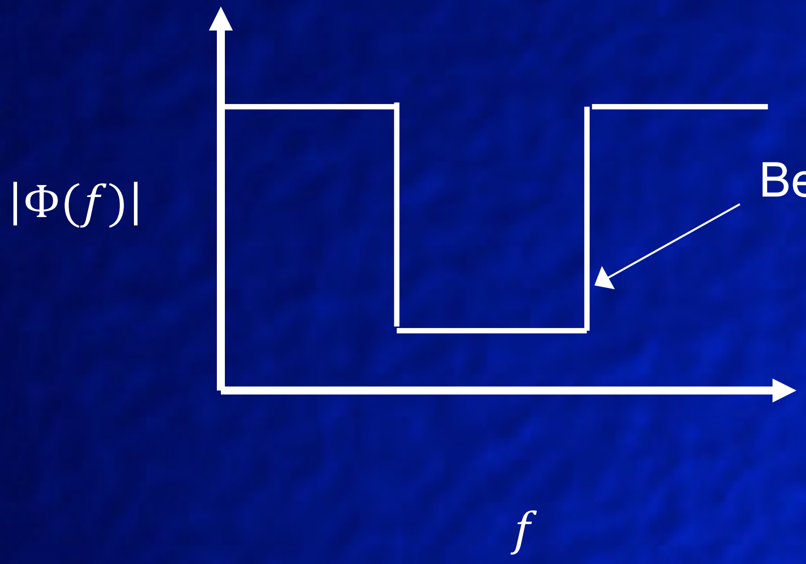
A notch or band gap filter rejects frequencies within a specified range and keeps those outside the range.



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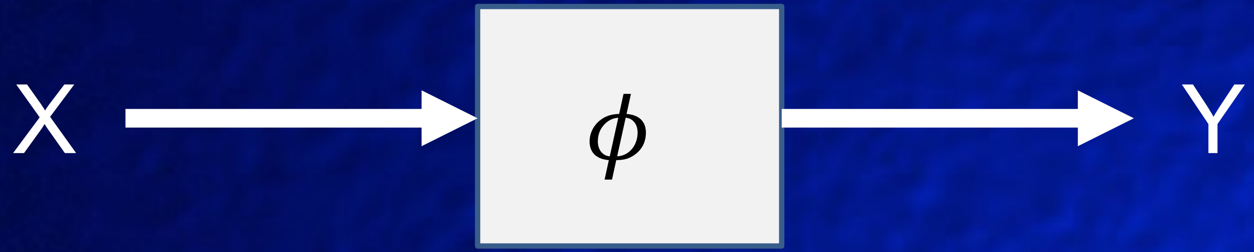

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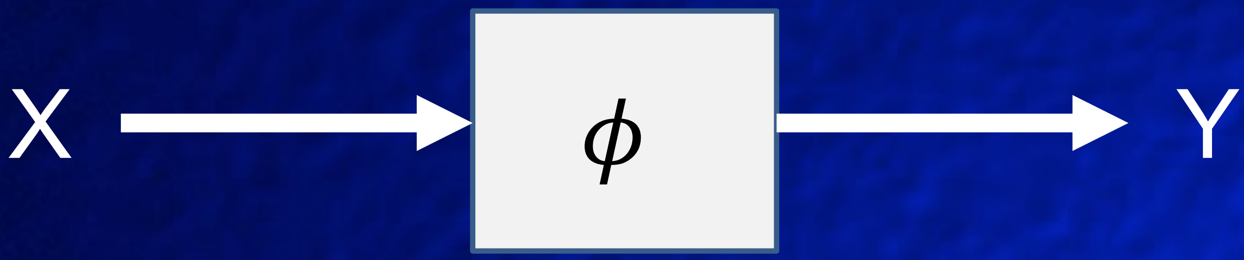

