Math 587/GEOP 505 - Homework 2

September 9, 2013

Due to Rick Aster by 5 pm on September 25. Hard copies can be handed in in class, in Rick's EES mailbox (MSEC 208), or under his office door (MSEC 356). Alternatively you can email a scan (be sure it is fully legible) to aster@ees.nmt.edu.

1) In studying earthquake sources, it is often especially desirable to have a highly accurate record of the true displacement history of the Earth at a seismographic station.

a) Write a MATLAB time-domain program that simulates and plots the first five seconds of the time-domain displacement response of three vertical seismometers with damping coefficients of $\zeta = 0.707\omega_s$ and natural free resonant periods of $T_s = 0.1$ s, 1.0 s, and 100 s, respectively, when the true displacement of the ground is that given in the 100 sample/s MATLAB data file *xtruedisp.mat* (available on the web site; the units for this time series are 10^{-3} m). Use a sampling rate of 100 samples/s throughout your time-domain calculations (the *conv* and *diff* functions in MATLAB should be used).

b) Comparing the true and instrumentally detected ground displacements (plot them on top of each other in different line styles and/or colors), what do you conclude about the ability of the three seismometers to study very-longperiod or permanent (zero frequency) ground offsets following large earthquakes; can you suggest a rule-of-thumb frequency threshold below which we may be unlikely to uncover accurate information in each case?

c) Use your program from parts (a) and (b) to plot the time domain response of the three seismometers to a displacement step input, again, comparing the true and instrumentally detected ground displacements (plot them on top of each other in different line styles and/or colors); explain the results. What would the resonant period of a seismographic instrument have to be to perfectly record a step function?

2) If the frequency response of a real-valued linear system is

$$\Phi(f) = \alpha(f) + i\beta(f) \tag{1}$$

show, using a frequency-domain derivation, followed by an inverse Fourier transform, that the convolution of a sine wave with the impulse response is given by

$$\phi(t) * \sin(2\pi f t) = |\Phi(f)| \cdot \sin(2\pi f t + \theta(f))$$
(2)

where

$$\theta(f) = \tan^{-1} \left(\frac{\beta(f)}{\alpha(f)} \right) \tag{3}$$

$$|\Phi(f)| = (\alpha^2(f) + \beta^2(f))^{\frac{1}{2}} .$$
(4)

3) Estimate power spectral density (PSD) for the 100 sample/s time series mysteryseries.mat that is posted on the class site. Label your time series and PSD plots and calculations appropriately (the units of the time series are volts).

a) Use a data segment length of $N_1 = 2^{16} = 16384$ samples (163.84 s). b) Use a data segment length of $N_2 = 2^{16} = 65536$ samples (655.36 s). c) Use a data segment length of $N_3 = 2^{18} = 262144$ samples (2621.44 s).

In each case, estimate the PSD using the *pwelch* MATLAB function with 16 subwindows (NFFT = $N_i/16$). Plot your one-sided (nonnegative frequency) PSD estimates on a decibels vs. log (base 10) frequency scale. Again, title your plots appropriately, and show the proper units for the PSD and frequency axes.

d) For (c), calculate total signal power in the time and frequency (PSD) domains by doing appropriate sums, and show that they are equivalent.

e) Estimate the relative signal power of any narrow-band spectral components that you observe in (c) by integrating your PSD estimate over appropriate ranges.

f) Evaluate and plot (c) above using the MATLAB *pmtm* multitaper PSD estimation program, the full signal length of $N_3 = 2^{18}$ samples, and a timebandwidth product of 4. Compare with results obtained using Welch's method and explain any differences/improvement/interpretations. You will need a reasonably fast computer with sufficient memory for this.

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