Pole Zero Potpourri.

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Steiner PZ Discussion

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The sound level is $\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$.

I_0 is a reference level and β is in units of decibels.

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Then ${}^{1}/_{x}$ will reduce by 6db for every doubling in x or 6db per octave.

An octave in music is the interval between one musical pitch and another with half or double the frequency.

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For each zero, the amplitude response rises by 6db/octave. So that a system with two zeros, both at the origin, will have a positive slope of 12db/octave. If a a zero is not at the origin, it produces an increase in slope of 6db/octave at the location (frequency) of the zero.

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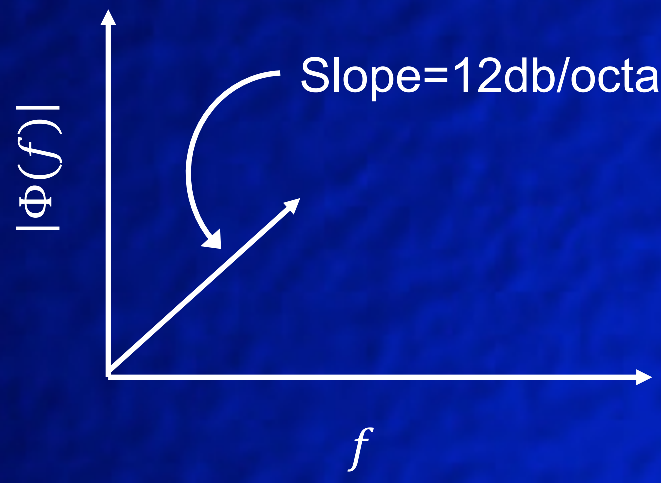

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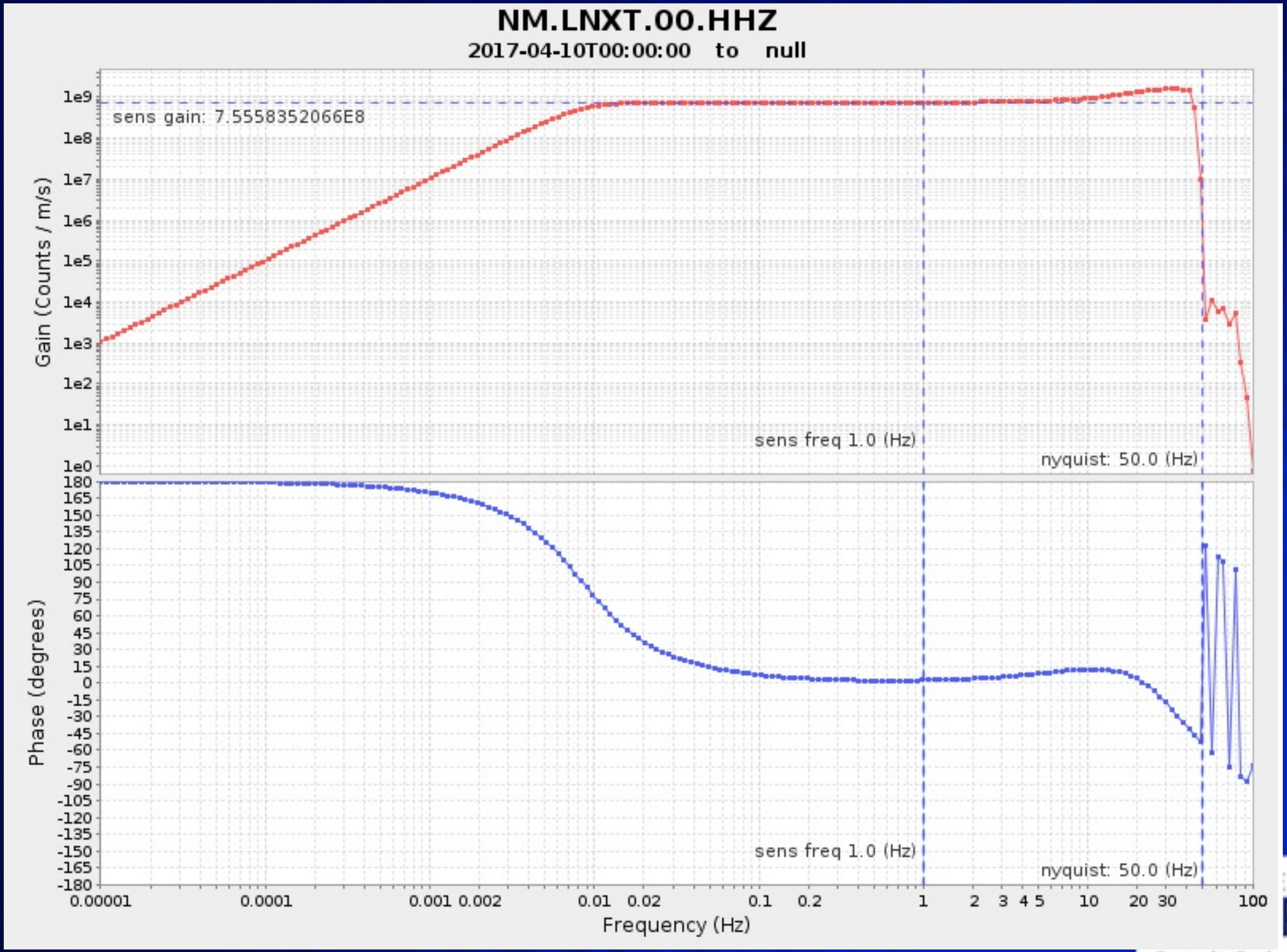
If np<nz, then the response is increase toward infinity.