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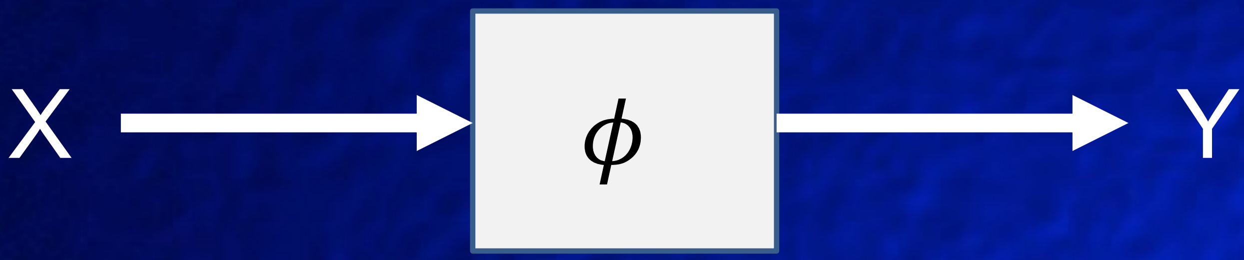

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Beware the unstable filter (no poles on the RHS of the S-plane).

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Run matlab command, filterdemo_mac

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Linear phase response → minimal distortion

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Both approaches accomplish the same thing, choosing M, but depend on which aspect is most important to the design; time-length or frequency resolution.

Consider the running mean filter, an easy and fast FIR filter.

$$w_n = \frac{1}{M}\Pi_m = \begin{cases} \frac{1}{M}, & |n| \le {}^{(M-1)}\!/_{2} \\ 0, & |n| > {}^{(M-1)}\!/_{2} \end{cases}$$

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We use the DFT to see the frequency domain response of this filter.

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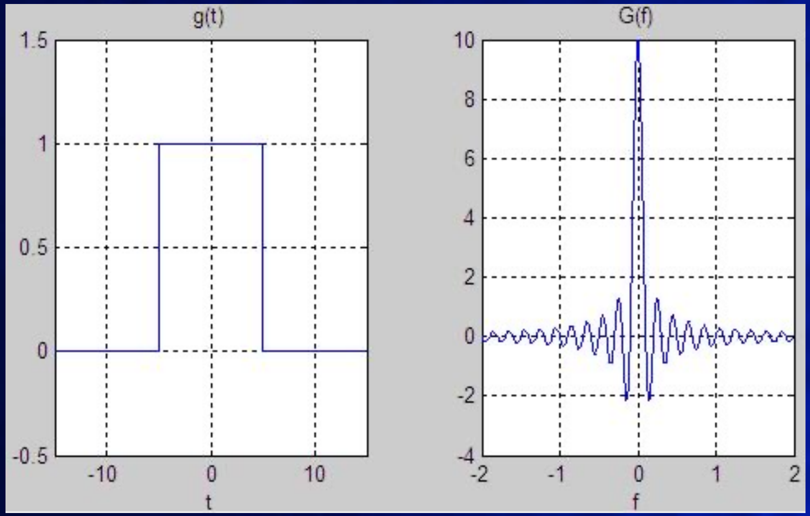

A low pass filter with a lot of ripple in the stop band.

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Recall the shift property of the FT

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Just cut the extra points if not needed, y=y(1:N);

The matlab **filter** command may also be used to implement your FIR filter. It’s designed for IIR filters (more later) which uses two vectors but works just as well with 1 vector.

```
y=filter(w,[1],x);
```

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It is frequently more intuitive to design a filter in the frequency domain.

The problem here is there may be an infinite number of non-zero weights, w_n.

