# Signal Processing for the Earth Sciences

CERI 7106-8106

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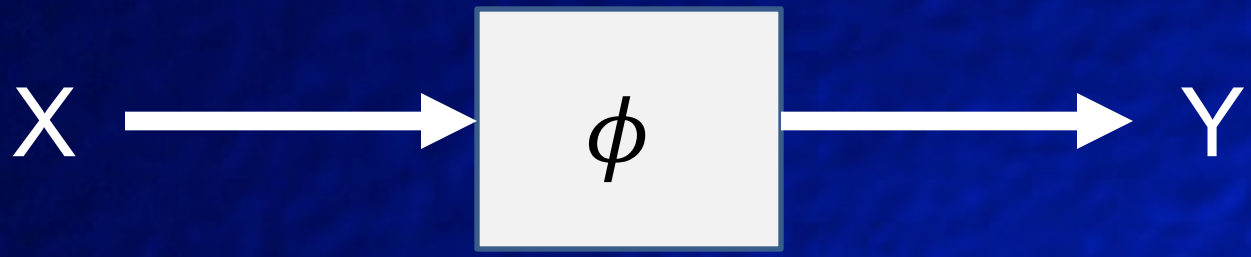
THE UNIVERSITY OF MEMPHIS
Center for Earthquake Research and Information

Diagram representing a Linear System. The system, $\phi$, operates on input X to produce output Y, frequently using an integral transform.




In this class we'll be studying tools for examining and manipulating series, frequently time series. Though the series can be time, space, objects or samples, people, etc. The math is all the same so long as its linear.

Syllabus: http://www.ceri.memphis.edu/people/mwithers/CERI7106/index.html

Grades

- 70% homework (four of them)
- 25% “Final”
- 5% punctuality and participation
- Late homework = 0
- Homework due at the beginning of class
- Matlab is required for homework
- Work together but don’t plagiarize
- Don’t be intimidated by the notes, ask questions

Aster Class notes (no longer maintained, see A&B Time Series Analysis for most recent version):
http://www.ceri.memphis.edu/people/mwithers/CERI7106/aster/GEOP505.html

If you're not familiar with matlab, please find a very useful primer at the link above. Even if you are familiar with matlab, the primer has examples useful for doing the homework.

# Introductions

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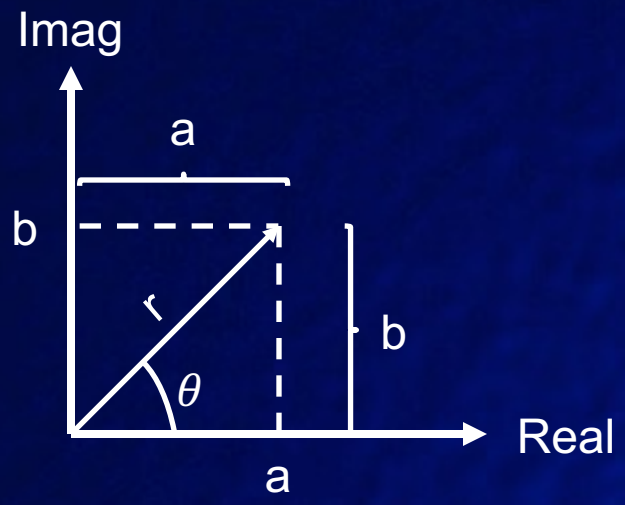
What number when multiplied by itself is -1?

i.e. $x \cdot x = -1 \implies x = \sqrt{-1} = i, not\ a\ real\ number$

i is said to be imaginary and is the basis for a field of mathematics called complex analysis.

