Math 587/GEOP 505 - Homework 1

Due to Rick Aster by 5 pm on September 4. Hard copies can be handed in in class, in Rick's EES mailbox (MSEC 208), or under his office door (MSEC 356). Alternatively you can email a scan (be sure it is fully legible) to aster@ees.nmt.edu.

1. Give analytical expressions for the following convolutions
   (a) $f(t) * (15\delta(t) - 2\delta(t-3) + \delta(t+1))$
   (b) $\Pi(t) * \Pi(t)$

2. Show that convolution of a function with the time derivative of the delta function is equivalent to time differentiation of the function. That is, $f'(t) = f(t) * \delta'(t)$. You will have to make some (reasonable) mathematical assumptions about $f(t)$ to solve this problem. Hint: one way do this problem involves integration by parts.

3. (a) Analytically evaluate the Fourier transform of
   $$\phi(t) = e^{-\alpha/\tau} H(t)$$
   where α and τ are constants.
   (b) Analytically evaluate the Fourier transform of
   $$\phi(t) = e^{\imath 2\pi f_0 t}$$
   (c) Analytically evaluate the Fourier transform of
   $$\phi(t) = \sin(2\pi f_0 t)$$

4. The spring element is found to be elastically extended by 10 cm when a force of 3 Newtons is imposed. A 2 kg mass is hung from the spring. The mass-spring system is attached to a viscous damper that produces a restraining force of 3 Newtons when the velocity of the mass is 5 m/sec. In testing this mechanical seismometer, an external force time function $f(t)$ (defined as positive upward) is applied to the mass.
   (a) Write down a differential equation model for $u(t)$, the upward displacement of the mass from its equilibrium position at time t.
   (b) Take the Fourier transform of this differential equation and solve for the transfer function, $\Phi(f)$.
   (c) Use MATLAB to plot graphs of the transfer function amplitude, $|\Phi(f)|$, and phase (in degrees), $\angle\Phi(f)$.
   (d) At what input frequency does the system respond most strongly to an input, and what is the amplitude and phase of $\Phi(f)$ there?

(e) How does this maximum response frequency compare to the natural resonant frequency of the undamped spring/mass system?

(f) Explain how you could, in principle, determine the time-domain impulse repsonse from $\Phi(f)$.