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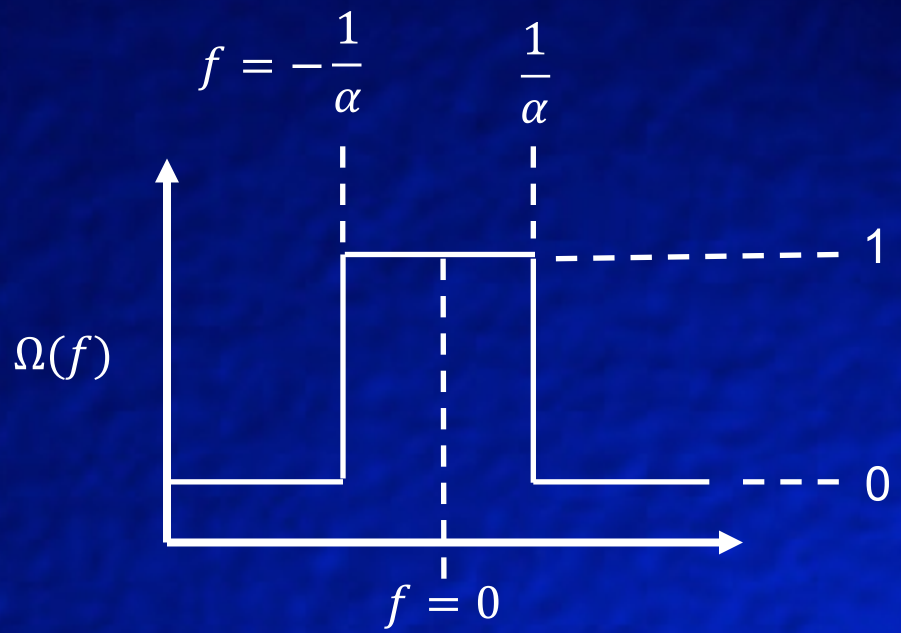
Transitions at $\frac{\alpha f}{2} = \pm\frac{1}{2}$



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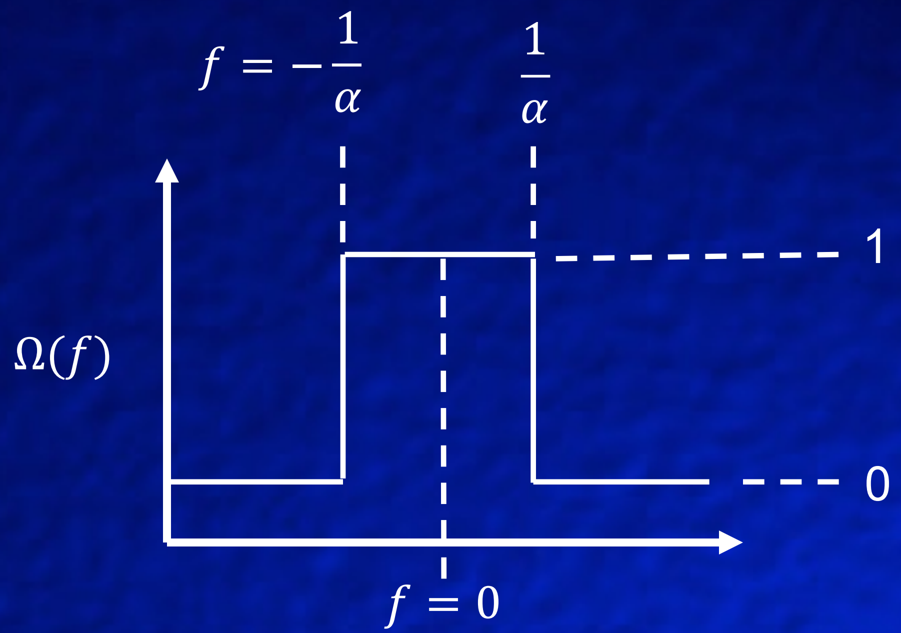

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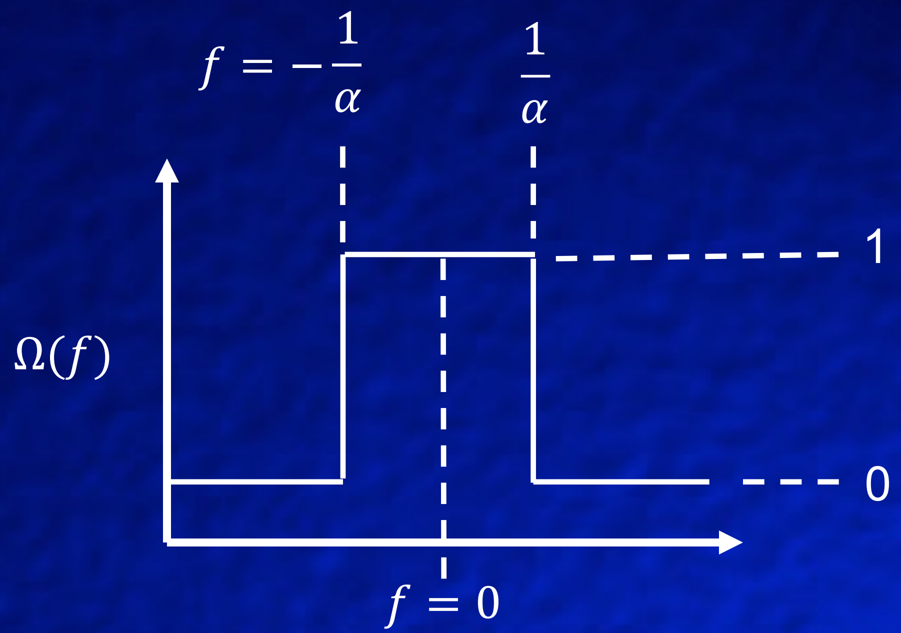