If np>nz, then the response is decreasing toward infinity.

Metadata available at the IRIS Metadata Aggregator (MDA)

For example, Trillium t-120 response for station LNXT as of April 2017.



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ZEROS 5	
+0.000000e+00	+0.000000e+00
+0.000000e+00	+0.000000e+00
-9.000000e+01	+0.000000e+00
-1.607000e+02	+0.000000e+00
-3.108000e+03	+0.000000e+00
POLES 7	
-3.852000e-02	+3.658000e-02
-3.852000e-02	-3.658000e-02
-1.780000e+02	+0.000000e+00
-1.350000e+02	+1.600000e+02
-1.350000e+02	-1.600000e+02
-6.710000e+02	+1.154000e+03
-6.710000e+02	-1.154000e+03
CONSTANT 2.339025e+14	

Note changes in slope (plots are in frequency and poles/zeros are in radians/s). For epoch beginning 4/10/2017 there are 3 zeros at zero and 1 zero at about 90 radians/s or about 14Hz. Note other changes in slope. Why such a large rolloff at 50Hz?

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In this case, the first complex conjugate pair represent the root that is the corner frequency of the T120.

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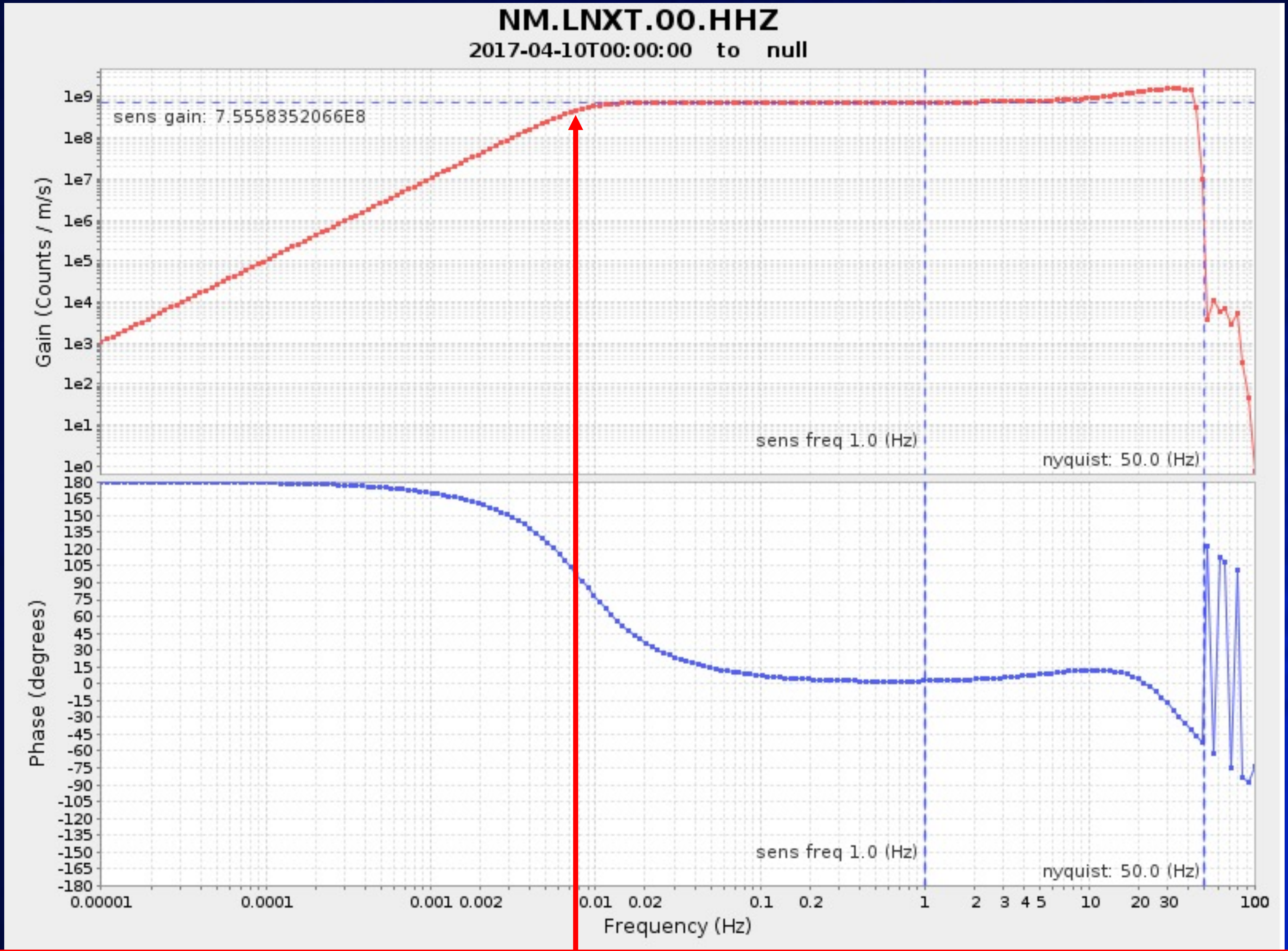
$$\sqrt{-0.03852^2 + 0.03658^2} = 0.05312\, radians = 0.008454\, Hz\; (recall\; f = \frac{\omega}{2\pi})$$

And $\frac{1}{f} = \lambda$, $\frac{1}{0.00845\, {}^{cycles}/_{second}} = 118.28\; {}^{seconds}/_{cycle}$



0.008455 Hz (118.28 seconds) the so-called 3db point aka the corner frequency or period of the seismometer. The T120 is called a 120 second seismometer.

