Signal Processing for the Earth Sciences

Homework 4; **Due November 22, 2024**

November 1, 2024

1) Obtain a formula for the infinite number of FIR filter coefficients (w_n in equation 9 of the notes) necessary to characterize a band-limited differentiator, which has the analog frequency response for a unit sampling rate $(F_s = 1, \alpha \geq 2)$ of

$$\Omega(f) = \begin{cases} 2\pi i f, (|f| \leq 1/\alpha) \\ 0, (|f| > 1/\alpha) \end{cases}$$

2) Write a MATLAB program that calculates the FIR weights. Plot the first 31 coefficients ($-15 \leq n \leq 15$) for $\alpha = 4$ and $\alpha = 2$.

3) For $\alpha = 2$, plot the dB-log f Nyquist interval amplitude and phase responses as functions of frequency for the 31 point FIR realization from (2). Do this for two cases:

(a) Where the FIR filter weights are simply truncated (rectangular window),

(b) Where the FIR filter weights are tapered with a Hamming window.

To facilitate easy comparison, use the same amplitude (dB scale; from -20 to 20 dB) and frequency scales for all plots. Compare these results with the analog results obtained in problem 1. Not you must calculate many more than 31 FIR coefficients in order to window for (a) and (b). Be sure to include the zero weighted FIR coefficients when converting from time to frequency.

4) Obtain the Laplace transform for $\varphi(t) = at - \sin(at)$ where $s > 0$ and a is a real constant > 0. If tables are used, please reference.

5) Without using tables, find the z-transform for the following series,

a) $x_n = \delta_n$ (the kronecker delta)

b) $x_n = h_n$ where h_n is the unit step function

c) $x_n = h_n e^{-anT}$ where a and T are constants greater than 0

d) $x_n = h_n a^n$ where a is a constant greater than 0