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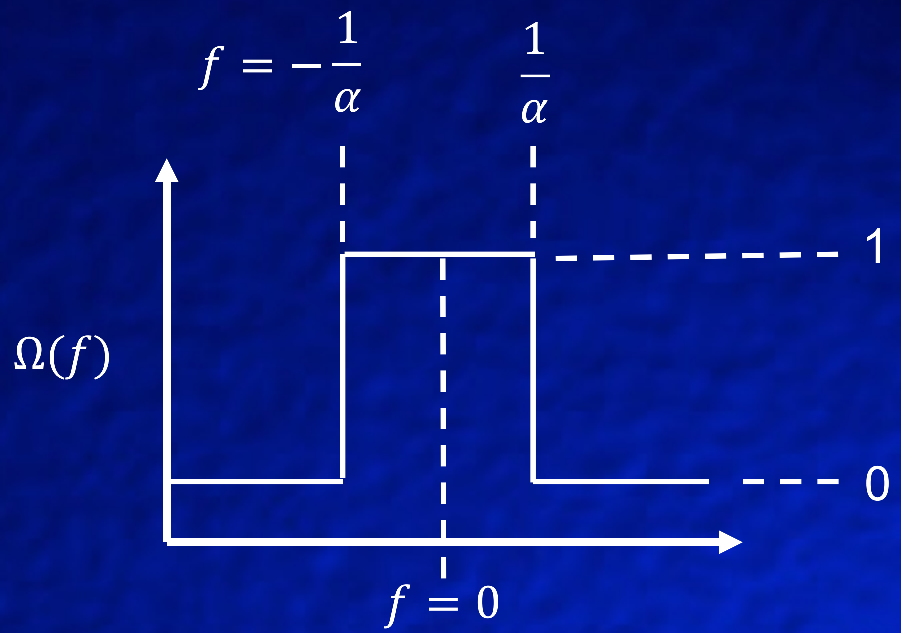

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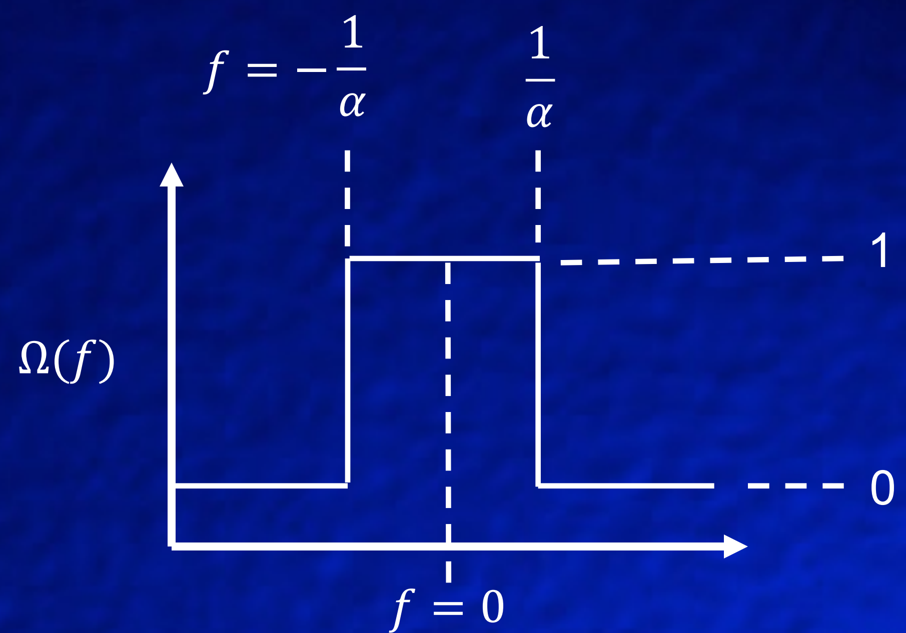