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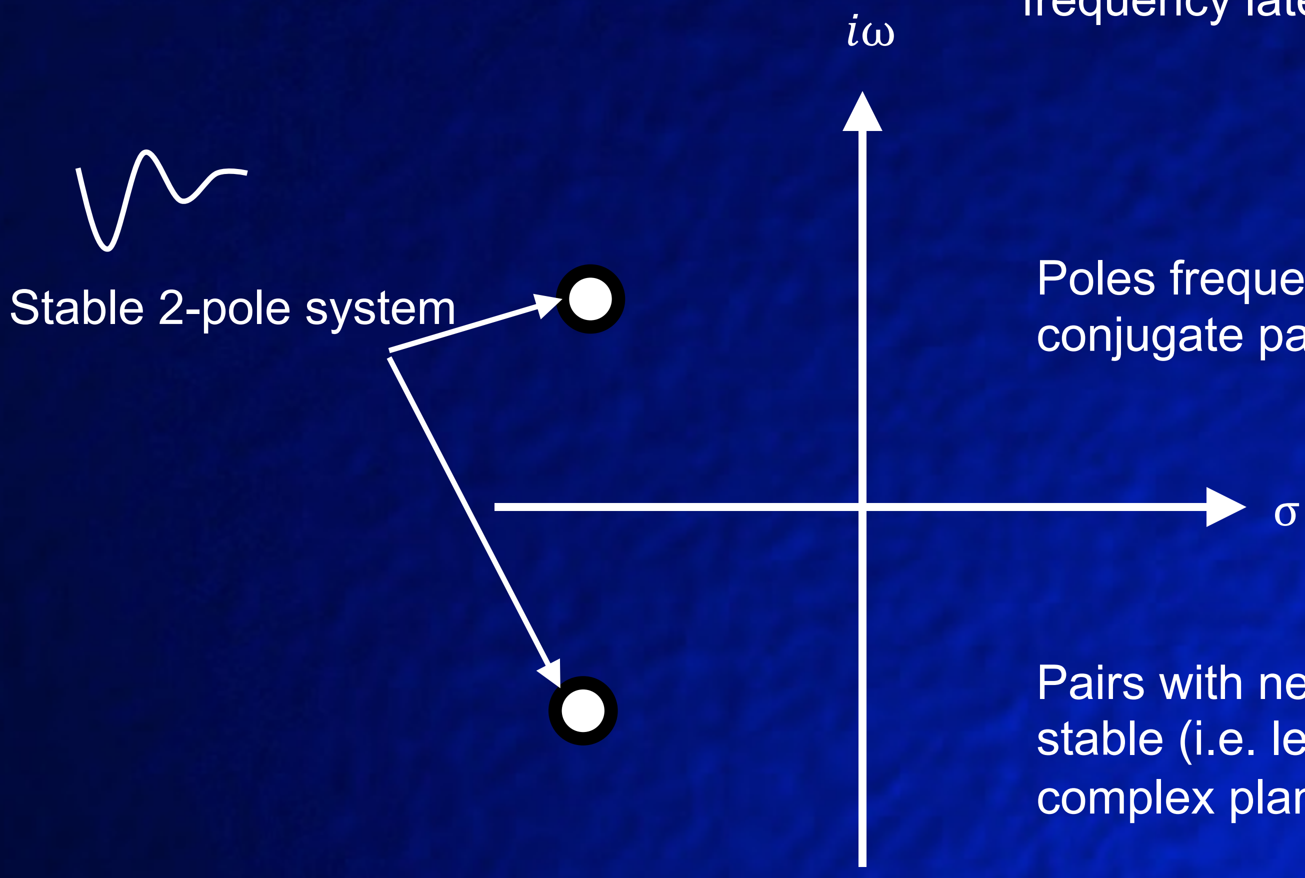
Aster Pole Zero Notes.