# Complex Numbers



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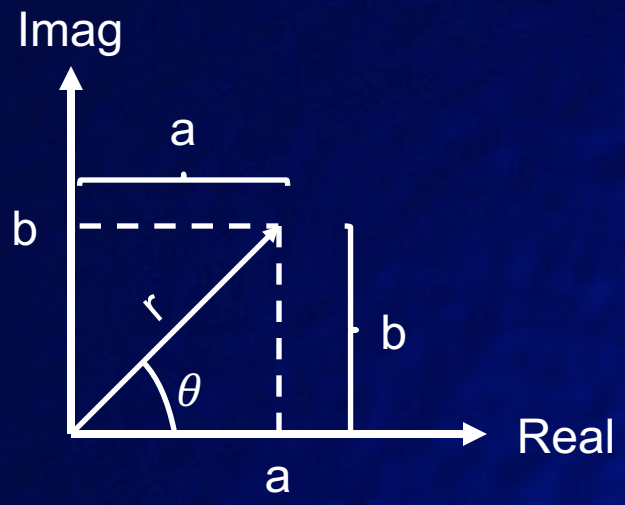


Recall

$$\tan(\theta) = \frac{opp}{adj}$$

$$\sin(\theta) = \frac{opp}{hyp}$$

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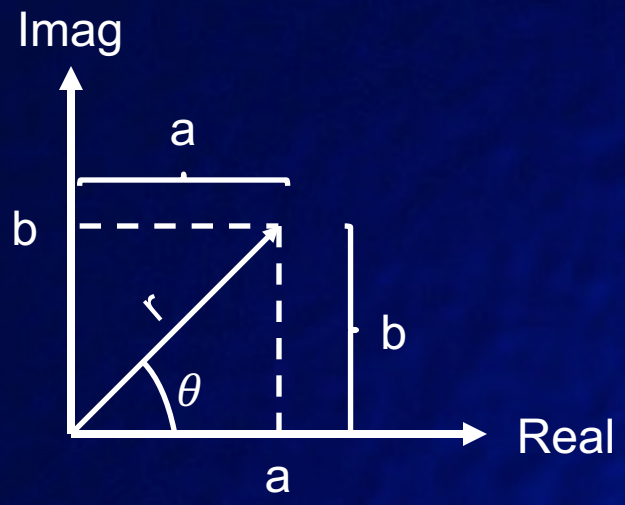


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$$r = \sqrt{a^2 + b^2}$$

$$\theta = tan^{-1}(b/a)$$

$$a = rcos(\theta)$$

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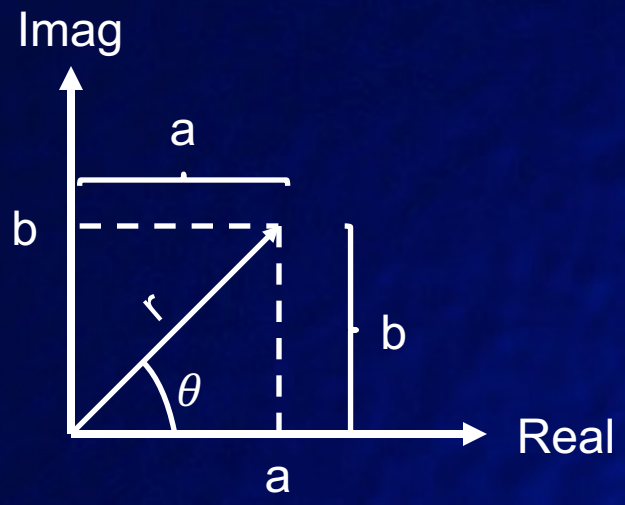


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Complex Conjugate

$$(a + ib)^* = \mathrm{a} - \mathrm{ib} = \mathrm{r}e^{-i\theta}$$

# Euler's number is *e*.

e is an irrational constant (pi is too) that is the base of the natural logarithm, ln. A surprising number of natural phenomena may be described using e.

$f(x) = e^x$ is the exponential function

One of several definitions for $e$ is

$$e \equiv \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \cong 2.71828$$

$\int e^x dx = e^x + C$ (though we'll ignore C)

Taking advantage of both *i* and *e* we have Euler's formula

$$e^{ix} = \cos(x) + isin(x)$$

$$e^{-ix} = \cos(x) - isin(x)$$

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Likewise by subtracting we get, $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$

Additional information for today's lecture.

More on e: https://mathworld.wolfram.com/e.html

Primer on Complex numbers See A&B TSA Appendix B.

Next time, linear systems in the time domain.  See A&B Chapter 1