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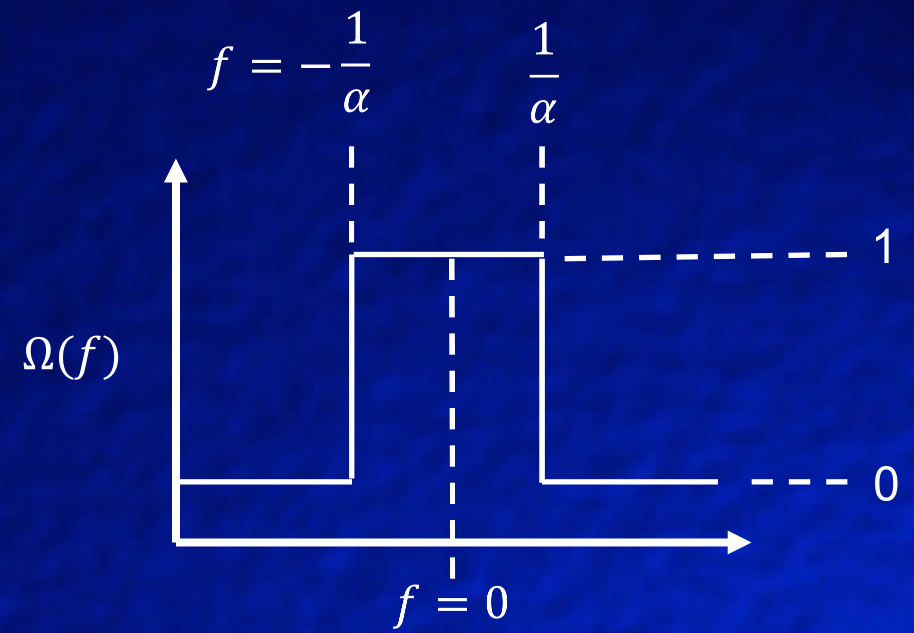

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We can truncate the weights using a boxcar from $\pm\, {}^{(M-1)}/_{2}$, but as we found with windowing in our discussion on power spectra, this creates spectral leakage in the frequency domain by convolving the ideal filter, $\Omega(f)$ with a sinc function.

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$w_n = \frac{2}{\alpha} sinc\left(\frac{2n}{\alpha}\right)$, are the time domain weights of our filter for the ideal low pass response.

The weights decay as $\frac{1}{n}$ and only goes to 0 in the limit as $n \rightarrow \infty$.

We can truncate the weights using a boxcar from $\pm\, {}^{(M-1)}/_{2}$, but as we found with windowing in our discussion on power spectra, this creates spectral leakage in the frequency domain by convolving the ideal filter, $\Omega(f)$ with a sinc function.

This leads us right back to windowing.

Run matlab program aster_fig54ff.m

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We designed our FIR filter by first selecting the desired ideal frequency response, $\Omega(f)$, then converting it to the discrete time domain with the integral transform,

$$w_n = \int_{-f_s/2}^{f_s/2} \Omega(f) e^{i2\pi f n} df$$

where f_s is the sample rate.

This, incidentally, is similar to our asymmetric transform pair we found in equation 3.32 in Aster and Borchers while developing the DFT.

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We then *windowed* w_n to the desired number of weights (128 points in our example).

