Intermediate Seismology
ESCI 7402
Homework #2
Due: February 27, 2012
Seismic Gradiometry/Anisotropy

1. In one dimension, find the relationship between the wave field gradient, wave field and time derivative of the wave field for the following cases:

    a. A plane wave

$$u(t,x) = a_0 f(t - p_0(x - x_0))$$

    b. A refracted body wave

$$u(t,x) = a_0 x^{-n} f(t - p(x)(x - x_0))$$

    c. A surface wave

$$u(t,x) = a_0 x^{-\frac{1}{2}} f(t - p(x - x_0))$$

2. Fill in the details for equations (20) through (32) in the "Wave Gradiometry in the Time Domain" paper, including finding the time derivative of the instantaneous phase to obtain equation (27).

3. An anisotropic elastic material with hexagonal symmetry (6-fold rotational symmetry about the x_3 axis) is said to be transversely isotropic. The modified elastic coefficient matrix is given by

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

where,

$$C_{66} = \frac{C_{11} - C_{12}}{2} .$$

The Christoffel Equation for a plane wave propagating in the $\hat{\eta} = (\eta_1, \eta_2, \eta_3)$ direction is

$$\left(\Lambda_{ik} - \rho v^2 \delta_{ik}\right)U_k = 0$$

where,

$$\Lambda_{11} = C_{11}\eta_1^2 + C_{66}\eta_2^2 + C_{44}\eta_3^2$$
$$\Lambda_{22} = C_{66}\eta_1^2 + C_{11}\eta_2^2 + C_{44}\eta_3^2$$
$$\Lambda_{33} = C_{44}\eta_1^2 + C_{44}\eta_2^2 + C_{33}\eta_3^2$$
$$\Lambda_{12} = \left(C_{11} - C_{66}\right)\eta_1\eta_2$$
$$\Lambda_{13} = \left(C_{13} + C_{44}\right)\eta_1\eta_3$$
$$\Lambda_{23} = \left(C_{13} + C_{44}\right)\eta_2\eta_3$$

Find the velocities for the quasi P wave and 2 quasi S waves in terms of the elastic constants and density for the following wave directions. Also identify which are the quasi SV and SH wave velocities.

a) A plane wave traveling in $\hat{\eta} = (1,0,0)$.
b) A plane wave traveling in $\hat{\eta} = (0,1,0)$.
c) A plane wave traveling in $\hat{\eta} = (0,0,1)$.