ESCI 7105/8105
Global Seismology
Homework Set #4
Time Domain/Frequency Domain
Due October 3, 2018

We will looking at time domain/frequency domain concepts using a number of simple MatLab modules for the Discrete Fourier series. These can be found at

/clangstn/matlab/mcs

on the Mac system and include

```
[f]=ricker(T,npts,fm)
[f]=trapezoid(T,npts,dt1,dt2,dt3)
[alpn,wn,n]=fcoeff(f,T,npts,m)
plotcoef(alpn,wn,T,n)
[fs]=fseries(f,alpn,wn,T,npts,n,m)
[a]=convolve(f,g,T,npts)
```

A description of input and output parameters can be found by looking at the source files directly or by typing, for example,

help ricker

while in the MatLab command window.

1. (10) Examine the spectral characteristics of a Ricker wavelet.

The Ricker wavelet is given mathematically by:

$$f(t) = \left(1 - 2\pi^2 f_m^2 t^2\right) e^{-(\pi f_m t)^2}$$

Assume the following parameters:

T=1 s
npts=100
f_m=5 Hz

Construct the Ricker wavelet using "ricker". Compute the complex Fourier series coefficient using n=20 and the module "fcoef". Plot the coefficients using "plotcoef".

a) What is the behavior of the real and imaginary parts of the coefficients?
b) What is the behavior of the amplitude spectrum? Does the observed peak frequency agree with the assumed value of f_m? What frequency components does the input signal primarily have?

c) What is the behavior of the phase spectrum? Are these values consistent with the assertion that the function is a "zero phase" function?

d) Now reconstruct the signal using the 20 coefficient pairs calculated above using "fseries". Use m=20 to completely reconstruct the signal. Does it work?

e) Progressively use m=1,2, ...20 to observe how the signal is built up by each sinusoid. For these partial reconstructions, does the frequency content of the reconstruction differ from the original signal? Can you make sense of the changing frequency content from the frequency components being summed into the reconstruction? Illustrate your discussion.

2. (10) Construct a trapezoidal time function using "trapezoid" and the following parameters:

T=5s
npts=100
dt1=0.3
dt2=0.5
dt3=0.8

This is an example of a "broadband" pulse.

Repeat the questions in (1) above for the trapezoidal time function. In addition, does the value at zero frequency agree with the time integral of the trapezoid?

3. (10) Convolution

Using the parameters for the Ricker wavelet, $f(t)$, in question (1) and trapezoidal function, $g(t)$, in question (2), perform the time domain convolution of $f(t)*g(t)$ using the module "convolve". Two graphics windows will appear. One will contain a lag-by-lag display of the operations in the convolution process. This window will show $f(t)$ and a flipped and time-lagged $g(t)$. The red asterisks will by the product of $f(\tau)g(t-\tau)$ which is then integrated to get the value of the convolution. The other window contains the updated convolution result. Press the "return" button on the keyboard for each time lag. Keep pressing until you get the MatLab prompt.

a) Observe the convolution process in action. Describe, in words, what is going on mathematically. Make plots of some intermediate results.

b) Why does the resulting convolution time series have twice as many points as the original two time series?

c) Now perform the convolution of $g(t)*f(t)$ (i.e, the other way around). Is it the same as $f(t)*g(t)$? Why?

d) Perform the convolution of $f(t)*g(t)$ by multiplying their two complex spectra together and reconstructing the time series from the spectra using "fseries". Compare the amplitude spectrum of the product and two original spectra. Does the resulting amplitude spectrum make sense in relation to the individual spectra of *f* and *g*? Compare the reconstructed time series to the time domain method result.

4. (10) The following differential equation relates pendulum displacement, *W*, to ground motion displacement, *u*:

$$\frac{\partial^2 W}{\partial t^2} + 2\xi\omega_0 \frac{\partial W}{\partial t} + \omega_0^2 W = -\frac{\partial^2 u}{\partial t^2}$$

where, ξ is the damping coefficient and ω_0 is the resonance frequency.

Transform this equation using the Fourier transform and solve for the pendulum displacement spectra. Plot the amplitude and phase spectra assuming $\xi = 1$ and $\omega_0 = 2\pi$. Assume that the ground displacement is a Dirac delta function. Your amplitude and phase spectra give the instrument <u>impulse response</u>. You can also think of this response as a <u>filter</u> acting on ground displacement.

5. (10) The Fourier Transform integrals are given by:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)\, e^{+i\omega t}\, d\omega$$

$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-i\omega t}\, d\omega$$

Find the Fourier transforms of the following time functions.

a. $f(t) = e^{-\gamma t}$ for $t \geq 0$ ($f(t)=0$ otherwise)

b.

$$B(t) = \begin{cases} 0 & \text{for } t<0 \\ 1 & \text{for } 0 \leq t \leq 1 \\ 0 & \text{for } t > 1 \end{cases}$$

(This is a "boxcar" function.)

c.

$$f(t) = \begin{cases} 0 & ,t<0 \\ \dfrac{t}{t_1} & ,0 \le t \le t_1 \\ 1 & ,\ t_1 \le t \le t_2 \\ \dfrac{t - t_3}{t_2 - t_3} & ,\ t_2 \le t \le t_3 \\ 0 & ,t > t_3 \end{cases}$$

(This is a trapezoidal function.)

6. (10) Evaluate your result from 5(c) and compare it to the Discrete Fourier transform result from question (2). What is needed to make the amplitude spectra agree?

7. (10) A synthetic reflection response is given by the time series

$$R(t) = \delta(t-1) - 0.7\ \delta(t-2.5) + 0.5\ \delta(t-4)$$

Given the following source function (note the limits of the time function!)

$$f(t) = \begin{cases} \sin(2\pi t) & \text{for } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

sketch (no MatLab!) the ground displacement given by

$$u(t) = f(t) * R(t)$$

8. (10) Wave Gradiometry

The displacement, *u(t,x)*, of a simple refracted body wave or surface wave propagating in a medium with slowly varying physical properties can be represented by

$$u(t,x) = G(x)\, f\left(t - p\left(x - x_0\right)\right) \qquad (1)$$

where, *G(x)* is the geometrical spreading, *p* the horizontal wave slowness, and x_0 a reference position. Here *p* is assumed to be varying with distance *x*. This form of wave displacement naturally models individual body wave rays in vertically inhomogeneous media and surface waves in media where the material property changes occur over scales greater than a wavelength. Differentiating equation (1) with respect to x gives

$$\frac{\partial u}{\partial x} = A(x)u + B(x)\frac{\partial u}{\partial t} \qquad (2)$$

where,

$$A(x) = \frac{G'(x)}{G(x)} \qquad (3)$$

and

$$B(x) = \frac{\partial}{\partial x}\left[-p(x-x_0)\right] = -\left[p + \frac{\partial p}{\partial x}(x-x_0)\right]. \qquad (4)$$

Fourier transform equation (2) and solve for $A(x)$ and $B(x)$ given u and $\frac{\partial u}{\partial x}$.