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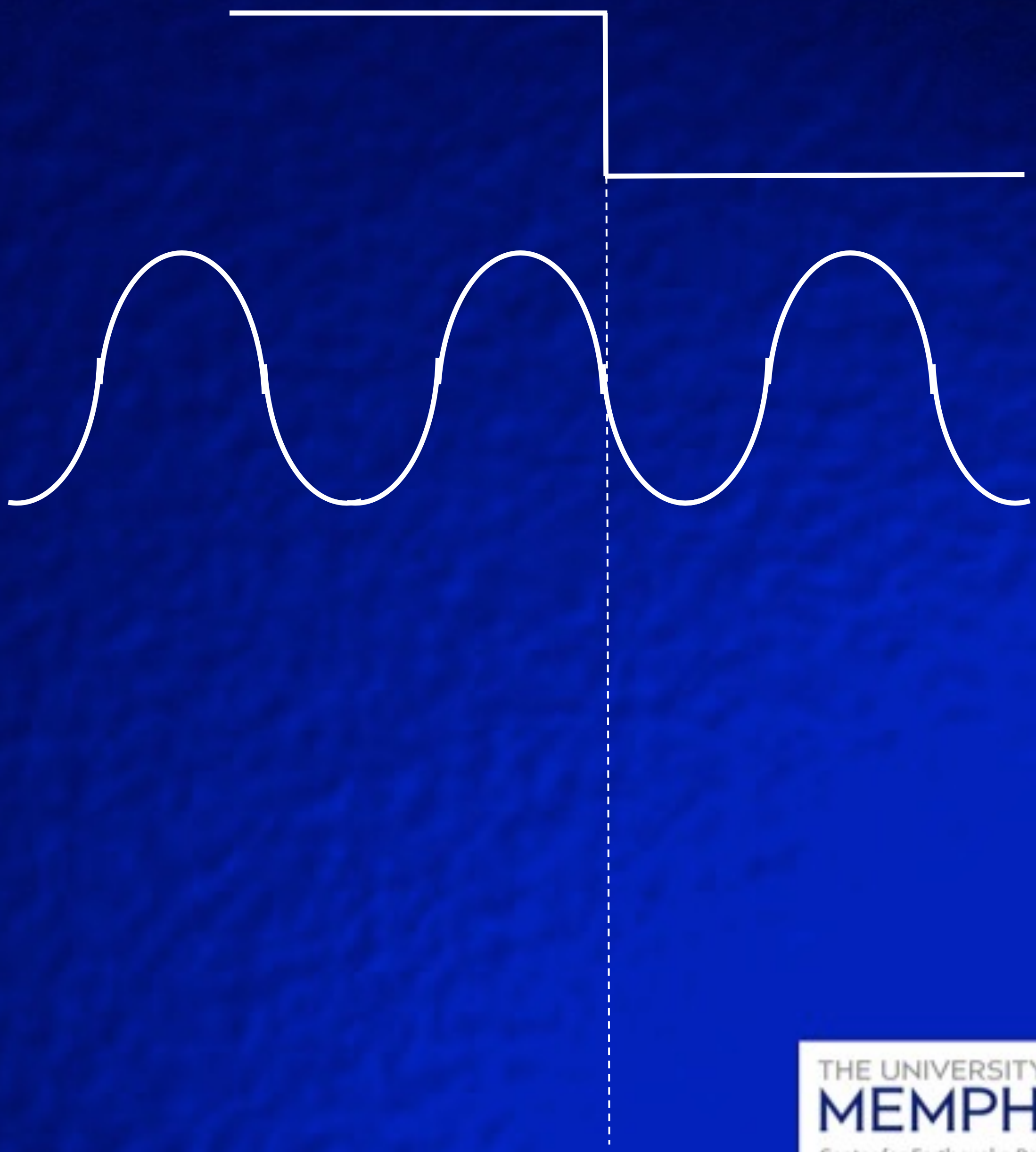
This is not the best way to design a FIR filter but it is the easiest.