S-plane (more on complex frequency later). $s = \sigma + i\omega$

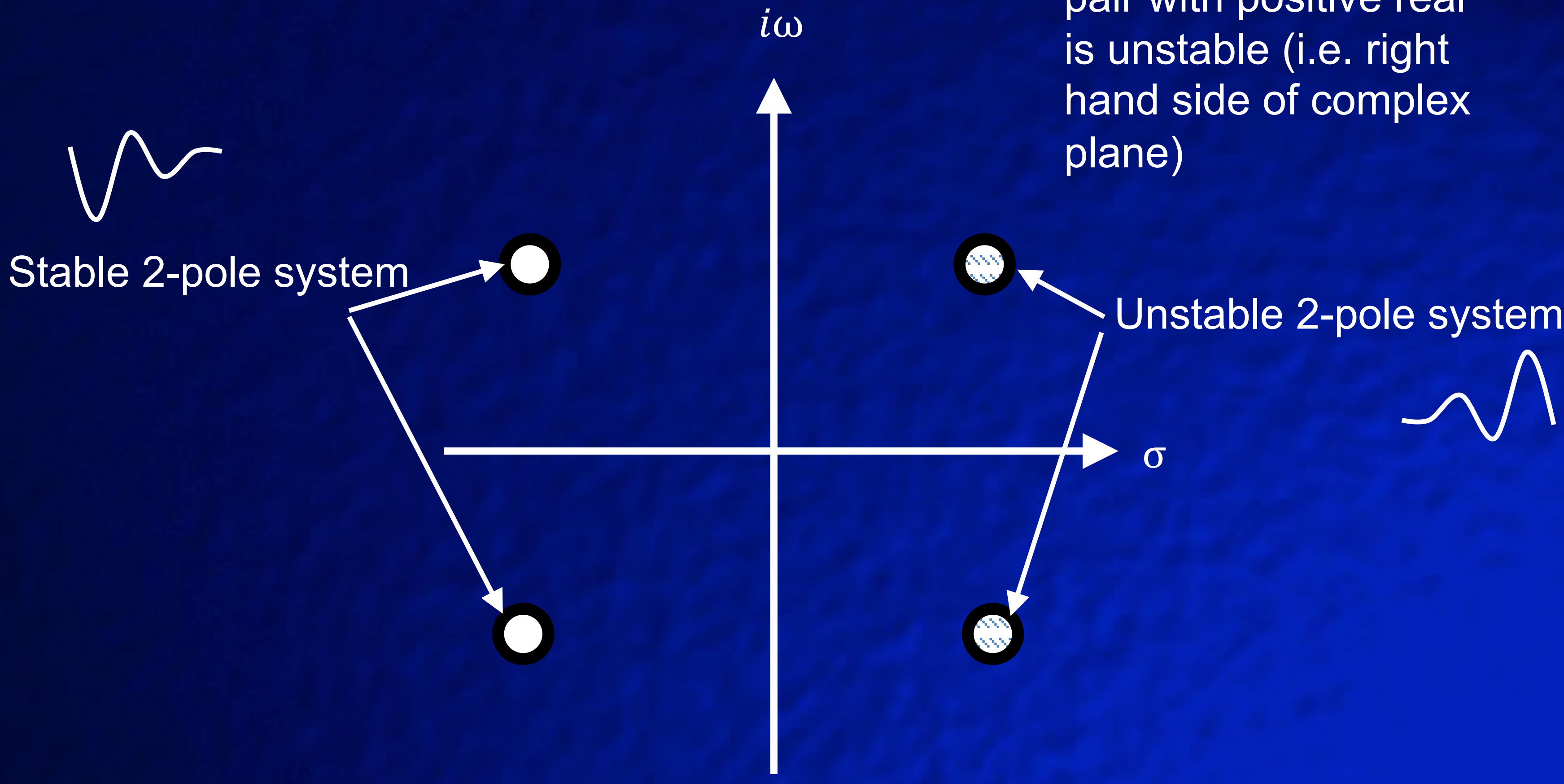


Poles frequently occur as complex conjugate pairs.

Pairs with negative real part are stable (i.e. left hand side of complex plane).

