Intermediate Seismology
ESCI 7402
Homework #1
Seismic Arrays and Beamforming

1. Derive the theoretical frequency/wavenumber response for a linear array of length, *D*, as a function of frequency. Assume a continuous number of observation points between 0 and *D*. Plot the array beam as a function of slowness for various frequencies.

   Method:
   Assume an impulsive plane wave

   $$u(x,t) = \delta(t - p_0 x) \quad .$$

   Fourier transform from time to circular frequency

   $$\hat{u}(x,\omega) = \int_{-\infty}^{+\infty} u(x,t) e^{-i\omega t} dt \quad .$$

   Fourier transform from space to horizontal wavenumber

   $$\hat{\hat{u}}(k,\omega) = \int_{-\infty}^{+\infty} \hat{u}(x,\omega) e^{+ikx} dx \quad .$$

   Derive the power spectrum

   $$\left|\hat{\hat{u}}(k,\omega)\right|^2 \quad .$$

   Make sensible plots of power vs wave slowness, *p*, for different frequencies assuming $p_0 = 0.1\ s/km$ and *D=1 km*. Remember that $k = \omega p$. For what frequency can p_0 be resolved to 10%? Estimate the error using the approximate halfwidth of the power spectrum. Make sure you find the maximum of the power spectrum to make sense of your result.

2. I have a set of MatLab programs that compute array responses for various array geometries. These routines can be found on the Mac system at

/pod0/clangstn/matlab/array

and the main program is called array_driver.m .

Design a regional array to resolve ambient ground motions to 10% in the frequency band 0.1 – 0.3 Hz that propagate at velocities of 3 km/s for arrivals at any azimuth. Use an existing array geometry to do the problem and then create one of your own.

Show your reasoning and assumptions in the array design and produce plots of the array geometry, co-array, and frequency/wavenumber response for such a wave that propagates from an azimuth of 135 degrees.