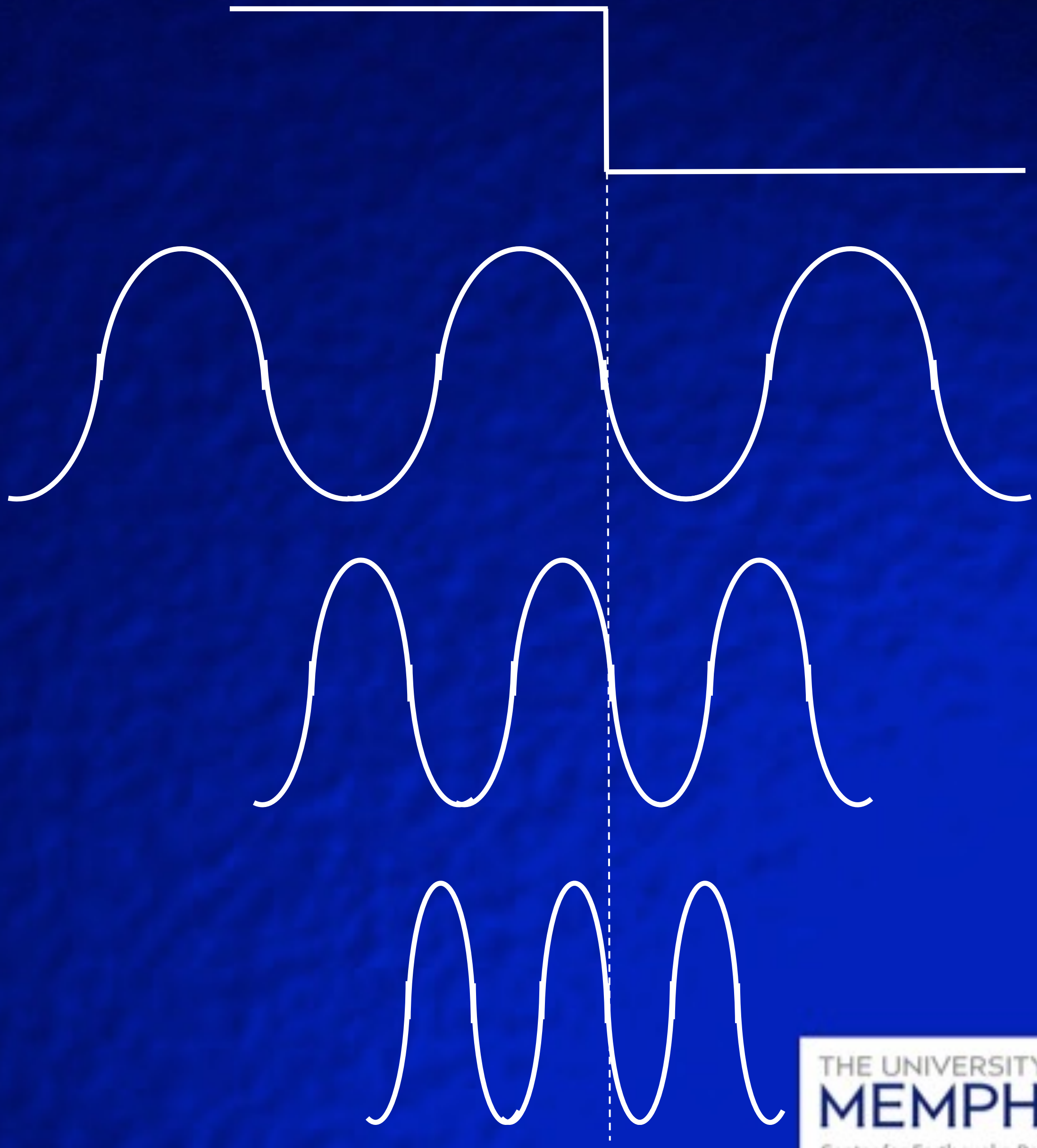
Run matlab program firpmdemo.m

Consider how successive superpositions of precisely aligned (think phase) cosines sum to create a discontinuity.



Consider how successive superpositions of precisely aligned (think phase) cosines sum to create a discontinuity.

Long periods are needed to make the flat parts and high frequencies are needed to construct the discontinuity.



