Math 587/GEOP 505 - Homework 3

September 27, 2013

Due to Rick Aster by 5 pm on October 14. Hard copies can be handed in in class, in Rick's EES mailbox (MSEC 208), or under his office door (MSEC 356). Alternatively you can email a scan (be sure it is fully legible) to aster@ees.nmt.edu.

1) Use MATLAB to create a 1 s long time sequence that has a uniform sampling rate of 100 samples/s characterizing an 0.8 s-long boxcar pulse with an amplitude of one. Place the boxcar pulse in the center of your (otherwise zeros) sequence.

a) Plot your function using a correct (time in s) x-axis and labels.

b) Plot the corresponding discrete spectrum of of your sequence with a correct (frequency in Hz) x-axis and labels.

c) Calculate and plot the convolution of this sequence with itself using the MATLAB *conv* function. Show correct time axes consistent with (a).

d) Calculate and plot the convolution of this sequence with itself using the discrete convolution theorem. Show correct time axes consistent with (a). Why is the result different that that obtained in (c)?

e) Modify your time series so that the result calculated using the methodology of (d) is equal to that found in (c). Explain what you did and why it worked.

2) Calculate the frequency and phase response of the FIR filter corresponding to the *diff* operation in MATLAB. Compare this response to the theoretical one for exact differentiation of a continuous functions and explain the differences.

3) A filter has the analog frequency response (for a unit sampling rate) of

$$\Omega(f) \;=\; \begin{cases} 2\pi i f & (|f| \le 1/\alpha) \\ 0 & (|f| > 1/\alpha) \end{cases} \tag{1}$$

a) Obtain a formula for the infinite number of FIR filter coefficients necessary to implement this *band-limited differentiator*.

b) Write a MATLAB program that calculates the FIR weights from your analytic formula in (a).

c) Tabulate and plot the first 31 coefficients ($-15 \le n \le 15$) for $\alpha = 4$ and $\alpha = 2$.

4) a) Express the bilinear z transform for an ideal *broadband* differentiator response, $\Phi(s) = s$.

b) Obtain the corresponding difference equation in terms of y_n, y_{n-1}, x_n, and x_{n-1}.

5) Plot amplitude versus linear frequency/dB and linear phase versus linear frequency Nyquist interval ($-1/2 \le f \le 1/2$) responses for:

a) The bilinear z transform realization from problem 3 for $\alpha = 2$.

b) The 31 point FIR realization from problem 2, for the following two cases:

i) Where the FIR filter weights are simply truncated (rectangular window).

ii) Where the FIR filter weights are tapered using a Hamming window.

To facilitate easy comparisons above, use the same amplitude (dB; e.g., from -20 to 20 dB) and nonnegative frequency (linear; from 0 to 0.5) scales for all plots. Finally, overlay the ideal analog response. Label all curves appropriately.