Signal Processing for the Earth Sciences

Homework 3; **Due November 1, 2024**

October 11, 2024

1) **a)** Show that the N-point DFT of a real sequence, x_n, is Hermitian, i.e., $X_{N-k} = X_k^*$.

b) Show that the N independent numbers of the real input sequence, x_n, produce only N/2+1 independent output numbers (N even) or (N-1)/2+1 independent output numbers (N odd) in this case. Why hasn't information been lost?

2) Estimate Power Spectral density for the 100 sample/s time series *mysteryseries.ascii* posted on the class website. The units of the time series are m/s.

a) Use a data length of $N_1 = 2^{14} =$ 16384 samples (163.84 s).

b) Use a data length of $N_2 = 2^{16} =$ 65536 samples (655.36 s).

c) Use a data length of $N_3 = 2^{18} =$ 262144 samples (2621.44 s).

In each case, estimate the PSD using the *pwelch* MATLAB function with 16 subwindows (NFFT=N/16). Plot your one-sided (nonnegative frequency) PSD estimates on a decibels vs. log (base 10) frequency scale. Title your plots appropriately, and show the proper units for the PSD and frequency axes.

d) For (c), calculate total signal power in the time and frequency (PSD) domains by doing appropriate sums, and show that they are equivalent.

e) Estimate the relative signal amplitudes of any narrow-band spectral components you observe in (c).

f) Using the convolution theorem, convolve the mystery time series with the velocity-to-velocity response of an underdamped $\omega_s = 4\pi$ Hz ($f_s = 2$ Hz) seismometer with damping $\zeta = \omega_s / \sqrt{2}$. Explain the observed changes in the PSD using the full N_3 samples of data, relative to the result of (c).