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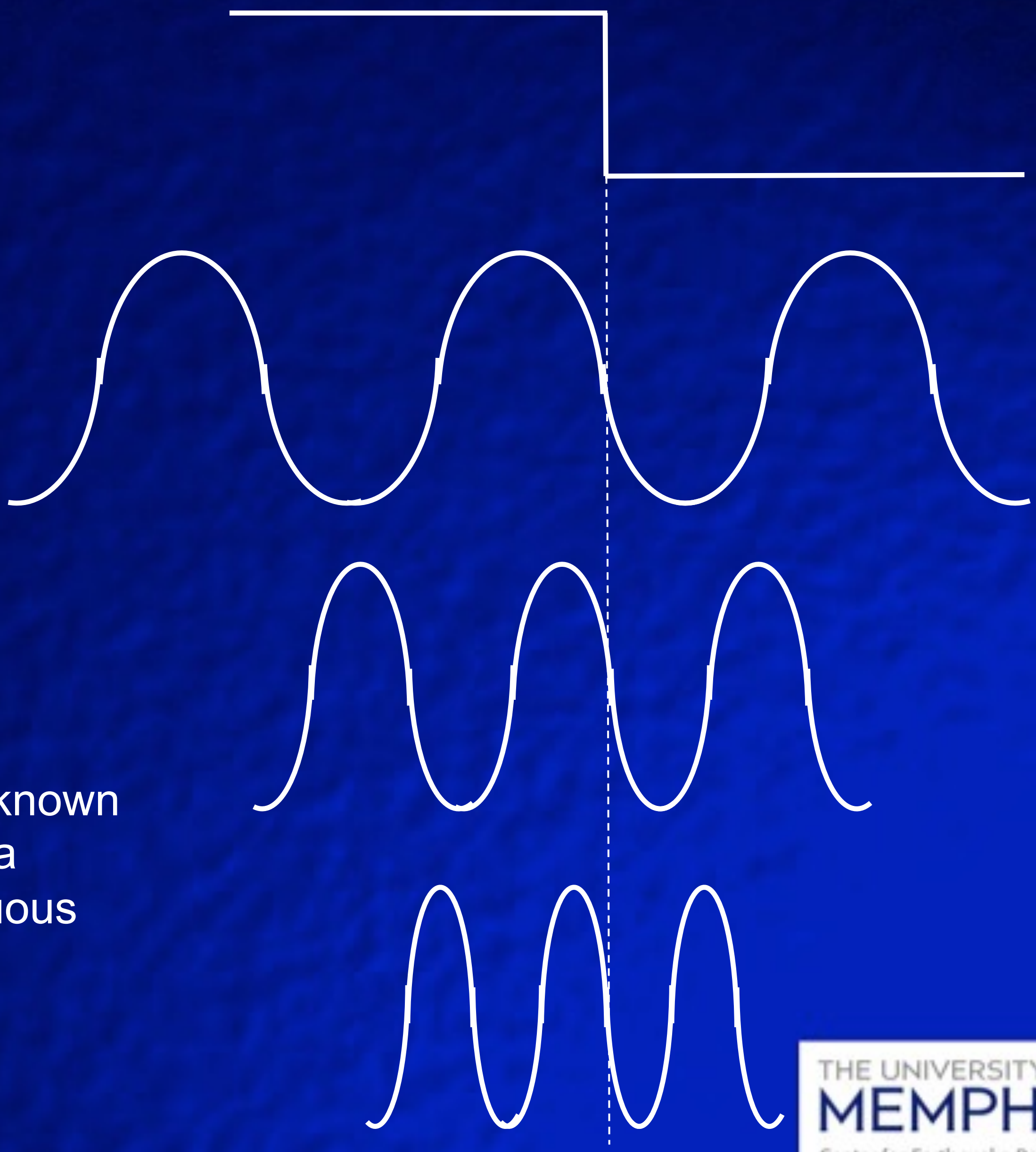

Long periods are needed to make the flat parts and high frequencies are needed to construct the discontinuity.

This leads us to a phenomena known as Gibbs Phenomena which is a result of constructing discontinuous signals with a finite number of discrete frequencies.

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On the one hand, we need the lower frequencies to construct the flat portion of the function, and high frequencies aligned to create the discontinuity.

This creates overshoot at the discontinuity. We can make the overshoot more narrow by adding additional higher frequencies but the amplitude of the overshoot remains at 9%. This is Gibb's phenomena.

Run gibbs.m