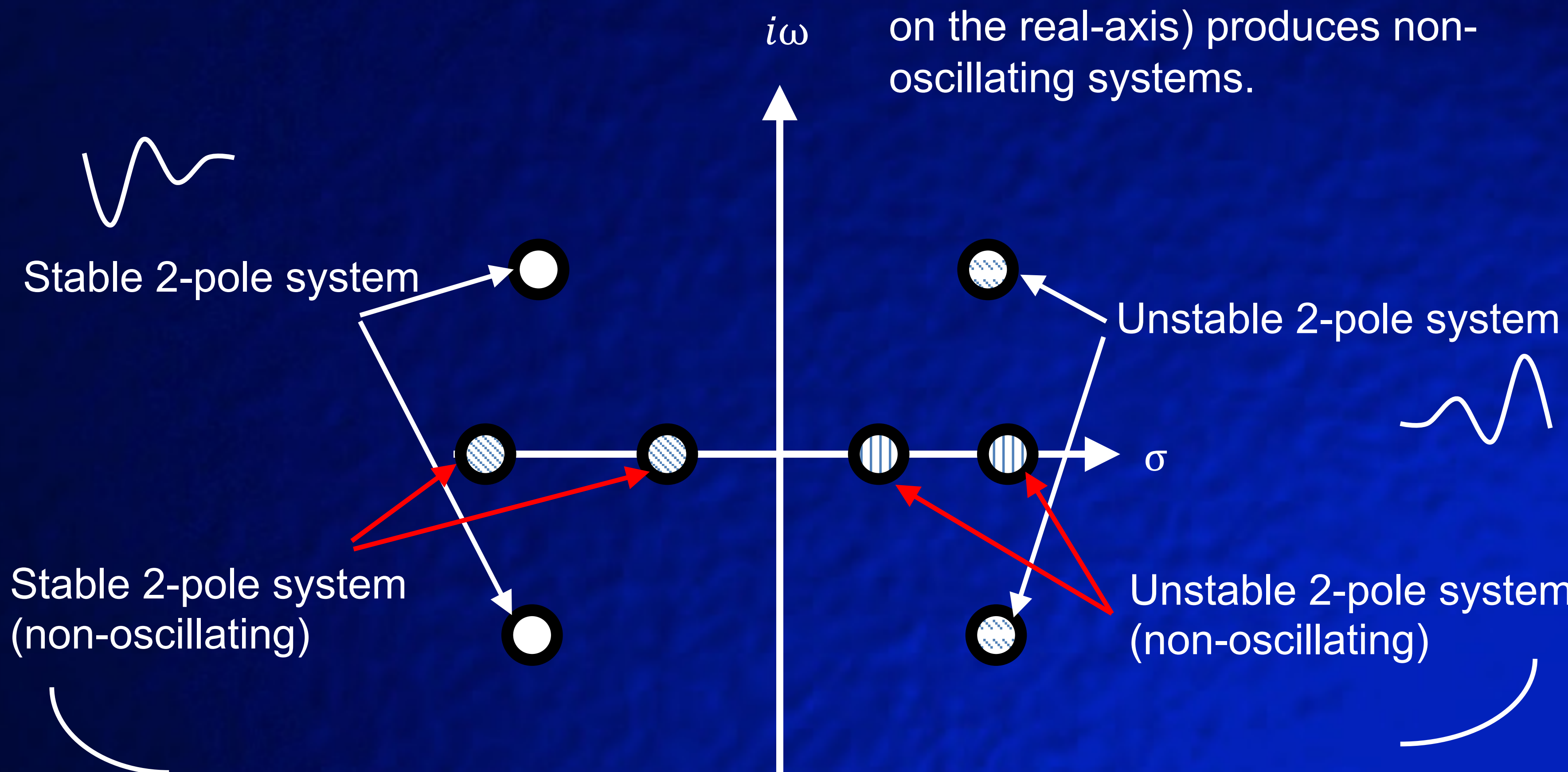
Aster Pole Zero Notes.

Complex conjugate pair with positive real is unstable (i.e. right hand side of complex plane)



Aster Pole Zero Notes.

A 0 imaginary component (i.e. poles lie on the real-axis) produces non-oscillating systems.



