www.ees.nmt.edu/Geop/Classes/GEOP505.html

Basis functions

Linear Systems

$\Rightarrow y(t) = \phi[x(t)]$ output

superposition: $\phi[x_1(t)+x_2(t)] = \phi[x_1(t)] + \phi[x_2(t)]$

scaling: $\phi[\alpha x(t)] = \alpha\,\phi[x(t)]$

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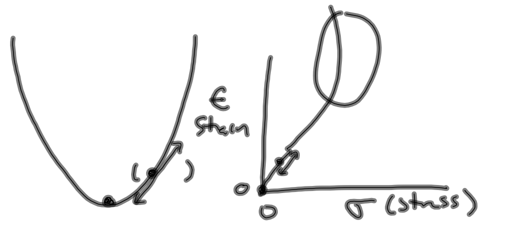

Corr to scaling relationship

$\phi[0] = 0$

$\sigma \propto \epsilon$ (linear version)

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* Time invariance

if $\phi[x(t)] = y(t)$

then $\phi[x(t-\tau)] = y(t-\tau)$

* Causality

if $x(t) = 0$ for $t<0$

then $\phi[x(t)] = y(t)$ is also $=0$ $t<0$

$\Rightarrow$ if a system is linear then the relationship between $x(t)$ and $y(t)$ is characterizable by an integral equation $\Rightarrow$ Convolution Integral.

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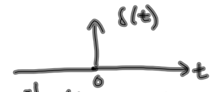
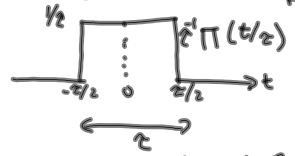
Delta function



$\delta(t) = \lim_{\tau \to 0} \tau^{-1}\Pi(t/\tau)$ Boxcar function



Sifting property of the delta function:

$$\Rightarrow \int_a^b f(t)\cdot\delta(t-t_0)\,dt = f(t_0) \quad a \le t_0 \le b$$

0 otherwise

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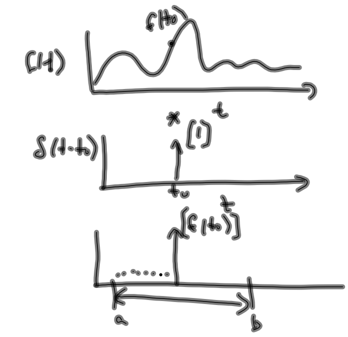

$\int_a^b f(t)\delta(t-t_0)dt = f(t_0)$

step function $H(t-t_0) = \int_{-\infty}^{t} \delta(\tau - t_0)\,d\tau$

=

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Impulse Response of a (linear) System

$\Rightarrow$ output $\phi[x(t)]$ when $x(t) = \delta(t)$

$h(t) = \phi[\delta(t)]$

$\Rightarrow$ Next: We show that output of any linear time invariant system can be written as a convolution

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$$\Rightarrow f(t) = \int_{-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\,d\tau \Leftarrow \quad [\phi[x_1+x_2] = \phi[x_1] + \phi[x_2]]$$

$$g(t) = \phi[f(t)] = \phi\left[\int_{-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\,d\tau\right]$$

$$g(t) = \phi\left[\lim_{\Delta\tau\to 0}\sum_{n=-\infty}^{\infty} f(\tau_n)\,\delta(t-\tau_n)\,\Delta\tau\right]$$

$$g(t) = \lim_{\Delta\tau\to 0}\sum_{n} \underbrace{f(\tau_n)}_{\text{weights}}\,\phi[\delta(t-\tau_n)]\,\Delta\tau$$

note: $\phi[\delta(t-\tau_n)] = h(t-\tau_n)$ when $h(t) = \phi[\delta(t)]$

$$\Rightarrow g(t) = \lim_{\Delta\tau\to 0}\sum_{n} f(\tau_n)\,h(t-\tau_n)\,\Delta\tau$$

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$$\Rightarrow g(t) = \int_{-\infty}^{\infty} f(\tau)\,h(t-\tau)\,d\tau$$ Time Domain (t)

$\boxed{\text{Convolution}}$ of $f(t)$ and $h(t)$

$$\boxed{g(t) = f(t) * h(t)}$$

$$\boxed{\int_{-\infty}^{\infty} f(\tau)\,H(t-\tau)\,d\tau} = \int_{-\infty}^{\infty} f(\tau)\int_{-\infty}^{t} \delta(\xi-\tau)\,d\xi\,d\tau$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{t} f(\tau)\,\delta(\xi-\tau)\,d\xi\,d\tau$$

Reverse Order of integration $= \int_{-\infty}^{t}\int_{-\infty}^{\infty} f(\tau)\,\delta(\xi-\tau)\,d\tau\,d\xi = \int_{-\infty}^{t} f(\tau)\,d\tau$

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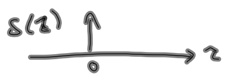
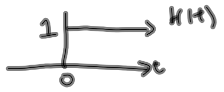
So if your linear operation is

$$\int_{-\infty}^{t} f(\tau)\,d\tau$$ what is the "impulse response"?

let $f(t) = \delta(t)$

$$\Rightarrow \int_{-\infty}^{t} \delta(\tau)\,d\tau = H(t)$$





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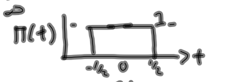
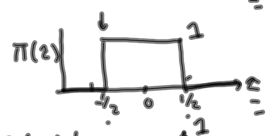
Consider

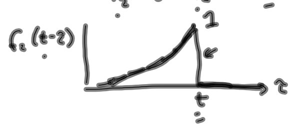
$$c(t) = f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)\,f_2(t-\tau)\,d\tau$$ integrate over τ, t varies

$f_1(t) = \Pi(t)$

$f_2(t) = e^{-t}H(t)$







• case (a) $t \le -1/2$

$c(t) = 0$

• case (b) $-1/2 \le t \le 1/2$

$$c(t) = \int_{-1/2}^{t} (1)\cdot e^{-(t-\tau)}\,d\tau$$

$$c(t) = 1 - e^{-(t+1/2)}$$

• case (c) $t > 1/2$

$$\int_{-1/2}^{1/2} (1)\,e^{-(t-\tau)}\,d\tau = e^{-(t-1/2)} - e^{-(t+1/2)}$$

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Correlation (vs. Convolution)

Corr $z(t) = f_1(t) \star f_2(t)$ — $f_1(t) = f_2(t)$ ⇒ autocorrelation

$$\boxed{= \int_{-\infty}^{\infty} f_1(\tau)\,f_2(t+\tau)\,d\tau}$$

Conv

$$c(t) = f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)\,f_2(t-\tau)\,d\tau$$

zero lag, for $f_1(t) \star f_1(t)$

$$z(0) = \int_{-\infty}^{\infty} f_1(\tau)\,f_1(\tau)\,d\tau = \int_{-\infty}^{\infty} f_1^2(\tau)\,d\tau$$

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Normalized Correlation

$$G(t) = \frac{\int_{-\infty}^{\infty} f_1(\tau)\,f_2(t+\tau)\,d\tau}{\sqrt{\int_{-\infty}^{\infty} f_1^2(\tau)\,d\tau \cdot \int_{-\infty}^{\infty} f_2^2(\tau)\,d\tau}}$$

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