Inverse Methods in Geophysics
CERI 7260/8260; CVL 7128/8128
Spring 2018
Problem Set #2
Due Wednesday 14 February 2018

1. (Aster et al Problem 1.3)

Find a journal article that discusses the solution of an inverse problem in a discipline of special interest to you. What are the data? Are the data discrete or continuous? Have the authors discussed possible sources of noise in the data? What is the model? Is the model continuous or discrete? What physical laws determine the forward operator **G**? Is **G** linear or nonlinear? Do the authors discuss any issues associated with existence, uniqueness, or instability of solutions?

(These will be discussed in class, as well.)

2. Let

$$A = \begin{bmatrix} -4 & 6 & 3 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & -1 & 0 \\ 3 & 1 & 0 \end{bmatrix}.$$

Calculate

a) $\mathbf{A} + \mathbf{B}$

b) $\mathbf{A} - \mathbf{B}$

c) $1.5\,\mathbf{A}$

d) $\mathbf{A}^{\mathrm{T}}$

e) AB^{T}

f) $(\mathbf{AB}^{\mathrm{T}})^{\mathrm{T}}$

3. Let

$$\mathbf{x} = \mathbf{A}\,\mathbf{y}$$
$$\mathbf{y} = \mathbf{B}\,\mathbf{z}$$

and

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

find **C**, where

$$\mathbf{x} = \mathbf{C}\,\mathbf{z}\ .$$

4. Let

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 4 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 2 & 1 \\ 6 & 4 & 2 \end{bmatrix}.$$

Determine if the following is possible. If so, then calculate the result by hand and Matlab. Show all your work.

a) **AB**

b) **BA**

c) $\mathbf{A}^{\mathrm{T}}\mathbf{B}$

d) $\mathbf{A}\mathbf{B}^{\mathrm{T}}$

5. This is an exercise with Matlab. Hand in copies of your Matlab scripts in addition to the requested answers and plots.

Consider the following system of equations.

$$\begin{aligned} x + y + z &= 5 \\ x - 2y + 5z &= 3 \\ 3x + y - 3z &= 2 \end{aligned}$$

a) Solve this set of equations using the “rref” Matlab function.

b) Plot a 3D view of these equations. Plot the solution as a point on the 3D view.

c) Show that the columns of the coefficient matrix of this set of equations are linearly independent vectors.

d) Replace the third equation by 2x + 2y + 2z = 10. Are the columns of this coefficient matrix linearly independent vectors? What are the solutions to this new set of equations? Plot them in 3D. How many are there?