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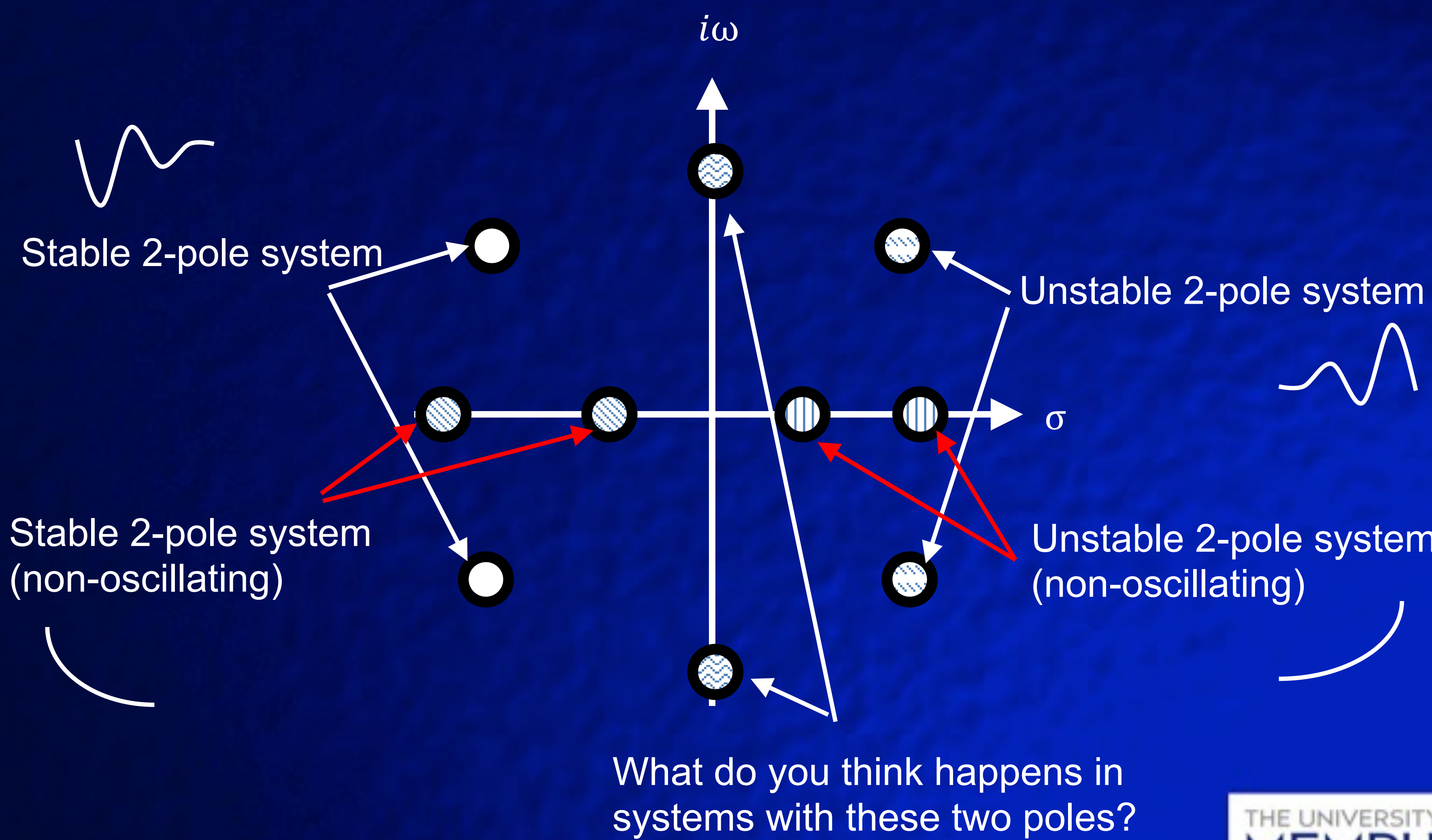

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