CERI7105/8105
Global Seismology
Homework Set #3
Elasticity and the Wave Equation
Due September 26, 2018

1. Typical crustal P and S wave velocities are 6.0 and 3.5 km/s. If the density is 2.7 gm/cc for crustal material, what are the values of Lame's constants, λ and μ, in Newton/m^2? What is the value of shear stress in the crust, τ_{23}, if a seismic shear strain, ε_{23}, is 10^{-5}?

2. Poisson's ratio is defined as

$$\sigma = \frac{-\varepsilon_{22}}{\varepsilon_{11}}$$

Using the stress-strain relationship for isotropic elastic material

$$\tau_{ij} = \lambda\, \varepsilon_{kk}\, \delta_{ij} + 2\mu\, \varepsilon_{ij}$$

derive Poisson's ratio in terms of Lame's constants λ and μ. Assume that τ_{11} is positive and that $\tau_{22}=\tau_{33}=0$.

3. Given the Helmholtz decomposition for displacement in terms of P and S wave potentials

$$\mathbf{u} = \nabla\phi + \nabla\times\mathbf{A}$$

and the vector wave equation

$$\rho\frac{\partial^2\vec{u}}{\partial t^2} = (\lambda+2\mu)\nabla(\nabla\bullet\vec{u}) - \mu(\nabla\times\nabla\times\vec{u})$$

show that it is <u>sufficient</u> that solutions for wave equations for the scalar P-wave potential, φ, and the vector S-wave potential, $\boldsymbol{A}$,

$$\frac{1}{\alpha^2}\frac{\partial^2\phi}{\partial t^2} = \nabla^2\phi$$

$$\frac{1}{\beta^2}\frac{\partial^2\vec{A}}{\partial t^2} = \nabla^2\vec{A}$$

are solutions for the vector wave equation. These equations will form the basis for much of our future work on plane waves, and much later, waves from point sources in plane-layered media. (Hint: Plug in the Helmholtz decomposition for **u** into the vector wave equation and group terms into divergence parts and curl parts for the P and S wave potentials, respectively. Create two equations by arbitrarily breaking it up into P and S wave equations. Further reduce these two equations into the simple wave equations above and wave your arms briskly by stating that solutions to the simple wave equations are solutions to the more complicated equations.)

4. Assume that a wave propagates in the x_1 - x_3 plane and has vector displacement in the x_2 direction only, e.g., the functional dependence of displacement is $u_2(x_1,x_3,t)$ with no motion u_1 or u_3. Directly show from the vector wave equation that such a wave is governed by its own wave equation:

$$\frac{1}{\beta^2}\frac{\partial^2 u_2}{\partial t^2} = \nabla^2 u_2$$

   We will use this result later to study the propagation of SH (horizontally polarized) shear waves.

5. "Synthetic Seismogram" is a term used for a theoretical computation of ground motion versus time from a wave propagation theory result. Compute synthetic seismograms for the <u>spherical</u> wave solution for the outgoing wave, where

$$\phi = \frac{1}{r} g\left(t - \frac{r}{\alpha}\right)$$

   and

$$\vec{u}_p = \nabla \phi \quad .$$

   Assume a Gaussian time history for g where

$$g\left(t - \frac{r}{\alpha}\right) = C_0 e^{-\gamma\left(t - \frac{r}{\alpha}\right)^2}$$

$$C_0 = 10^3 m^3$$

   Compute synthetic seismograms for radial displacement at r = 5, 10, and 20km, for γ = 7.5/s^2 and α = 5 km/s. Also assume a time series 10 seconds long and a sampling interval of 0.01 seconds. Displacement units should be in meters.