Signal Processing for the Earth Sciences

Homework 2; **Due October 11, 2023**

September 22, 2023

(1) If the frequency response of a linear system is,

$\Phi(f) = \alpha(f) + i\beta(f)$

and is Hermitian, i.e., $\Phi(f) = \Phi^*(-f)$, show that,

$\phi(t) * \sin(2\pi ft) = |\Phi(f)| \cdot \sin(2\pi ft + \theta(f))$

where,

$\theta(f) = \tan^{-1}(\frac{\beta(f)}{\alpha(f)})$

and

$|\Phi(f)| = (\alpha^2(f) + \beta^2(f))^{\frac{1}{2}}$

(2) In studying earthquake sources, it is especially desirable to have a highly accurate record of the true displacement history of the ground.

a) Write a MATLAB program that simulates and plots the first five seconds of the time-domain displacement response of three vertical seismometers with damping coefficients of $\varsigma = 0.707\omega_s$ and natural free resonant periods $T_s = 0.1$ s, 1.0 s, and 100 s, respectively, when the true displacement of the ground is that given in the 100 sample/s file *xtruedisp.ascii* (available on the class web site). Use a sampling rate of 100 samples/s throughout your time-domain calculations. Do your convolutions in the time domain.

b) Comparing the true and recorded ground displacements, what do you conclude about the ability of the three seismometers to say meaningful things about very long-period or permanent ground offsets following large earthquakes; can you suggest a frequency threshold below which we are unlikely to uncover useful information in each case?

c) Use your program to plot the time and frequency (amplitude and phase) domain response of the three seismometers to a unit delta input and explain your results.

(3) Use MATLAB to estimate the equilibrium crustal topography response beneath and near a North American Pleistocene ice sheet lobe with a width of 200 km, a circular arc profile, a maximum elevation of 2 km, and an approximate density (assumed uniform throughout) of $\rho_l = 1000$ kg/ m^3. Assume that the rigid lithosphere

is 100 km thick and is uniform with Poisson's ratio $\upsilon = 0.25$, and Young's modulus $E = 5 \times 10^{10} Pa$. Also assume that the lithosphere floats on a displaced uniform mantle with density $\rho_m = 3.3 g/cm^3$. Use your program to plot the impulse response q, the input hl, and the response w of the plate to the input. Be sure to extend the horizontal axis in your plots to at least +-500km and include units in your axes labels. Calculate Hint: the equation for a circle centered at (x_0, z_0) is $r^2 = (x - x_0)^2 + (z - z_0)^2$.

Show your work for all questions and include both your MATLAB plots and scripts.