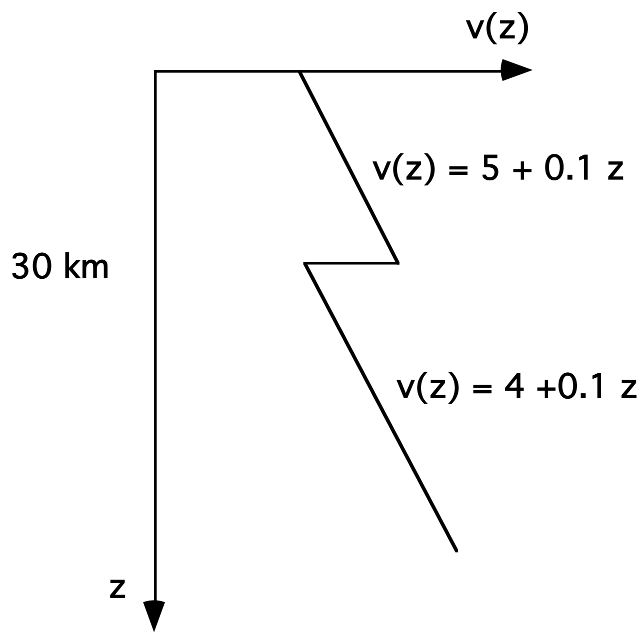
Global Seismology CERI 7105/8105
Homework Set #5
Due Monday, October 22, 2018
Ray Theory Travel-time Computations and Inversion

1. (7105/8105 students) (30)Assume the following velocity profile, velocities are in km/sec:



Do the following for

- the direct turning ray above 30 km depth,
- the direct turning ray in the structure below 30 km depth,
- and the reflected ray from the interface.

a. Derive the travel time and distance formulas as a function of incidence angle at the surface.

b. Write a computer program to evaluate the integrals to create a plot of the traveltime - distance curves to a distance of 400 km. Also plot the travel times using a reducing velocity of 7 km/s.

c. Accurately sketch out the paths of important rays in this structure. (You do not have to make a ray-tracing program – just sketch the important rays, where they bottom, and explain the *T-x*, and *p-x* behavior.)

2. (8105 students, extra credit for 7105 students) (20) Ray bottoming depth is given by the Herglotz-Weichert integral:

$$Z_1 = \frac{1}{\pi}\int_0^{x_1} \cosh^{-1}\left(\frac{p(x)}{p(x_1)}\right)dx$$

where,

$$p(x) = \frac{dT}{dx}$$

$$\cosh^{-1} y = \ln\left(y + \sqrt{y^2 - 1}\right) .$$

Numerically, *p(x)* can be found be differencing adjacent points on the travel time curve. For example, at x_i and T_i

$$p(x_i) = \frac{T_i - T_{i+1}}{x_i - x_{i+1}} .$$

The integral over x can be approximated by the trapezoidal rule:

$$Z_N = \frac{1}{\pi}\sum_{j=1}^{N-1}\left(\cosh^{-1}\left[\frac{p(x_j)}{p(x_N)}\right] + \cosh^{-1}\left[\frac{p(x_{j+1})}{p(x_N)}\right]\right)\left(\frac{x_{j+1} - x_j}{2}\right)$$

Using your results from problem 1, calculate a "synthetic" data set of travel times and distances for the turning ray in the upper layer of the structure shown above. Using this data set, calculate the ray parameter - distance relation and then numerically evaluate the Herglotz-Weichert integral to determine bottoming depth for the ray at each distance. Compare your inverted velocity values with the actual values. Try different sampling intervals in x and evaluate the accuracy of your answer.